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GENERALIZED PLANE WAVES IN A ROTATING THERMOELASTIC DOUBLE POROUS SOLID

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The propagation of plane waves in a rotating homogeneous, isotropic, thermoelastic solid with double porosity following Lord-Shulman's theory of thermoelasticity has been investigated. It is assumed that the medium rotates about an axis normal to the surface with a uniform angular velocity. There may exist five coupled waves that evolved due to the longitudinal, transverse disturbance, voids of type-I and type-II, and temperature change in the medium. The secular equation for the model under consideration has been derived with the help of formal solutions and boundary conditions. The amplitude of displacements, temperature change and volume fraction fields for voids of type-I and type-II have also been computed analytically. Finally, numerical computations have been carried out for magnesium crystal material to understand the behavior of amplitude of phase velocity, penetration depth, specific loss, displacement components, temperature change, and volume fraction field due to type-I and type-II voids corresponding to the different rotation rates. Various graphs have been plotted to support the analytical findings. The study may be used in the development of rotation sensors, material design and thermal efficiency.

Key words: voids of type-I and type-II, rotation, plane waves; homogeneous isotropic, generalized thermoelasticity.

1. Introduction

Almost every substance on our planet is porous, and the thermoelastic effect is present in almost all of them; as a result, when a medium is heated, the voids present in the medium expand and contract when the medium is cooled. He fact that particles try to migrate back to their original form plays a vital role in contraction and expansion. The study of the propagation of waves through multiple porous media influenced by the presence of voids and temperature in the medium is the backbone in many areas of the petroleum industry, geophysics, and chemical engineering. The theory of thermoelasticity was formulated by Lord and Shulman (LS) [1] who came up with a wave-type heat equation that includes a time derivative and a heat flow vector. By adding the existence of pores in an elastic continuum, Nunziato and Cowin [2] established the nonlinear theory of elastic materials with voids, which allocated an extra degree of freedom to each material particle. Wilson and Aifanits [3] investigated the idea of dual-porosity consolidation and concluded that if one kind of a pore space is reduced to zero, the outcomes are similar to the traditional idea of single porosity consolidation. Iesan [4] developed some theorems about the uniqueness of solution, reciprocity connection, and the variational characterization of solution in a linear theory of thermoelastic materials with voids. A heat-flux dependent theory in which a new set of independent variables is added, including the heat flow vector, has been proposed by Dhaliwal and Wang [5].

A mechanical model for a hydrothermoelastic medium with double porosity has been analyzed by Khalili and Selvadurai [6], in which the governing equations are obtained using a systematic macroscopic approach that adheres to the necessary conservation. Sharma and Pathania [7, 8] demonstrated the propagation

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of waves on a homogeneous, thermally conducting isotropic plate bordered on both sides by layers or half-spaces of an inviscid liquid using extended theories of thermoelasticity. The impact of rotation on temperature profiles and displacement of a micro-polar thermoelastic half-space under five theories has been examined by Othman and Singh [9]. Sharma and Grover [10] analyzed the propagation of body waves in a homogeneous isotropic rotating generalized thermoelastic solid. The effect of rotation, voids and thermal relaxation time on the propagation of Rayleigh waves in a rotating thermoelastic half-space with voids was investigated by Sharma and Kaur [11]. Othman and Abbas [12] observed the effect of rotation on the plane waves in a rotating thermoelastic half-space using the LS theory of thermoelasticity. Darcy's law to establish the uniqueness of the solution by extending the Nunziato–Cowin theory to doubly porous materials was used by Iesan and Quantinilla [13]. Kumar *et al.* [14] studied the boundary value problem for the thermoelastic solid with double porosity using the state space approach. The effect of rotation on the surface wave propagation in magneto-thermoelastic materials with voids was studied by Farhan and Alla [15], who found that the Coriolis force is the cause of damping thermoelastic voids. Barak and Kaliraman [16, 17] examined the imperfect interface between fluid-saturated porous solid half space and micropolar elastic solid half-space for the reflection and transmission of elastic waves.

The propagation of plane harmonic waves in partly saturated soils was studied using Christoffel equations by Barak *et al.* [18]. Reflection of plane waves from the free surface of a rotating orthotropic magneto-thermoelastic solid half-space with diffusion, fractional-order initially stressed, was studied by Yadav [19, 20, 21, 22, 23, and 24]. Barak and Dhankhar [25] evaluated the effect of inclined load on a functionally graded fiber-reinforced thermoelastic medium with temperature-dependent properties, and the effect of Lamb-type waves in a porothermoelastic plate immersed in the inviscid fluid was analyzed by Pathania and co-researchers [26, 27]. Kumari *et al.* [28, 29] studied the distinctive characteristics of reflection coefficients and energy sharing at both open-pore and sealed-pore border surfaces and measured the horizontal and vertical motion at the surface of partially saturated soils. The aim of the present study is to determine the effect of angular velocity, porosity, and thermal variation on the propagation of waves in a rotating homogeneous isotropic thermoelastic solid medium with double porosity for the stress-free boundaries as depicted graphically in Figs [2-10].

2. Formulation of the problem

Let us consider a space which is initially at a uniform temperature T_0 and rotating uniformly with an angular velocity $\vec{\Omega} = (0, \Omega, 0)$. The surface $z = 0$ is considered thermally insulated, isothermal, stress-free, and has no fractional volume change on the boundary surface (Fig.1).

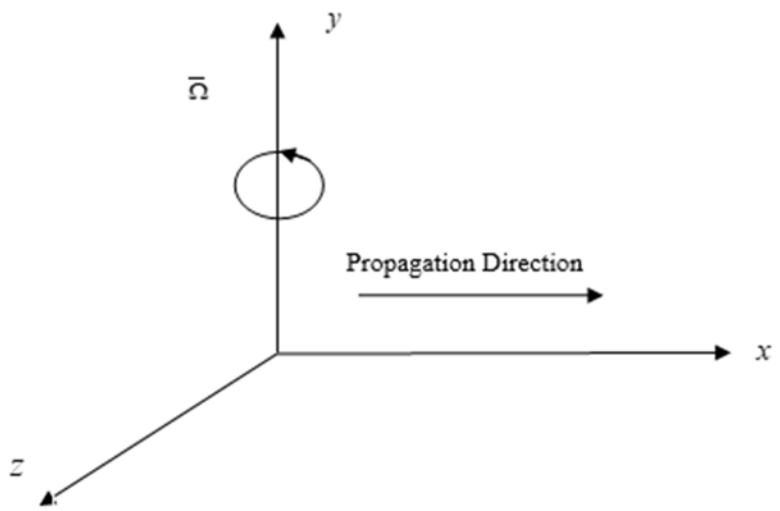

Fig.1. Geometry of the problem.

The $x-$axis is chosen in the direction of wave propagation so that all the field quantities are independent of the $y-$axis. The constitutive equations include two additional terms, the part of centripetal force $\vec{\Omega}\times(\vec{\Omega}\times\vec{u})$ (depends on time) and the Coriolis acceleration $2(\vec{\Omega}\times\dot{\vec{u}})$ due to rotation, where $\vec{u}=(u,0,w)$ is the displacement vector. The disturbance must be confined in the neighborhood of the free surface $z=0$, and it fades away as $z\to\infty$.

3. Constitutive relations

The constitutive relations established by Lord and Shulman [1] for a rotating, isotropic homogeneous double porous thermoelastic solid in the absence of body forces, equilibrated forces, and heat sources in the $xz-$plane considered by Iesan and Quantinilla [13], Sharma and Kaur [11] are as follow:

$$\tau_{ij}=\lambda\delta_{ij}e_{kk}+2\mu e_{ij}+b\delta_{ij}\varphi+d\delta_{ij}\psi-\beta\delta_{ij}T, \tag{3.1}$$

$$\sigma_i^1=\alpha\varphi_{,i}+b_1\psi_{,i}\,, \tag{3.2}$$

$$\sigma_i^2=b_1\varphi_{,i}+\gamma\psi_{,i}\,, \tag{3.3}$$

$$\xi=-be_{jj}-\alpha_1\varphi-\alpha_3\psi+\gamma_1 T\,, \tag{3.4}$$

$$\zeta=-de_{jj}-\alpha_3\varphi-\alpha_2\psi+\gamma_2 T\,, \tag{3.5}$$

$$\rho\eta=\beta e_{jj}+\gamma_1\varphi+\gamma_2\psi+aT\,. \tag{3.6}$$

The Fourier's law of heat conduction

$$Q_i+t_0\dot{Q}_i=KT_{,i}\,. \tag{3.7}$$

The equations of motion

$$\tau_{ij,j}=\rho\left(\ddot{u}_i+\Omega_j u_j\Omega_i-\Omega^2 u_i+2\in_{ijk}\Omega_j\dot{u}_k\right). \tag{3.8}$$

The equilibrated stress equations of motion

$$\sigma_{j,j}^1+\xi=\chi_1\ddot{\varphi}\,, \tag{3.9}$$

$$\sigma_{j,j}^2+\zeta=\chi_2\ddot{\psi}\,. \tag{3.10}$$

The energy equation

$$\rho T_0\left(\dot{\eta}+t_0\ddot{\eta}\right)=Q_{j,j}\,. \tag{3.11}$$

Using the above constitutive relations in the two-dimensional form, we have

$$(\lambda+2\mu)u_{,xx}+\lambda w_{,xz}+\mu\left(u_{,zz}+w_{,xz}\right)+b\varphi_{,x}+d\psi_{,x}-\beta T_{,x}=\rho\left(\ddot{u}-\Omega^2 u+2\Omega\dot{w}\right), \tag{3.12}$$

$$(\lambda+2\mu)w_{,zz}+\mu w_{,xx}+(\lambda+\mu)u_{,xz}+b\varphi_{,z}+d\psi_{,z}-\beta T_{,z}=\rho\left(\ddot{w}-\Omega^2 w-2\Omega\dot{u}\right), \tag{3.13}$$

$$\alpha\left(\varphi_{,xx}+\varphi_{,zz}\right)+b_1\left(\psi_{,xx}+\psi_{,zz}\right)-b\left(u_{,x}+w_{,z}\right)-\alpha_1\varphi-\alpha_3\psi+\gamma_1 T=\chi_1\ddot{\varphi}, \tag{3.14}$$

$$b_1\left(\varphi_{,xx}+\varphi_{,zz}\right)+\gamma\left(\psi_{,xx}+\psi_{,zz}\right)-d\left(u_{,x}+w_{,z}\right)-\alpha_3\varphi-\alpha_2\psi+\gamma_2 T=\chi_2\ddot{\psi}, \tag{3.15}$$

$$\begin{aligned}&K\left(T_{,xx}+T_{,zz}\right)-\beta T_0\left(\dot{u}_{,x}+\dot{w}_{,z}+t_0\left(\ddot{u}_{,x}+\ddot{w}_{,z}\right)\right)+\\&-\gamma_1 T_0\left(\dot{\varphi}+t_0\ddot{\varphi}\right)-\gamma_2 T_0\left(\dot{\psi}+t_0\ddot{\psi}\right)=\rho C_e\left(\dot{T}+t_0\ddot{T}\right).\end{aligned} \tag{3.16}$$

Consider the dimensionless parameters

$$\begin{aligned}&x'=\frac{\omega^* x}{c_1},\quad z'=\frac{\omega^* z}{c_1},\quad u'=\frac{\rho\omega^* c_1 u}{\beta T_0},\quad w'=\frac{\rho\omega^* c_1 w}{\beta T_0},\quad t'=\omega^* t,\quad t'_0=\omega^* t_0,\\&T'=\frac{T}{T_0},\quad \varphi'=\frac{\omega^{*2}\chi_1\varphi}{c_1^2},\quad \psi'=\frac{\omega^{*2}\chi_1\psi}{c_1^2},\quad \Omega'=\frac{\Omega}{\omega^*},\quad \omega'=\frac{\omega}{\omega^*},\quad c'=\frac{c}{c_1}.\end{aligned} \tag{3.17}$$

where, $\beta=(3\lambda+2\mu)\alpha_t$, $\rho C_e=aT_0$, Q_i denotes the heat flux, η is the entropy per unit mass, $\in_{ijk}$ is the Levi-Civita term, a is the thermal capacity of the material, c_1 and ω^* represent the compressional wave velocity and characteristic frequency. Here dot '·' represents the differentiation with respect to time 't' and ',' indicates the partial differential with respect to space coordinates.

Putting the non-dimensional quantities (3.17) in Eqs (3.12)-(3.16) and omitting the primes, the non-dimensional version of the governing equations have been obtained as:

$$u_{,xx}+\delta^2 u_{,zz}+\left(1-\delta^2\right)w_{,xz}+a_1\varphi_{,x}+a_2\psi_{,x}-T_{,x}=\ddot{u}-\Omega^2 u+2\Omega\dot{w}, \tag{3.18}$$

$$\left(1-\delta^2\right)u_{,xz}+\delta^2 w_{,xx}+w_{,zz}+a_1\varphi_{,z}+a_2\psi_{,z}-T_{,z}=\ddot{w}-\Omega^2 w-2\Omega\dot{u}, \tag{3.19}$$

$$\varphi_{,xx}+\varphi_{,zz}+a_3\left(\psi_{,xx}+\psi_{,zz}\right)-a_4\left(u_{,x}+w_{,z}\right)-a_5\varphi-a_6\psi+a_7 T=\frac{1}{\delta_1^2}\ddot{\varphi}, \tag{3.20}$$

$$\varphi_{,xx}+\varphi_{,zz}+a_8\left(\psi_{,xx}+\psi_{,zz}\right)-a_9\left(u_{,x}+w_{,z}\right)-a_{10}\varphi-a_{11}\psi+a_{12} T=\frac{1}{\delta_2^2}\ddot{\psi}, \tag{3.21}$$

$$T_{,xx}+T_{,zz}-\left(\dot{T}+t_0\ddot{T}\right)=\varepsilon_T\left(\dot{u}_{,x}+\dot{w}_{,z}+t_0\left(\ddot{u}_{,x}+\ddot{w}_{,z}\right)\right)+a_{13}\left(\dot{\varphi}+t_0\ddot{\varphi}\right)+a_{14}\left(\dot{\psi}+t_0\ddot{\psi}\right) \tag{3.22}$$

where

$$a_1=\frac{bc_1^2}{\omega^{*2}\chi_1\beta T_0},\quad a_2=\frac{dc_1^2}{\omega^{*2}\chi_1\beta T_0},\quad a_3=\frac{b_1}{\alpha},\quad a_4=\frac{\beta T_0\chi_1 b}{\rho c_1^2\alpha},\quad a_5=\frac{\alpha_1 c_1^2}{\alpha\omega^{*2}},$$

$$a_6=\frac{\alpha_3 c_1^2}{\alpha\omega^{*2}},\qquad a_7=\frac{\gamma_1 T_0\chi_1}{\alpha},\qquad a_8=\frac{\gamma}{b_1},\qquad a_9=\frac{\beta T_0\chi_1 d}{\rho c_1^2 b_1},\qquad a_{10}=\frac{\alpha_3 c_1^2}{\omega^{*2} b_1},$$

$$a_{11}=\frac{\alpha_2 c_1^2}{\omega^{*2} b_1},\qquad a_{12}=\frac{\gamma_2 T_0\chi_1}{b_1},\qquad a_{13}=\frac{\gamma_1 c_1^4}{K\omega^{*3}\chi_1},\qquad a_{14}=\frac{\gamma_2 c_1^4}{K\omega^{*3}\chi_1},$$

$$\varepsilon_T=\frac{\beta^2 T_0}{\rho C_e(\lambda+2\mu)},\qquad c_1^2=\frac{\lambda+2\mu}{\rho},\qquad c_2^2=\frac{\mu}{\rho},\qquad c_3^2=\frac{\alpha}{\chi_1},\qquad c_4^2=\frac{b_1}{\chi_2},$$

$$\omega^*=\frac{C_e\rho c_1^2}{K},\qquad \delta^2=\frac{c_2^2}{c_1^2},\qquad \delta_1^2=\frac{c_3^2}{c_1^2},\qquad \delta_2^2=\frac{c_4^2}{c_1^2}.$$

Here ε_T is the thermo-mechanical coupling parameter, c_2, c_3, and c_4 are the velocities of the transverse wave, change in type-I voids, and change in type-II voids, respectively. The displacement components using the Helmholtz decomposition theorem may be presented as

$$u=G,_x+H,_z\,,\quad w=G,_z-H,_x \tag{3.23}$$

where $G(x,z,t)$ is the scalar potential and $H(x,z,t)$ is the vector potential. Plugging Eq.(3.23) in Eqs (3.18) to Eq.(3.22), we get

$$G,_{xx}+G,_{zz}+a_1\varphi+a_2\psi-T=\ddot{G}-\Omega^2G-2\Omega\dot{H}\,, \tag{3.24}$$

$$\delta^2\left(H,_{zz}+H,_{xx}\right)=\ddot{H}-\Omega^2H+2\Omega\dot{G}\,, \tag{3.25}$$

$$\varphi,_{xx}+\varphi,_{zz}+a_3\left(\psi,_{xx}+\psi,_{zz}\right)-a_4\left(G,_{xx}+G,_{zz}\right)-a_5\varphi-a_6\psi+a_7T=\frac{1}{\delta_1^2}\ddot{\varphi}\,, \tag{3.26}$$

$$\varphi,_{xx}+\varphi,_{zz}+a_8\left(\psi,_{xx}+\psi,_{zz}\right)-a_9\left(G,_{xx}+G,_{zz}\right)-a_{10}\varphi-a_{11}\psi+a_{12}T=\frac{1}{\delta_2^2}\ddot{\psi}\,, \tag{3.27}$$

$$T,_{xx}+T,_{zz}-\left(\dot{T}+t_0\ddot{T}\right)=\varepsilon_T\left(\dot{G},_{xx}+\dot{G},_{zz}+t_0\left(\ddot{G},_{xx}+\ddot{G},_{zz}\right)\right)+a_{13}\left(\dot{\varphi}+t_0\ddot{\varphi}\right)+a_{14}\left(\dot{\psi}+t_0\ddot{\psi}\right). \tag{3.28}$$

4. Plane wave solution

To discuss the plane wave propagation in a linear homogeneous double porous thermoelastic material the solutions of Eqs (3.24) to (3.28) are assumed of the form

$$\left(G,H,\varphi,\psi,T\right)=\left\{\bar{G}(z),\bar{H}(z),\bar{\varphi}(z),\bar{\psi}(z),\bar{T}(z)\right\}\exp\left\{\imath k\left(x-ct\right)\right\}. \tag{4.1}$$

Here $\bar{G}(z)\,,\bar{H}(z)\,,\bar{\varphi}(z)\,,\bar{\psi}(z)\,,\ \bar{T}(z)$ are the functions representing the amplitude of waves, $c=\omega/k$ is the phase velocity, ω is the frequency, k is the wavenumber. Putting the solution (4.1) in Eqs (3.24) to (3.28), the system of coupled equations after suppressing bars has been obtained as

$$(m^2 - \alpha^{*2})G - 2\iota\Gamma\omega^2 H + a_1\varphi + a_2\psi - T = 0\,, \tag{4.2}$$

$$(2\Gamma\iota\omega^2 / \delta^2)G + (m^2 - \beta^{*2})H = 0\,, \tag{4.3}$$

$$-a_4(m^2 - k^2)G + (m^2 - \gamma^{*2})\varphi + \left[a_3(m^2 - k^2) - a_6\right]\psi + a_7 T = 0\,, \tag{4.4}$$

$$-a_9(m^2 - k^2)G + (m^2 - k^2 - a_{10})\varphi + a_8(m^2 - \delta^{*2})\psi + a_{12}T = 0\,, \tag{4.5}$$

$$\varepsilon_T\tau_0\omega^2(m^2 - k^2)G + a_{13}\tau_0\omega^2\varphi + a_{14}\tau_0\omega^2\psi + [m^2 - k^2(1 - \tau_0 c^2)]T = 0 \tag{4.6}$$

where

$$m = \frac{\partial}{\partial z}, \qquad \Gamma = \frac{\Omega}{\omega}, \qquad \delta^* = k^2\left[1 - c^2\left(\frac{1}{a_8\delta_2^2} - \frac{a_{11}}{a_8\omega^2}\right)\right], \qquad \alpha^{*2} = k^2(1 - c^2(1 + \Gamma^2))\,,$$

$$\beta^{*2} = k^2\left[1 - \frac{c^2}{\delta^2}(1 + \Gamma^2)\right], \qquad \gamma^{*2} = k^2\left[1 - c^2\left(\frac{1}{\delta_1^2} - \frac{a_5}{\omega^2}\right)\right], \qquad \tau_0 = t_0 + \iota\omega^{-1}\,.$$

The homogeneous system of Eqs (4.2) to Eq.(4.6) has a non-trivial solution if the determinant of its coefficient matrix vanishes which gives characteristic roots as

$$m_j^2 = k^2\left(1 - \lambda_j^2 c^2\right)$$

where $\lambda_j^2;\ j = 1,2,3,4,5$ are the roots of the characteristic equation

$$L_0 D^{10} + L_1 D^8 + L_2 D^6 + L_3 D^4 + L_4 D^2 + L_5 = 0 \tag{4.7}$$

The values of L_0, L_1, L_2, L_3, L_4 and L_5 are given in Appendix A.

Equation (4.7) is of degree five in D^2, which gives the basic to study the impact of rotation on various wave characteristics. These waves are longitudinal, shear, thermal, and volume fraction of type-I and type-II voids, which get modified due to rotation and propagate possibly in the double porous thermoelastic solid about an axis normal to its plane. The motion must be restricted to the free surface $z = 0$. Therefore, the radiation conditions $\mathrm{Re}(m_j) \geq 0;\ j = 1,2,3,4,5$ satisfied by the formal solution (4.1) are given by

$$\left(G, H, \varphi, \psi, T\right) = \sum_{q=1}^{5}\left(1, R_q, V_q, W_q, S_q\right)U_q e^{ik(x-ct) - m_q z}\,. \tag{4.8}$$

Putting Eq.(4.8) in Eqs (3.13) to (3.16) we obtained the set of non-homogeneous systems in a matrix form (4.9), where the coupling parameters R_q, V_q, W_q, S_q are obtained after solving Eq.(4.9). These coupling parameters show the effect of various interacting fields on the waves.

$$\begin{bmatrix} -2\iota\Gamma\omega^2 & a_1 & a_2 & -1 \\ m_q^2-\beta^{*2} & 0 & 0 & 0 \\ 0 & m_q^2-\gamma^{*2} & a_3(m_q^2-k^2)-a_6 & a_7 \\ 0 & m_q^2-k^2-a_{10} & a_8(m_q^2-\delta^{*2}) & a_{12} \end{bmatrix}\begin{bmatrix} R_q \\ V_q \\ W_q \\ S_q \end{bmatrix}=\begin{bmatrix} -m_q^2+\alpha^{*2} \\ -2\Gamma\iota\omega^2/\delta^2 \\ a_4(m_q^2-k^2) \\ a_9(m_q^2-k^2) \end{bmatrix}. \tag{4.9}$$

5. Boundary conditions

At the plane surface $z=0$ of a rotating thermoelastic solid with double porosity, the mechanical and thermal boundary conditions are given by

(i) The normal and shear stress are vanishing, i.e. $\tau_{zz}, \tau_{xz}=0$

$$\ddot{G}-\Omega^2 G-2\Omega\dot{H}-2\delta^2\left(G,_{xx}+H,_{xz}\right)=0, \tag{5.1}$$

$$\ddot{H}-\Omega^2 H+2\Omega\dot{G}+2\delta^2\left(G,_{xz}-H,_{xx}\right)=0. \tag{5.2}$$

(ii) Equilibrated stress tensor for type-I and type-II voids is zero, i.e. $\sigma_i^1, \sigma_i^2=0$

$$\alpha\varphi,_z+b_1\psi,_z=0, \tag{5.3}$$

$$b_1\varphi,_z+\gamma\psi,_z=0. \tag{5.4}$$

(iii) The thermal boundary condition is

$$T,_z+H^*T=0 \tag{5.5}$$

where H^* indicates the surface heat transfer coefficient; the boundary is thermally insulated if $H^*\to 0$ and isothermal if $H^*\to\infty$.

6. Derivation of the secular equation

In this section, the main aim is to obtain the secular equation of the medium under the effect of rotation, using Eq.(4.8) with boundary conditions (5.1) to (5.5) at the surface $z=0$. A homogeneous system of equations in U_q; $q=1, 2, 3, 4, 5$ has been obtained as

$$\sum_{q=1}^{5}\left(1+Qf_qR_q\right)U_q=0, \tag{6.1}$$

$$\sum_{q=1}^{5}\left(R_q - Qf_q\right)U_q = 0\,, \tag{6.2}$$

$$\sum_{q=1}^{5} m_q\left(V_q + a_3 W_q\right)U_q = 0\,, \tag{6.3}$$

$$\sum_{q=1}^{5} m_q\left(V_q + a_8 W_q\right)U_q = 0\,, \tag{6.4}$$

$$\sum_{q=1}^{5}\left(-m_q + H^*\right)S_q U_q = 0\,, \tag{6.5}$$

here

$$Q = \frac{2\iota k}{P}, \quad P = k^2\left(2 - \left(1+\Gamma^2\right)\frac{c^2}{\delta^2}\right), \quad f_q = m_q + \frac{\Gamma k c^2}{\delta^2}; \quad q = 1, 2, 3, 4, 5\,.$$

A non-trivial solution exists for the systems (6.1) to Eq.(6.5) only if the determinant of the coefficient matrix vanishes. The determinant of Eqs (6.1) to Eq.(6.5) leads to the following secular equation for the propagation of waves in the medium

$$\begin{aligned}&\left(m_1 S_1 F_1 - m_2 S_2 F_2 + m_3 S_3 F_3 - m_4 S_4 F_4 + m_5 S_5 F_5\right) + \\ &-H^*\left(S_1 F_1 - S_2 F_2 + S_3 F_3 - S_4 F_4 + S_5 F_5\right) = 0\end{aligned} \tag{6.6}$$

where

$$F_1 = \begin{vmatrix} 1+Qf_2R_2 & 1+Qf_3R_3 & 1+Qf_4R_4 & 1+Qf_5R_5 \\ R_2 - Qf_2 & R_3 - Qf_3 & R_4 - Qf_4 & R_5 - Qf_5 \\ m_2\left(V_2 + a_3W_2\right) & m_3\left(V_3 + a_3W_3\right) & m_4\left(V_4 + a_3W_4\right) & m_5\left(V_5 + a_3W_5\right) \\ m_2\left(V_2 + a_8W_2\right) & m_3\left(V_3 + a_8W_3\right) & m_4\left(V_4 + a_8W_4\right) & m_5\left(V_5 + a_8W_5\right) \end{vmatrix},$$

$$F_2 = \begin{vmatrix} 1+Qf_1R_1 & 1+Qf_3R_3 & 1+Qf_4R_4 & 1+Qf_5R_5 \\ R_1 - Qf_1 & R_3 - Qf_3 & R_4 - Qf_4 & R_5 - Qf_5 \\ m_1\left(V_1 + a_3W_1\right) & m_3\left(V_3 + a_3W_3\right) & m_4\left(V_4 + a_3W_4\right) & m_5\left(V_5 + a_3W_5\right) \\ m_1\left(V_1 + a_8W_1\right) & m_3\left(V_3 + a_8W_3\right) & m_4\left(V_4 + a_8W_4\right) & m_5\left(V_5 + a_8W_5\right) \end{vmatrix},$$

$$F_3 = \begin{vmatrix} 1+Qf_1R_1 & 1+Qf_2R_2 & 1+Qf_4R_4 & 1+Qf_5R_5 \\ R_1 - Qf_1 & R_2 - Qf_2 & R_4 - Qf_4 & R_5 - Qf_5 \\ m_1\left(V_1 + a_3W_1\right) & m_2\left(V_2 + a_3W_2\right) & m_4\left(V_4 + a_3W_4\right) & m_5\left(V_5 + a_3W_5\right) \\ m_1\left(V_1 + a_8W_1\right) & m_2\left(V_2 + a_8W_2\right) & m_4\left(V_4 + a_8W_4\right) & m_5\left(V_5 + a_8W_5\right) \end{vmatrix},$$

$$F_4 = \begin{vmatrix} 1+Qf_1R_1 & 1+Qf_2R_2 & 1+Qf_3R_3 & 1+Qf_5R_5 \\ R_1-Qf_1 & R_2-Qf_2 & R_3-Qf_3 & R_5-Qf_5 \\ m_1(V_1+a_3W_1) & m_2(V_2+a_3W_2) & m_3(V_3+a_3W_3) & m_5(V_5+a_3W_5) \\ m_1(V_1+a_8W_1) & m_2(V_2+a_8W_2) & m_3(V_3+a_8W_3) & m_5(V_5+a_8W_5) \end{vmatrix},$$

$$F_5 = \begin{vmatrix} 1+Qf_1R_1 & 1+Qf_2R_2 & 1+Qf_3R_3 & 1+Qf_4R_4 \\ R_1-Qf_1 & R_2-Qf_2 & R_3-Qf_3 & R_4-Qf_4 \\ m_1(V_1+a_3W_1) & m_2(V_2+a_3W_2) & m_3(V_3+a_3W_3) & m_4(V_4+a_3W_4) \\ m_1(V_1+a_8W_1) & m_2(V_2+a_8W_2) & m_3(V_3+a_8W_3) & m_4(V_4+a_8W_4) \end{vmatrix}.$$

Equation (6.6) provides full information regarding the wavenumber, phase velocity, and attenuation coefficient of the plane waves in the considered medium.

7. Special cases of the secular equation

Some special cases of the secular equation (6.6) have been discussed below.

(i) **Thermally insulated case:** For the thermally insulated case $H^* \to 0$

$$m_1S_1F_1 - m_2S_2F_2 + m_3S_3F_3 - m_4S_4F_4 + m_5S_5F_5 = 0. \tag{7.1}$$

(ii) **Isothermal case:** For the isothermal case $H^* \to \infty$

$$S_1F_1 - S_2F_2 + S_3F_3 - S_4F_4 + S_5F_5 = 0. \tag{7.2}$$

(iii) **Non-rotating double porous thermoelasticity:** In the absence of rotation, i.e. $\Gamma = 0$, the secular equation (6.6), with reduced values of characteristics roots m_q and amplitude ratios R_q, V_q, W_q S_q has been obtained. Also, f_q involved in determinants F_q will be replaced by m_q; $q = 1,2,3,4,5$ and $P = k^2\left(2 - \frac{c^2}{\delta^2}\right)$.

(iv) **Coupled thermoelasticity:** The compact secular equation (6.6) for a thermoelastic solid half-space with double porosity in the context of the coupled theory of thermoelasticity has been obtained by substituting the thermal relaxation time $t_0 = 0$ in the expressions of characteristics of roots m_j and amplitude ratios R_q, V_q, W_q, S_q.

8. Solution of the secular equation

The phase velocity of the waves is complex in nature because the characteristics of roots m_j^2 are also complex in nature as investigated by [26,27,30]. After obtaining the complex roots m_j^2, we get

$$c_j = \pm \frac{1}{m_j}; j = 1,2,3,4,5 . \tag{8.1}$$

These five pairs give us five distinct types of dispersive and attenuated waves that can propagate in the medium, which rotates about the axis normal to its plane. We write

$$c_j^{-1} = v_j^{-1} + \iota\omega^{-1} Q_j^* \tag{8.2}$$

where $k = R_j^* + \iota\omega^{-1} Q_j^*$, and $R_j^* = \omega/\nu_j$, R_j^* and Q_j^* are real quantities. The plane wave solutions components given by Eq.(4.1) become $-Q_j^* x + \iota R_j^*(x - \nu_j t)$. This shows that v_j and Q_j^* is the phase velocity and the attenuation coefficient of the waves. By using expression (8.2) in the secular equation (6.6), the complex roots λ_j^2; $j = 1,2,3,4,5$ of Eq.(4.7) are calculated with the help of MATLAB. These roots are further used to compute the complex characteristics of roots $m_j^2; j = 1,2,3,4,5$ from the relation $m_j^2 = k^2 \left(1 - \lambda_j{}^2 c^2\right)$. So, the phase velocity (v_j) and attenuation coefficient (Q_j^*) are obtained as $\nu_j = 1/\mathrm{Re}(m_j)$, $Q_j^* = \omega \mathrm{Im}(m_j)$ using relation (8.2) in Eq.(8.1). The five distinct real values of phase velocity ν_j correspond to five distinct waves. These waves are attenuated in space, having the attenuation coefficient Q_j^*, and get modified due to the volume fraction of type-I and type-II voids as well as thermal variations and rotation effect.
The specific loss (SL) and penetration depth (PD) have been obtained by Pathania and Joshi [27] and Sharma *et al.* [30] as follows:

$$SL = 4\pi \left| \frac{\mathrm{Im}(k)}{\mathrm{Re}(k)} \right| = 4\pi \left| \frac{Q_j^*}{R_j^*} \right| = 4\pi \left| \frac{\nu_j Q_j^*}{\omega} \right| , \; PD = \frac{1}{|\mathrm{Im}(k)|} = \frac{1}{\left|Q_j^*\right|} . \tag{8.3}$$

9. Displacement amplitude, volume fraction field, and temperature change

The amplitudes of the x and z component of displacements, volume fraction field of type-I and type-II voids, and temperature change on the surface $z = 0$ of plane wave propagation have been developed by using Eqs (3.23) and (4.8) as

$$\begin{gathered} \tilde{U} = U\chi \exp\left\{ \iota R_j^* \left(x - v_j t\right) \right\}, \quad \tilde{W} = W\chi \exp\left\{ \iota R_j^* \left(x - v_j t\right) \right\}, \\ \tilde{\varphi} = \Phi\chi \exp\left\{ \iota R_j^* \left(x - v_j t\right) \right\}, \quad \tilde{\Psi} = \Psi\chi \exp\left\{ \iota R_j^* \left(x - v_j t\right) \right\}, \quad \tilde{T} = \Theta\chi \exp\left\{ \iota R_j^* \left(x - v_j t\right) \right\} . \end{gathered} \tag{9.1}$$

Here,

$$u = \tilde{U} , \quad w = \tilde{W} , \quad \varphi = \tilde{\varphi} , \quad \psi = \tilde{\psi} , \quad T = \tilde{T} , \quad \chi = U_1 \exp\left\{ -Q_j^* x \right\} .$$

Also,

$$U = \left(U_1 B_1 + U_2 B_2 + U_3 B_3 + U_4 B_4 + U_5 B_5\right) / U_1 ,$$

$$W = -\left(U_1 D_1 + U_2 D_2 + U_3 D_3 + U_4 D_4 + U_5 D_5\right) / U_1 ,$$

$$\Phi = \left(U_1 V_1 + U_2 V_2 + U_3 V_3 + U_4 V_4 + U_5 V_5\right) / U_1 ,$$

$$\Psi = \left(U_1 W_1 + U_2 W_2 + U_3 W_3 + U_4 W_4 + U_5 W_5\right) / U_1 ,$$

$$\Theta = \left(U_1 S_1 + U_2 S_2 + U_3 S_3 + U_4 S_4 + U_5 S_5\right) / U_1$$

where $B_j = \imath k - m_j R_j$, $D_j = \imath k R_j + m_j$.

Using Eqs (9.1) in Eqs (6.1)-(6.5), the set of equations has been written in a matrix form as

$$A = \begin{bmatrix} 1 + Qf_2 R_2 & 1 + Qf_3 R_3 & 1 + Qf_4 R_4 & 1 + Qf_5 R_5 \\ R_2 - Qf_2 & R_3 - Qf_3 & R_4 - Qf_4 & R_5 - Qf_5 \\ m_2\left(V_2 + a_3 W_2\right) & m_3\left(V_3 + a_3 W_3\right) & m_4\left(V_4 + a_3 W_4\right) & m_5\left(V_5 + a_3 W_5\right) \\ m_2\left(V_2 + a_8 W_2\right) & m_3\left(V_3 + a_8 W_3\right) & m_4\left(V_4 + a_8 W_4\right) & m_5\left(V_5 + a_8 W_5\right) \end{bmatrix},$$

$$X = \begin{bmatrix} U_2 / U_1 \\ U_3 / U_1 \\ U_4 / U_1 \\ U_5 / U_1 \end{bmatrix}, \qquad B = \begin{bmatrix} -\left(1 + Qf_1 R_1\right) \\ -\left(R_1 - Qf_1\right) \\ -m_1\left(V_1 + a_3 W_1\right) \\ -m_1\left(V_1 + a_8 W_1\right) \end{bmatrix}.$$

The terms $U_i / U_1 ; i = 2,3,4,5$ are obtained from the relation $AX = B$.

10. Numerical discussion

To compute the numerical results for this model, MATLAB software has been used for the magnesium crystal material as defined by Singh *et al.* [31], and physical values of these parameters are taken as shown in Table.1.

Table 1. Values of the material constants.

Symbol	Value	Symbol	Value	Symbol	Value
λ	$1.5\times10^{10}\ Nm^{-2}$	b	$2\times10^{8}\ Nm^{-2}$	μ	$7.5\times10^{9}\ Nm^{-2}$
ρ	$2\times10^{3}\ Kgm^{-3}$	d	$2.1\times10^{8}\ Nm^{-2}$	α	$8\times10^{9}\ N$
α_1	$1.2\times10^{10}\ Nm^{-2}$	K	$1.7\times10^{2}\ Wm^{-1}K^{-1}$	α_2	$2.21\times10^{10}\ Nm^{-2}$
α_3	$1.23\times10^{6}\ Nm^{-2}$	χ_2	$330\,kgm^{-1}$	α_t	$1.78\times10^{-5}\ K^{-1}$
γ	$8.2\times10^{9}\ N$	b_1	$8.1\times10^{6}\ N$	γ_1	$2\times10^{6}\ Nm^{-2}K^{-1}$
γ_2	$3.11\times10^{6}\ Nm^{-2}$	C_e	$1.809\times10^{6}\ m^2s^2K^{-1}$	T_0	$293\,K$
χ_1	$320\,kgm^{-1}$	ω	$0.01Hz$		

Using the numerical value of the parameters, the graphs of the attenuation coefficient, phase velocity, penetrating depth, specific loss, surface displacements, surface temperature change, and the surface volume fraction for type-I and type-II voids with respect to the wavenumber have been plotted. The influence of angular velocity $(\Omega = 0, 0.2, 0.4, 0.6)$ on various physical properties (from Figs 2-10) has been examined.

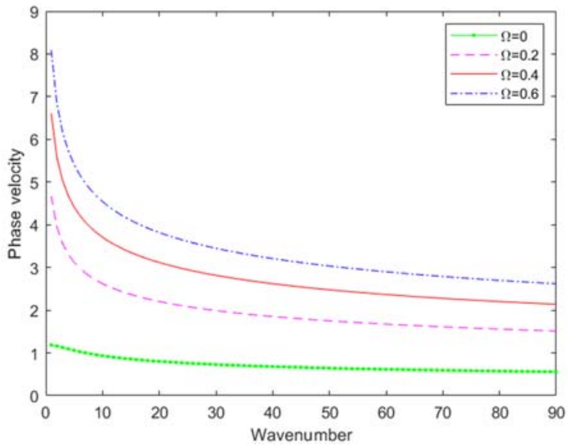

Fig.2. Variation of phase velocity with wavenumber.

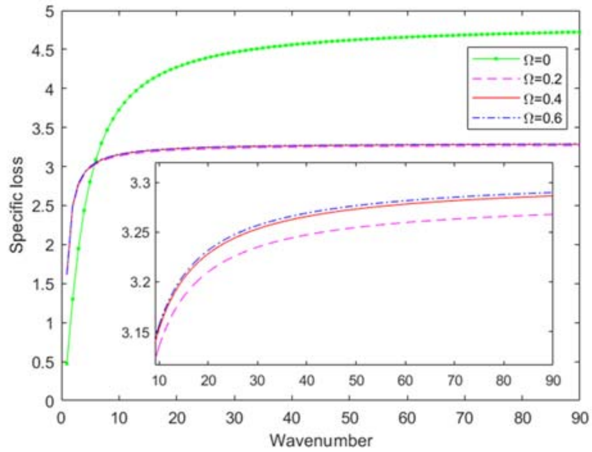

Fig.3. Variation of specific loss with wavenumber.

Figure 2 depicts the variation of phase velocity against wavenumber for different values of angular velocity and it is observed that when the medium is rotating, the magnitude of phase velocity is quite high at vanishing wavenumber, and it decreases with an increase in the wavenumber. The magnitude of phase velocity is small in the case of the non-rotating medium as compared to the medium in rotation. The rotation rate also affects the phase velocity; if the number of rotation-rate increases, the magnitude of phase velocity also increases. The variation is studied in Fig.3 when the medium is non-rotating or rotating with different angular velocities. It is noticed that if the medium is non-rotating, the specific loss increases rapidly, and after that, it increases at a deliberate rate. It is also noticed that when the rotation rate increases the amplitude of specific loss also increases.

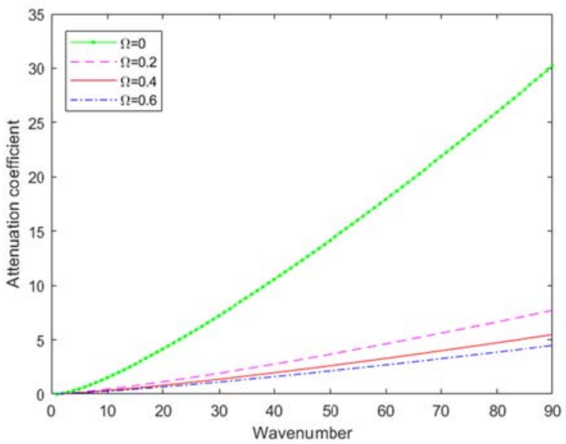

Fig.4. Variation of attenuation coefficient with wavenumber.

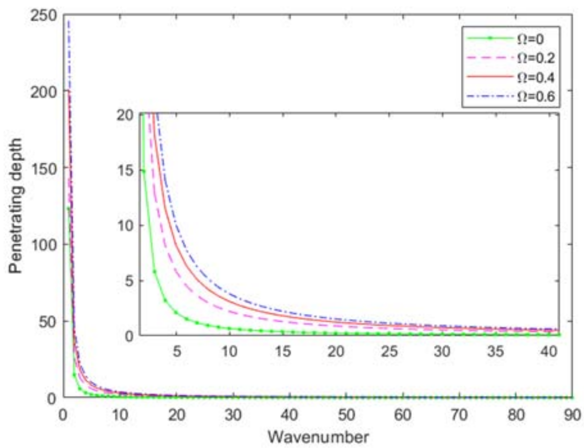

Fig.5. Variation of penetrating depth with wavenumber.

Figure 4 depicts the variation of the attenuation coefficient against the wavenumber under the effect of rotation of the medium. The amplitude of the attenuation coefficient is much higher when the medium is non-rotating, i.e. $(\Omega = 0)$. If the rotation rate increases from $\Omega = 0.2$ to $\Omega = 0.6$, the magnitude of the attenuation coefficient decreases. The variation of the penetrating depth with the wavenumber for different values of the rotating parameters shown in Fig.5. It is also observed that as the wavenumber approaches zero, the magnitude of the penetrating depth is high and then slashes down sharply with an increase of the wavenumber for the non-rotating case, the amplitude of the penetrating depth is small as compared to the rotating case. The magnitude of the penetrating depth in case of $\Omega = 0.6$ is higher than that of $\Omega = 0.2$. So it has been concluded that as the rotation rate increases, the penetrating depth also slightly increases.

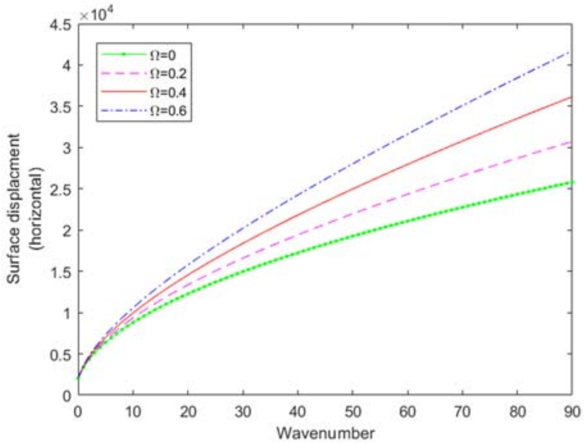

Fig.6. Variation of surface displacement (horizontal) with wavenumber.

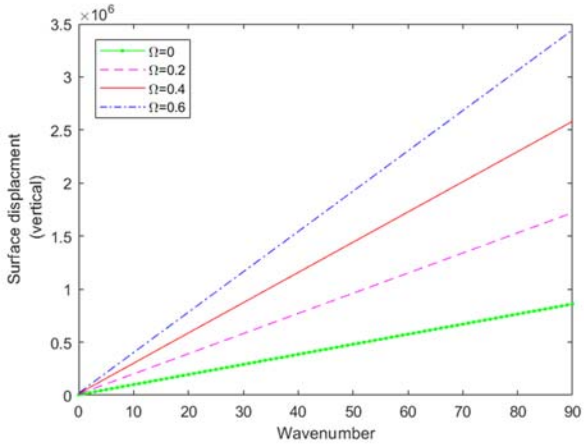

Fig.7. Variation of surface displacement (vertical) with wavenumber.

Figures 6 and 7 illustrate the effect of wavenumber on the horizontal and vertical surface displacement components under the effect of rotation of the medium. The magnitude of both the horizontal and vertical surface displacement components increases with an increase of the wavenumber. The amplitude of both displacement components has a small magnitude in the case of a non-rotating medium. Besides, there is a significant effect of rotation rate on the amplitude of displacement components. It is observed that if the rotation rate of the medium decreases, the profile of horizontal and vertical displacement components also decreases.

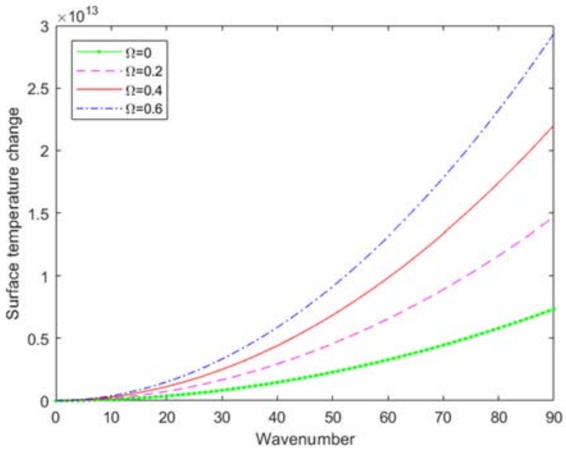

Fig.8. Variation of surface temperature change with wavenumber.

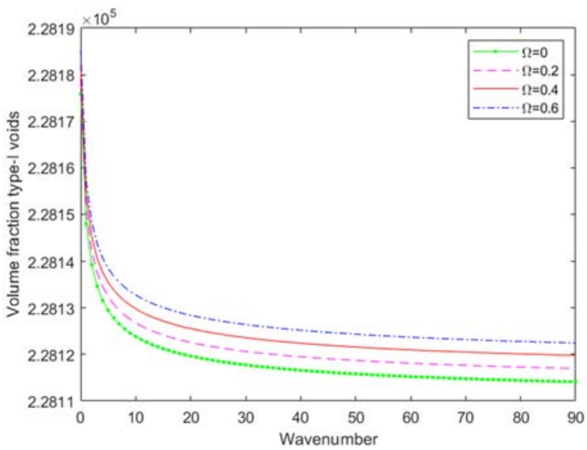

Fig.9. Variation of volume fraction type-I voids with wavenumber.

Figure 8 graphically shows the effect of wavenumber on the surface temperature change. The effect of rotation is noticed on the surface temperature change as the profiles show dispersive behavior for increasing the value of wavenumber. It can be seen that the magnitude of surface temperature change is high for the rotating medium in comparison with the non-rotating medium. When the rotation rate is *0.2*, the magnitude of surface temperature change is small as compared to the rotation rate $\Omega = 0.4$ and $\Omega = 0.6$. Figures 9 and 10 depict the variation of wavenumber on the volume fraction field due to type-I voids and type-II voids for different values of rotation parameters. The magnitude of volume fraction of type-I and type-II voids decreases with an increase in the wavenumber. The significant effect of the rotation parameters on the volume fraction field due to type-I and type-II voids is observed. If the rotation rate increases, the amplitude of the volume fraction field due to both types of voids increases.

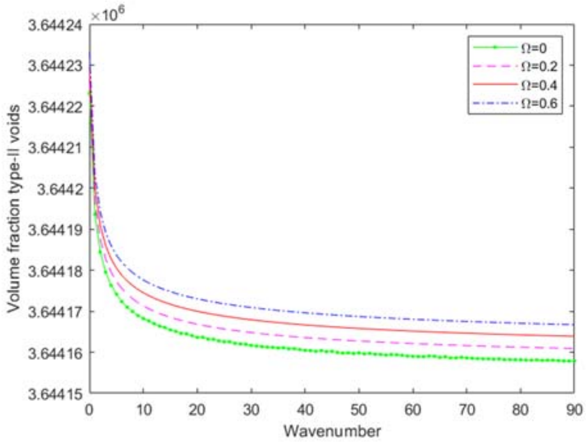

Fig.10. Variation of volume fraction type-II voids with wavenumber.

12. Conclusions

The propagation of plane waves in a thermoelastic double porous solid medium under the effect of rotation is studied. The secular equation has been derived in the simplest form to analyze various physical properties of the waves. After the lengthy and difficult algebraic calculations in the background, the following conclusions may be drawn:

1. There may exist five coupled plane waves propagating with different phase speeds. The presence of type-I and type-II voids, thermal, and rotation parameters are responsible for the coupling in the waves.
2. In the absence of type-I and type-II voids and thermal variation, the classical longitudinal and transverse waves are coupled through the rotation parameter of the medium.
3. The effect of rotation is significant in the considered problem as the amplitude of various physical quantities has increasing and decreasing trends. The amplitude of phase velocity, penetrating depth, and specific loss increases with an increase of the rotation rate, but the amplitude of attenuation coefficient decreases.
4. The displacement components, temperature change, and volume faction field due to type-I and type-II voids increase with an increase in the rate of rotation.
5. All the wave profiles are significantly affected by type-I and type-II voids and thermal variations.

The current study may be used in a variety of fields, including earthquake engineering, soil dynamics, petroleum industry, geophysics, and chemical engineering. The study may be used in the development of rotation sensors and gyroscopic devices. Adding the double porous parameters to the rotating thermoelastic material makes this study more realistic.

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Nomenclature

ρ	– density of the medium
λ,μ	– Lame's constant
σ_i^1,σ_i^2	– equilibrated stress tensor for type-I, and type-II voids
φ,ψ	– volume fraction change of type-I and type-II voids
b,d	– parameters of type-I and type-II voids
$T(x,z,t)$	– temperature change in medium
t_0	– thermal relaxation time
ξ,ζ	– intrinsic equilibrated body forces for type-I and type-II voids
α_t	– coefficient of linear thermal expansion
χ_1,χ_2	– coefficient of the equilibrated inertia for type-I and type-II voids
C_e	– specific heat at constant strain
$\alpha,\alpha_1,\alpha_2,\alpha_3,b_1,\gamma$	– constitutive coefficients corresponding to voids properties
γ_1,γ_2	– constitutive coefficients corresponding to thermal properties
K	– thermal conductivity

Appendix A

$$L_0 = a_8 - a_3\ ,$$

$$L_1 = d_{44} - d_{34} - a_2(a_4 - a_9) - a_3(d_{43} + d_{51} + d_{55}) + a_8(d_{51} + d_{55}) +$$
$$+\alpha^{*2}(a_3 - a_8) + \beta^{*2}(a_3 - a_8) - a_8\gamma^{*2} - a_1(a_3a_9 - a_4a_8),$$

$$L_2 = (a_8 - a_3)(\alpha^{*2}\beta^{*2} + \beta^{*2}\gamma^{*2} - d_{21}d_{12} + d_{52}) + (\alpha^{*2} + \beta^{*2})(d_{34} - d_{44} + a_3d_{43} + a_3d_{55} - a_8d_{55}) + \beta^{*2}(a_2a_4 - a_2a_9 +$$
$$+a_3d_{51} - a_8d_{51} + a_1a_3a_9 - a_1a_4a_8) - \gamma^{*2}(d_{44} + a_2a_9 + a_8d_{51} + a_8d_{55}) + d_{54}(a_7 - a_4 + a_9 - a_{12}) - d_{34}(d_{43} + d_{51} +$$
$$+d_{55}) + d_{44}(d_{51} + d_{55} + a_1a_9) - a_3d_{43}(d_{51} + d_{55}) + d_{53}(a_4a_8 - a_3a_9 + a_3a_{12} + a_7a_8) + d_{31}(a_2 - a_1a_8) + d_{41}(a_1a_3 - a_2) +$$
$$+d_{51}(a_7a_8 - a_3a_{12} - a_7 + a_{12}) + d_{55}(a_4a_8 - a_3a_9 - a_4 + a_9) - a_2a_4d_{43} - a_1a_9d_{34},$$

$$L_3 = d_{21}d_{12}(d_{34} - d_{44} + a_3d_{43} + (a_3 - a_8)d_{55}) + d_{54}(d_{31} - d_{41}) + d_{52}(d_{44} - d_{34}) + a_2d_{52}(a_{12} - a_7) - d_{34}d_{43}(d_{51} + d_{55}) +$$
$$+d_{44}((a_4 - a_7)d_{53} - a_1d_{31}) + d_{31}(a_2(d_{43} + d_{55}) - a_8d_{53}) + d_{43}((a_7 - a_4)d_{54} - a_3d_{52}) + d_{53}((a_{12} - a_9)d_{34} + a_3d_{41}) +$$
$$+a_1a_3d_{41} + a_1a_3(d_{41}d_{55} - a_{12}d_{52}) + a_1a_7(a_8d_{52} + a_9d_{54} + d_{44}d_{51}) + a_2d_{53}(a_4a_{12} - a_7a_9) - a_1d_{55}(a_9d_{34} + a_8d_{31}) +$$
$$+a_1a_4(d_{44}d_{55} - a_{12}d_{54}) - a_1a_{12}d_{34}d_{51} - a_2a_4d_{43}d_{55} + \alpha^{*2}(d_{54}(a_{12} - a_7) + d_{43}(d_{34} + a_3d_{55}) + d_{55}(d_{34} - d_{44}) +$$
$$+d_{53}(a_7a_8 - a_3a_{12})) + \beta^{*2}(d_{31}(a_1a_8 - a_2) + d_{41}(a_2 - a_1a_3) + d_{51}(d_{34} - d_{44} + a_2a_7 - a_2a_{12} + a_3d_{43} + a_1a_3a_{12} - a_1a_7a_8) +$$
$$+d_{55}(d_{34} - d_{44} + a_2a_4 - a_2a_9 + a_2d_{43} + a_1a_3a_9 - a_1a_4a_8) + d_{53}(a_3a_9 - a_4a_8 - a_3a_{12} + a_7a_8) + d_{52}(a_3 - a_8) +$$
$$+d_{54}(a_4 - a_7 - a_9 + a_{12}) + d_{34}(d_{43} + a_1a_9) - a_4(a_1d_{44} - a_2d_{43})) + \gamma^{*2}(d_{51}(-d_{44} - a_2a_{12}) + d_{55}(-a_2a_9 - d_{44}) +$$
$$+d_{54}(a_{12} - a_9) + a_2d_{41} + a_8(d_{12}d_{21} - d_{52})) + \alpha^{*2}\beta^{*2}(d_{44} - d_{34} - a_3d_{43} + d_{55}(a_8 - a_3)) + \beta^{*2}\gamma^{*2}(d_{44} + a_2a_9 +$$
$$+a_8(d_{51} + d_{55})) + \alpha^{*2}\gamma^{*2}(d_{44} + a_8d_{55}) - a_8\alpha^{*2}\beta^{*2}\gamma^{*2},$$

$$L_4 = d_{31}(d_{43}d_{54} - d_{44}d_{53}) + d_{34}(d_{41}d_{53} - d_{43}d_{52}) + d_{12}d_{21}(a_3(d_{43}d_{55} - a_{12}d_{53}) + a_7(a_8d_{53} - d_{54}) + a_{12}d_{54} +$$
$$+d_{34}d_{43} + d_{55}(d_{34} - d_{44})) + a_2a_7(d_{41}d_{53} - d_{43}d_{52}) + a_1a_{12}(d_{31}d_{54} - d_{34}d_{52}) + a_1a_7(d_{41}d_{54} + d_{44}d_{52}) +$$
$$+a_1d_{55}(d_{34}d_{41} - d_{31}d_{44}) + a_2d_{31}(d_{43}d_{55} - d_{31}d_{53}) + \alpha^{*2}(d_{34}(d_{43}d_{55} - a_{12}d_{53}) + a_7(d_{44}d_{53} - d_{43}d_{54})) +$$
$$+\beta^{*2}(d_{31}(a_4d_{44} - d_{54} - a_2d_{43} - a_2d_{55} + a_8d_{53} + a_1a_8d_{55}) + d_{52}(d_{34} - d_{44} - a_2a_{12} + a_2a_7 + a_3d_{43} +$$
$$+a_1a_3a_{12} + a_1a_7a_8) + d_{53}(a_9d_{34} - a_3d_{41} - a_{12}d_{34} - a_4d_{44} + a_7d_{44} - a_2a_4a_{12} + a_2a_7a_9) + d_{54}(d_{41} + a_4d_{43} +$$
$$-a_7d_{43} + a_1a_4a_{12} - a_1a_7a_9) + d_{51}(d_{34}d_{43} + a_1a_{12}d_{34} - a_1a_7d_{44} + a_2a_7d_{43}) + d_{55}(a_2d_{41} + d_{34}d_{43} + a_1a_9d_{34} +$$
$$-a_1a_4d_{44} + a_2a_4d_{43}) - a_1d_{34}d_{41}) + \gamma^{*2}(d_{41}(d_{54} + a_2d_{55}) - d_{52}(d_{44} + a_2a_{12}) + d_{12}d_{21}(d_{44} + a_8d_{55})) +$$
$$\alpha^{*2}\beta^{*2}(d_{54}(a_7 - a_{12}) - d_{43}(d_{34} + a_3d_{55}) + d_{55}(d_{44} - d_{34}) + d_{53}(a_3a_{12} - a_7a_8)) + \beta^{*2}\gamma^{*2}(a_8d_{52} - a_2d_{41} +$$
$$+d_{54}(a_9 - a_{12}) + d_{51}(d_{44} + a_2a_{12}) + d_{55}(a_2a_9 + d_{44})) + \alpha^{*2}\gamma^{*2}(d_{44}d_{55} - a_{12}d_{54}) - \alpha^{*2}\beta^{*2}\gamma^{*2}(d_{44} + a_8d_{55}),$$

$$L_5 = \beta^{*2}(d_{31}d_{53}(d_{44} + a_{12}a_2) - d_{31}d_{43}d_{54} + d_{34}(d_{41}d_{53} + d_{43}d_{52}) + a_1a_{12}(d_{34}d_{52} - d_{31}d_{54}) + a_7d_{41}(a_1d_{54} +$$
$$-a_2d_{53}) + a_7d_{52}(a_2d_{43} - a_1d_{44}) + d_{55}(d_{31}d_{44}a_1 - a_1d_{34}d_{41} - a_2d_{31}d_{43})) + (\alpha^{*2}\beta^{*2} - d_{21}d_{12})(a_7(d_{43}d_{54} +$$
$$-d_{44}d_{53}) + d_{34}(a_{12}d_{53} - d_{43}d_{55})) + \gamma^{*2}\beta^{*2}(d_{44}d_{52} - d_{41}d_{54} + a_2a_{12}d_{52} - a_7d_{41}d_{51}) + \gamma^{*2}d_{12}d_{21}(d_{44}d_{55} +$$
$$-a_{12}) + \alpha^{*2}\beta^{*2}\gamma^{*2}(a_2d_{54} - d_{44}d_{55}).$$

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