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## Safety of critical infrastructure exposed to operation and weather condition changes

### Keywords

safety, critical infrastructure, operation process related to weather changes

### Abstract

*The problem of evaluating the safety of critical infrastructures impacted by extreme operation and weather hazards is very important in the face of the climate changing. The model introduced in the chapter enables determining indicators describing safety and resilience of critical infrastructures with considering the impact of changing weather and operating conditions. The chapter also includes the safety and resilience indicators evaluation for an example of the real critical infrastructure which is the port oil piping transportation critical infrastructure.*

### 1. Introduction

A complex system with inside and outside system dependencies that have a noticeable negative impact on the health, safety and security, economics and social conditions of human communities and territory areas in the case of its degradation, is a definition of a critical infrastructure (Laugé, et al., 2015; Schultz, et al., 2010). A critical infrastructure is often influenced by outside processes, for example its operation process and the weather change process at its operating area. These processes may have negative impact on the critical infrastructure safety. The safety analysis of the critical infrastructure influenced by those processes can be prepared by the multistate critical infrastructures safety modelling used with the semi-Markov modelling of considered outside processes (Kołowrocki, 2014; Kołowrocki & Soszyńska-Budny, 2011; Torbicki, 2019). When the critical infrastructure is influenced by its operation process and the weather change process at its operating area, this approach leads to the construction of the joint general safety model of the

critical infrastructure impacted by the operation process and the weather change process (Kołowrocki, Soszyńska-Budny, Torbicki, 2018; Torbicki, 2019). Moreover, the created model allow to define safety and resilience indicators of critical infrastructures (Kołowrocki, Soszyńska-Budny, Torbicki, 2018; Torbicki, 2019), practically important for their users and operators. These safety and resilience indicators are used to the safety analysis of a real critical infrastructure which is the port oil piping transportation critical infrastructure (Torbicki, 2019).

### 2. Theoretical background

We define the critical infrastructure operation process  $Z(t)$ ,  $t \geq 0$ , with discrete operation states  $z_1, z_2, \dots, z_v$  where  $v, v \in \mathbb{N}$ , is the number of the operation states. Moreover, we assume that there are  $\kappa, \kappa \in \mathbb{N}$ , independent and different weather change processes  $C_j(t)$ ,  $t \geq 0, j = 1, 2, \dots, \kappa$ , at the critical infrastructure operating area. They are taking  $w_j, w_j \in \mathbb{N}, j = 1, 2, \dots, \kappa$ , different weather states  $c_{j1}, c_{j2}, \dots, c_{jw_j}$ . We assume that the critical

infrastructure operation process  $Z(t)$ ,  $t \geq 0$ , and weather change processes  $C_j(t)$ ,  $t \geq 0$ ,  $j = 1, 2, \dots, \kappa$ , are semi-Markov and periodic processes.

Then, long term proportions of their sojourn times at their particular states are their limit transient probabilities defined in (Torbicki, 2019) denoted as follows:

- $p_b$ ,  $b = 1, 2, \dots, v$ , for the critical infrastructure operation process  $Z(t)$ ,  $t \geq 0$ ,
- $q_{jl}$ ,  $l = 1, 2, \dots, w_j$ ,  $j = 1, 2, \dots, \kappa$ , for weather change processes  $C_j(t)$ ,  $t \geq 0$ ,  $j = 1, 2, \dots, \kappa$ .

The operation process related to weather changes is denoted by  $\bar{Z}C(t)$ ,  $t \geq 0$ . We assume that this process is at the operation and weather state  $\bar{z}c_{b,(\beta_1, \beta_2, \dots, \beta_\kappa)}$ ,  $b = 1, 2, \dots, v$ ,  $\beta_j = 1, 2, \dots, w_j$ ,  $j = 1, 2, \dots, \kappa$ , at the moment  $t \geq 0$ , if and only if at that moment, the operation process  $Z(t)$  is at the operation state  $z_b$  and the weather change processes  $C_j(t)$  are respectively at the weather states  $c_{1\beta_1}, c_{2\beta_2}, \dots, c_{\kappa\beta_\kappa}$ .

We assume that the critical infrastructure is composed of assets  $A_i$ ,  $i = 1, 2, \dots, n$ , impacted by the operation process  $Z(t)$ ,  $t \geq 0$ , and weather change processes  $C_j(t)$ ,  $t \geq 0$ ,  $j = 1, 2, \dots, \kappa$ .

Evaluating the critical infrastructure safety with considering the operation process related to weather changes impact is necessary to determine with experts values of coefficients

$$[\rho_1^1(u)]^{(b)} \geq 1, u = 1, 2, \dots, z,$$

of the operation process impact on asset  $A_i$ ,  $i = 1, 2, \dots, n$ , intensities of ageing (the intensities of the assets departure from the safety state subsets  $\{u, u + 1, \dots, z\}$ ,  $u = 1, 2, \dots, z$ ) at the operation process  $Z(t)$ ,  $t \geq 0$ , states  $z_b$ ,  $b = 1, 2, \dots, v$ , and the coefficients

$$[\rho_j^2(u)]^{(\beta_j)} \geq 1, u = 1, 2, \dots, z,$$

of the weather impact on asset  $A_i$ ,  $i = 1, 2, \dots, n$ , intensities of ageing at the weather change processes  $C_j(t)$ ,  $t \geq 0$ ,  $j = 1, 2, \dots, \kappa$ , states  $c_{j\beta_j}$ ,  $\beta_j = 1, 2, \dots, w_j$ ,  $j = 1, 2, \dots, \kappa$ .

We assume that the values of the operation process related to weather changes  $\bar{Z}C(t)$ ,  $t \geq 0$ , impact coefficients on asset  $A_i$ ,  $i = 1, 2, \dots, n$ , intensities of ageing at the operation and weather states  $\bar{z}c_{b,(\beta_1, \beta_2, \dots, \beta_\kappa)}$ ,  $b = 1, 2, \dots, v$ ,  $\beta_j = 1, 2, \dots, w_j$ ,  $j = 1, 2, \dots, \kappa$ , are given by

$$\begin{aligned} & [\rho_i^3(u)]^{b,(\beta_1, \beta_2, \dots, \beta_\kappa)} \\ &= [\rho_i^1(u)]^{(b)} \cdot \prod_{j=1}^{\kappa} [\rho_j^2(u)]^{(\beta_j)} \end{aligned} \quad (1)$$

for  $u = 1, 2, \dots, z$ .

Next, we assume that the conditional asset safety function is a vector

$$\begin{aligned} & [S_i^3(t, \cdot)]^{b,(\beta_1, \beta_2, \dots, \beta_\kappa)} = \\ & [1, [S_i^3(t, 1)]^{b,(\beta_1, \dots, \beta_\kappa)}, \dots, [S_i^3(t, z)]^{b,(\beta_1, \dots, \beta_\kappa)}], \end{aligned} \quad (2)$$

for  $t \geq 0$ ,  $i = 1, 2, \dots, n$ ,  $b = 1, 2, \dots, v$ ,  $\beta_j = 1, 2, \dots, w_j$ ,  $j = 1, 2, \dots, \kappa$ ,

with exponential coordinates

$$[S_i^3(t, u)]^{b,(\beta_1, \beta_2, \dots, \beta_\kappa)} \quad (3)$$

$$= P([T_i^3(u)]^{b,(\beta_1, \beta_2, \dots, \beta_\kappa)} > t |$$

$$|\bar{Z}C(t) = \bar{z}c_{b,(\beta_1, \beta_2, \dots, \beta_\kappa)})$$

$$= \exp[-[\rho_i^3(u)]^{b,(\beta_1, \beta_2, \dots, \beta_\kappa)} \cdot \lambda_i^0(u)t],$$

$u = 1, 2, \dots, z$ ,

where  $[T_i^3(u)]^{b,(\beta_1, \beta_2, \dots, \beta_\kappa)}$ ,  $i = 1, 2, \dots, n$ ,  $b = 1, 2, \dots, v$ ,  $\beta_j = 1, 2, \dots, w_j$ ,  $j = 1, 2, \dots, \kappa$ , is the conditional lifetime of an asset  $A_i$  in the safety state subset  $\{u, u + 1, \dots, z\}$  while the operation process related to weather changes  $\bar{Z}C(t)$ ,  $t \geq 0$ , is at the operation and weather state  $\bar{z}c_{b,(\beta_1, \beta_2, \dots, \beta_\kappa)}$ ,  $\lambda_i^0(u)$ ,  $u = 1, 2, \dots, z$ , are the intensities of the asset  $A_i$  departure from the safety state subset  $\{u, u + 1, \dots, z\}$  without any impact and  $[\rho_i^3(u)]^{b,(\beta_1, \beta_2, \dots, \beta_\kappa)}$ ,  $u = 1, 2, \dots, z$ , are operation process related to weather changes  $\bar{Z}C(t)$  impact coefficients on asset  $A_i$  intensities of ageing at the operation and weather states  $\bar{z}c_{b,(\beta_1, \beta_2, \dots, \beta_\kappa)}$ ,  $b = 1, 2, \dots, v$ ,  $\beta_j = 1, 2, \dots, w_j$ , defined by (1).

We assume that the operation process  $Z(t)$ ,  $t \geq 0$ , and the weather change processes  $C_j(t)$ ,  $t \geq 0$ ,  $j = 1, 2, \dots, \kappa$ , at the critical infrastructure operating area are independent for simplicity our future safety and resilience evaluation.

$[T^3(u)]^{b, (\beta_1, \beta_2, \dots, \beta_\kappa)}$ ,  $u = 1, 2, \dots, z$ ,  $b = 1, 2, \dots, v$ ,  $\beta_j = 1, 2, \dots, w_j$ ,  $j = 1, 2, \dots, \kappa$ , is denoted the critical infrastructure conditional lifetime in the safety state subset  $\{u, u + 1, \dots, z\}$  while the operation process related to weather changes  $\bar{Z}C(t)$  is at the operation and weather state  $\bar{z}C_{b, (\beta_1, \beta_2, \dots, \beta_\kappa)}$ . Then, the conditional safety function of the critical infrastructure is defined as the vector

$$[\mathbf{S}^3(t, \cdot)]^{b, (\beta_1, \beta_2, \dots, \beta_\kappa)} = [1, [\mathbf{S}^3(t, 1)]^{b, (\beta_1, \dots, \beta_\kappa)}, \dots, [\mathbf{S}^3(t, z)]^{b, (\beta_1, \dots, \beta_\kappa)}] \quad (4)$$

$$t \geq 0, b = 1, 2, \dots, v, \beta_j = 1, 2, \dots, w_j, j = 1, 2, \dots, \kappa,$$

with the coordinates

$$[\mathbf{S}^3(t, u)]^{b, (\beta_1, \beta_2, \dots, \beta_\kappa)} = P\left([T^3(u)]^{b, (\beta_1, \beta_2, \dots, \beta_\kappa)} > t \mid \bar{Z}C(t) = \bar{z}C_{b, (\beta_1, \beta_2, \dots, \beta_\kappa)}\right), \quad (5)$$

$$u = 1, 2, \dots, z.$$

Similarly,  $T^3(u)$ ,  $u = 1, 2, \dots, z$ , is the critical infrastructure impacted by its operation process related to weather changes  $\bar{Z}C(t)$ ,  $t \geq 0$ , unconditional lifetime in the safety state subset  $\{u, u + 1, \dots, z\}$ .

## 2.1. Safety indicators

The first safety indicator (**SafI1**) is the unconditional safety function of the critical infrastructure related to the process  $\bar{Z}C(t)$ ,  $t \geq 0$ , defined by the vector

$$\mathbf{S}^3(t, \cdot) = [1, \mathbf{S}^3(t, 1), \dots, \mathbf{S}^3(t, z)], t \geq 0, \quad (6)$$

with the coordinates

$$\mathbf{S}^3(t, u) = P(T^3(u) > t), t \geq 0, \quad (7)$$

$$u = 1, 2, \dots, z.$$

The coordinates of the first safety indicator can be evaluated by

$$\mathbf{S}^3(t, u) \cong \sum_{b=1}^v \sum_{\substack{\beta_1=1,2,\dots,w_1 \\ \beta_2=1,2,\dots,w_2 \\ \dots \\ \beta_\kappa=1,2,\dots,w_\kappa}} [\mathbf{S}^3(t, u)]^{b, (\beta_1, \dots, \beta_\kappa)} \cdot p_b \prod_{j=1}^{\kappa} q_j \beta_j, \quad (8)$$

$$t \geq 0, u = 1, 2, \dots, z,$$

where  $[\mathbf{S}^3(t, u)]^{b, (\beta_1, \dots, \beta_\kappa)}$  is defined by (4)–(5), when the critical infrastructure operation time  $\bar{\theta}C$  is large enough.

The explanation for this condition is as follows. Under the assumption about periodicity of the operation process related to weather changes and independence of the weather and operation processes, the probability of the process  $\bar{Z}C(t)$  staying at the operation and weather state  $\bar{z}C_{b, (\beta_1, \beta_2, \dots, \beta_\kappa)}$  at the moment  $t \geq 0$  can be approximated by its limit transient probabilities i.e.

$$P(\bar{Z}C(t) = \bar{z}C_{b, (\beta_1, \beta_2, \dots, \beta_\kappa)}) \approx p_b \prod_{j=1}^{\kappa} q_j \beta_j$$

the bigger  $t \geq 0$  is, the more it increases the accuracy of this approximation.

Moreover, using the law of total probability, we receive

$$\mathbf{S}^3(t, u) = \sum_{b=1}^v \sum_{\substack{\beta_1=1,2,\dots,w_1 \\ \beta_2=1,2,\dots,w_2 \\ \dots \\ \beta_\kappa=1,2,\dots,w_\kappa}} ([\mathbf{S}^3(t, u)]^{b, (\beta_1, \dots, \beta_\kappa)} \cdot P(\bar{Z}C(t) = \bar{z}C_{b, (\beta_1, \beta_2, \dots, \beta_\kappa)})).$$

As it is very hard to determine the probability  $P(\bar{Z}C(t) = \bar{z}C_{b, (\beta_1, \beta_2, \dots, \beta_\kappa)})$ , we use instead its approximation  $p_b \prod_{j=1}^{\kappa} q_j \beta_j$ . However, it will be accurate only for large  $t$ . Thus, we have to assume that the critical infrastructure was operating without a damage for the enough long period of a time  $\bar{\theta}C$  before starting of our calculation (before  $t = 0$ ). The safety function  $\mathbf{S}^3(t + \bar{\theta}C, u)$  of the critical infrastructure at the moment  $t + \bar{\theta}C$

knowing that for the period of time  $\bar{\theta}C$  it was operating without damage, and under the assumption of exponential coordinates of its assets conditional safety functions is equal exactly  $S^3(t, u)$  due to the memorylessness of the exponential distribution.

The second safety indicator (**SafI2**) is the risk function defined by

$$r^3(t) = 1 - S^3(t, r), t \geq 0, \tag{9}$$

where  $S^3(t, r)$ ,  $t \geq 0$ , is the coordinate of the critical infrastructure unconditional safety function evaluated by (8) for  $u = r$  and  $r$  is the critical safety state.

The third safety indicator (**SafI3**) is the graph of the critical infrastructure risk function called the critical infrastructure fragility curve.

Moreover, we distinguish other useful safety factors:

- the mean value of the critical infrastructure lifetime in the safety state subset  $\{u, u + 1, \dots, z\}$ , defined by

$$\begin{aligned} \mu^3(u) &= \int_0^{+\infty} S^3(t, u) dt \\ &\cong \sum_{b=1}^v \sum_{\substack{\beta_1=1,2,\dots,w_1 \\ \beta_2=1,2,\dots,w_2 \\ \dots \\ \beta_\kappa=1,2,\dots,w_\kappa}} [\mu^3(u)]^{b,(\beta_1,\dots,\beta_\kappa)} \cdot p_b \prod_{j=1}^\kappa qj_{\beta_j}, \end{aligned} \tag{10}$$

$$u = 1, 2, \dots, z,$$

where  $[\mu^3(u)]^{b,(\beta_1,\dots,\beta_\kappa)}$  is defined by

$$\begin{aligned} &[\mu^3(u)]^{b,(\beta_1,\beta_2,\dots,\beta_\kappa)} \\ &= \int_0^\infty [S^3(t, u)]^{b,(\beta_1,\beta_2,\dots,\beta_\kappa)} dt, \end{aligned} \tag{11}$$

- the standard deviation of the critical infrastructure lifetime in the safety state subset  $\{u, u + 1, \dots, z\}$

$$\sigma^3(u) = \sqrt{2 \int_0^\infty t \cdot S^3(t, u) dt - [\mu^3(u)]^2}, \tag{12}$$

$$u = 1, 2, \dots, z,$$

- the mean lifetimes  $\bar{\mu}^3(u)$ ,  $u = 1, 2, \dots, z$ , of the critical infrastructure in the particular safety states:

$$\begin{aligned} \bar{\mu}^3(u) &= \mu^3(u) - \mu^3(u + 1), \\ u &= 1, 2, \dots, z - 1, \\ \bar{\mu}^3(z) &= \mu^3(z), \end{aligned} \tag{13}$$

where  $\mu^3(u)$  is defined by (10).

The other safety indicators (**SafI4**)–(**SafI8**) are:

- the fourth safety indicator (**SafI4**), the mean value  $\mu^3(r)$  of the critical infrastructure impacted by its operation process related to weather changes lifetime  $T^3(r)$  up to exceeding the critical safety state  $r$ , given by (10) for  $u = r$ ,
- the fifth safety indicator (**SafI5**), the standard deviation  $\sigma^3(r)$  of the critical infrastructure impacted by its operation process related to weather changes lifetime  $T^3(r)$  up to exceeding the critical safety state  $r$ , given by (12) for  $u = r$ ,
- the sixth safety indicator (**SafI6**), the moment  $t^3$  of exceeding an acceptable value of the critical infrastructure impacted by its operation process related to weather changes risk function permitted level  $d$ ,  $d \in [0, 1]$ , defined by

$$\tau^3 = (r^3)^{-1}(d), \tag{14}$$

where  $(r^3)^{-1}(t)$ ,  $t \geq 0$ , if it exists, is the second safety indicator  $r^3(t)$ ,  $t \geq 0$ , defined by (9);

- the seventh safety indicator (**SafI7**), the intensities of degradation (ageing) of the critical infrastructure impacted by its operation process related to weather changes, defined as the coordinates of the vector

$$\lambda^3(t, \cdot) = [0, \lambda^3(t, 1), \dots, \lambda^3(t, z)], t \geq 0, \tag{15}$$

where

$$\lambda^3(t, u) = -\frac{\frac{d}{dt}[S^3(t, u)]}{S^3(t, u)}, t \geq 0, u = 1, 2, \dots, z, \tag{16}$$

and  $S^3(t, u)$ ,  $t \geq 0$ ,  $u = 1, 2, \dots, z$ , are evaluated by (8),

- the eighth safety indicator (**SafI8**), the coefficients of the operation process related to weather changes impact on the critical infrastructure intensities of ageing, defined as the coordinates of the vector

$$\rho^3(t, \cdot) = [0, \rho^3(t, 1), \dots, \rho^3(t, z)], t \geq 0, \quad (17)$$

where

$$\rho^3(t, u) = \frac{\lambda^3(t, u)}{\lambda^0(t, u)}, u = 1, 2, \dots, z, \quad (18)$$

and  $\lambda^0(t, u), t \geq 0, u = 1, 2, \dots, z$ , are the intensities of degradation of the critical infrastructure without of any impact and  $\lambda^3(t, u), t \geq 0, u = 1, 2, \dots, z$ , are the intensities of degradation of the critical infrastructure with of its operation process related to weather changes impact.

## 2.2. Resilience indicators

We distinguish three critical infrastructure resilience indicators to the operation process related to weather changes impact:

- the first resilience indicator (**ResI1**) defined by

$$RI1^3(t) = \frac{1}{\rho^3(t, r)}, t \geq 0, \quad (19)$$

where  $\rho^3(t, r), t \geq 0$ , is defined by (18) for  $u = r$ ,

- the second resilience indicator (**ResI2**) defined by

$$RI2^3 = \lim_{t \rightarrow +\infty} \frac{\lambda^0(t, r)}{\lambda^3(t, r)}, \quad (20)$$

where  $\lambda^3(t, r), t \geq 0$ , for  $u = r$  is given by (16) and  $\lambda^0(t, r)$  is the intensity of degradation of the critical infrastructure free of any outside impact,

- the third resilience indicator (**ResI3**) defined by

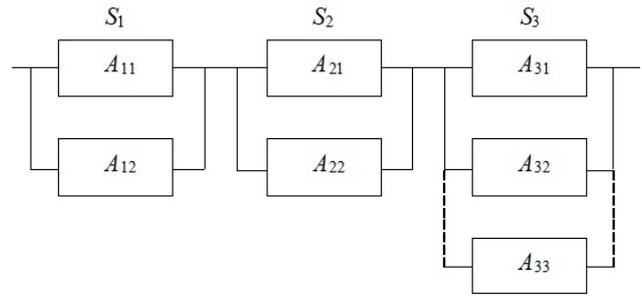
$$RI3^3 = \frac{\mu^3(r)}{\mu^0(r)}, \quad (21)$$

where  $\mu^3(r)$  is the fourth safety indicator and  $\mu^0(r)$  is the mean value of the critical infrastructure free of any outside impact lifetime up to exceeding the critical safety state  $r$ .

## 3. Evaluation of piping critical infrastructure safety and resilience

### 3.1. Necessary data

The considered in previous section critical infrastructure safety model is applied to evaluation of safety and resilience indicators of the port oil piping transportation critical infrastructure. It is operating at the Baltic Oil Terminal that is designated for the reception from ships, the storage and sending by carriages or cars the oil products. It is also designated for receiving from carriages or cars, the storage and loading the tankers with oil products such like petrol and oil. The Baltic Oil Terminal is composed of three parts *A, B* and *C*, linked by the piping transportation critical infrastructure with the pier (Kołowrocki & Soszyńska-Budny, 2011; Torbicki, 2019). The port oil pipeline critical infrastructure safety structure is presented in Figure 1.



**Figure 1.** General scheme of the port oil pipeline critical infrastructure safety structure.

According to Figure 1, the piping critical infrastructure structure is “ $m_i$  out of  $l_i$ ”-series and it is composed of two parallel subcritical infrastructures  $S_1, S_2$ , each containing two pipelines (assets) and one “2 out of 3” subcritical infrastructure  $S_3$  containing 3 pipelines (assets).

The port oil piping critical infrastructure and its assets safety parameters are as follows:

- the number of safety states  $z = 2$  (excluding safety state 0),
- the definitions of safety states:
  - a safety state 2 – the port oil piping critical infrastructure and its assets operation is fully safe,
  - a safety state 1 – the port oil piping critical infrastructure and its assets operation is less safe and more dangerous because of the possibility of environment pollution,

- a safety state 0 – the port oil piping critical infrastructure and its assets are destroyed,
- there are possible the transitions between the piping and its assets safety states only from better to worse ones,
- the critical safety state  $r = 1$ ,
- the port oil piping critical infrastructure risk function permitted level  $d = 0.05$ ,
- the piping assets intensities of ageing:
  - for the asset  $A_{11}$

$$\lambda_{11}^0(1) = 0.00362, \lambda_{11}^0(2) = 0.00540,$$

- for the asset  $A_{12}$

$$\lambda_{12}^0(1) = 0.00362, \lambda_{12}^0(2) = 0.00540,$$

- for the asset  $A_{21}$

$$\lambda_{21}^0(1) = 0.01444, \lambda_{21}^0(2) = 0.02163,$$

- for the asset  $A_{22}$

$$\lambda_{22}^0(1) = 0.01444, \lambda_{22}^0(2) = 0.02163,$$

- for the asset  $A_{31}$

$$\lambda_{31}^0(1) = 0.00730, \lambda_{31}^0(2) = 0.00912,$$

- for the asset  $A_{32}$

$$\lambda_{32}^0(1) = 0.00730, \lambda_{32}^0(2) = 0.00912,$$

- for the asset  $A_{33}$

$$\lambda_{33}^0(1) = 0.00874, \lambda_{33}^0(2) = 0.00984.$$

The piping operation process is described in (Kołowrocki & Soszyńska-Budny, 2011; Torbicki, 2019). It has 7 operation states with the below vector of the limit values of the piping operation process transient probabilities at the operation states

$$[p_b]_{1 \times 7} = [0.403, 0.055, 0.003, 0.002, 0.199, 0.057, 0.281].$$

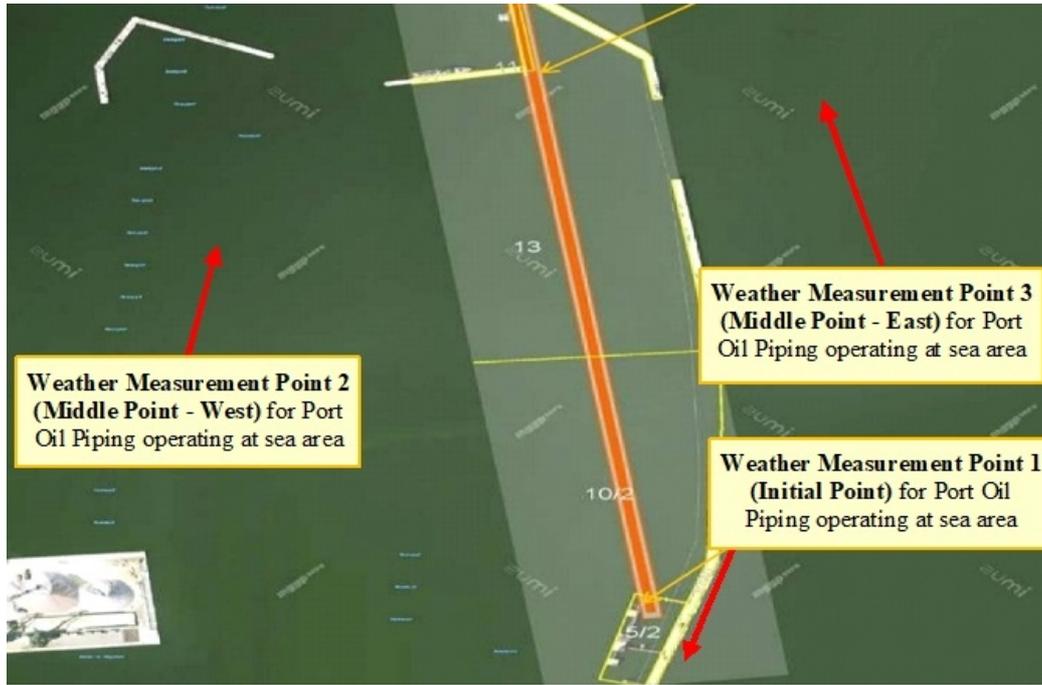
We divide the piping operating area into two parts. The first part is the sea operating area and the second is the land operating area. There are distinguished two different weather change processes for these two areas:

- the weather change process  $C1(t), t \geq 0$ , at the sea operating area (Figures 2–3, the measurement points 1–3),
- the weather change process  $C2(t), t \geq 0$ , at the land operating area (Figure 2, the measurement point 4).

The statistical weather data from points 1–3 are joined and analyzed together due to successful uniformity tested (Kuligowska & Torbicki, 2017).



**Figure 2.** The port oil piping transportation critical infrastructure operating area.



**Figure 3.** The sea area of the port oil piping transportation critical infrastructure.

The definitions of weather change process  $C1(t)$  states are given as follows:

- the weather state  $c1_1$  – the wave height from 0 up to 2 m and the wind speed from 0 m/s up to 17 m/s,
- the weather state  $c1_2$  – the wave height from 2 m up to 5 m and the wind speed from 0 m/s up to 17 m/s,
- the weather state  $c1_3$  – the wave height from 5 m up to 14 m and the wind speed from 0 m/s up to 17 m/s,
- the weather state  $c1_4$  – the wave height from 0 up to 2 m and the wind speed from 17 m/s up to 33 m/s,
- the weather state  $c1_5$  – the wave height from 2 m up to 5 m and the wind speed from 17 m/s up to 33 m/s,
- the weather state  $c1_6$  – the wave height from 5 m up to 14 m and the wind speed from 17 m/s up to 33 m/s.

The weather change process  $C1(t)$  limit transient probabilities are given in the vector:

$$[q1_{\beta1}]_{1 \times 16} = [0.841, 0.151, 0.001, 0, 0.006, 0.001].$$

The definitions of weather change process  $C2(t)$  states are given as follows:

- the weather state  $c21$  – the air temperature from  $-25^\circ\text{C}$  up to  $-15^\circ\text{C}$  and the soil temperature from  $-30^\circ\text{C}$  up to  $-5^\circ\text{C}$ ,

- the weather state  $c2_2$  – the air temperature from  $-15^\circ\text{C}$  up to  $5^\circ\text{C}$  and the soil temperature from  $-30^\circ\text{C}$  up to  $-5^\circ\text{C}$ ,
- the weather state  $c2_3$  – the air temperature from  $5^\circ\text{C}$  up to  $25^\circ\text{C}$  and the soil temperature from  $-30^\circ\text{C}$  up to  $-5^\circ\text{C}$ ,
- the weather state  $c2_4$  – the air temperature from  $25^\circ\text{C}$  up to  $35^\circ\text{C}$  and the soil temperature from  $-30^\circ\text{C}$  up to  $-5^\circ\text{C}$ ,
- the weather state  $c2_5$  – the air temperature from  $-25^\circ\text{C}$  up to  $-15^\circ\text{C}$  and the soil temperature from  $-5^\circ\text{C}$  up to  $5^\circ\text{C}$ ,
- the weather state  $c2_6$  – the air temperature from  $-15^\circ\text{C}$  up to  $5^\circ\text{C}$  and the soil temperature from  $-5^\circ\text{C}$  up to  $5^\circ\text{C}$ ,
- the weather state  $c2_7$  – the air temperature from  $5^\circ\text{C}$  up to  $25^\circ\text{C}$  and the soil temperature from  $-5^\circ\text{C}$  up to  $5^\circ\text{C}$ ,
- the weather state  $c2_8$  – the air temperature from  $25^\circ\text{C}$  up to  $35^\circ\text{C}$  and the soil temperature from  $-5^\circ\text{C}$  up to  $5^\circ\text{C}$ ,
- the weather state  $c2_9$  – the air temperature from  $-25^\circ\text{C}$  up to  $-15^\circ\text{C}$  and the soil temperature from  $5^\circ\text{C}$  up to  $20^\circ\text{C}$ ,
- the weather state  $c2_{10}$  – the air temperature from  $-15^\circ\text{C}$  up to  $5^\circ\text{C}$  and the soil temperature from  $5^\circ\text{C}$  up to  $20^\circ\text{C}$ ,
- the weather state  $c2_{11}$  – the air temperature from  $5^\circ\text{C}$  up to  $25^\circ\text{C}$  and the soil temperature from  $5^\circ\text{C}$  up to  $20^\circ\text{C}$ ,

- the weather state  $c_{212}$  – the air temperature from 25°C up to 35°C and the soil temperature from 5°C up to 20°C,
- the weather state  $c_{213}$  – the air temperature from –25°C up to –15°C and the soil temperature from 20°C up to 37°C,
- the weather state  $c_{214}$  – the air temperature from –15°C up to 5°C and the soil temperature from 20°C up to 37°C,
- the weather state  $c_{215}$  – the air temperature from 5°C up to 25°C and the soil temperature from 20°C up to 37°C,
- the weather state  $c_{216}$  – the air temperature from 25°C up to 35°C and the soil temperature from 20°C up to 37°C.

The weather change process  $C2(t)$  limit transient probabilities are given in the vector:

$$[q_{2\beta 2}]_{1 \times 16} = [0, 0.026, 0, 0, 0, 0.277, 0.014, 0, 0, 0.008, 0.612, 0, 0, 0, 0.062, 0.001].$$

The coefficients of the operation process  $Z(t)$  impact on the piping assets intensities of ageing at the operation states  $z_b$ ,  $b = 1, 2, \dots, 7$ , are as follows:

- for asset  $A_{11}$

$$[\rho_{11}^1(1)]^{(b)} = 1.00, [\rho_{11}^1(2)]^{(b)} = 1.00, b = 1, 2, 7,$$

$$[\rho_{11}^1(1)]^{(b)} = 1.20, [\rho_{11}^1(2)]^{(b)} = 1.20, b = 3, 4, 5, 6,$$

- for asset  $A_{12}$

$$[\rho_{12}^1(1)]^{(b)} = 1.00, [\rho_{12}^1(2)]^{(b)} = 1.00, b = 1, 2, 7,$$

$$[\rho_{12}^1(1)]^{(b)} = 1.20, [\rho_{12}^1(2)]^{(b)} = 1.20, b = 3, 4, 5, 6,$$

- for asset  $A_{21}$

$$[\rho_{21}^1(1)]^{(b)} = 1.00, [\rho_{21}^1(2)]^{(b)} = 1.00, b = 1, 2, 7,$$

$$[\rho_{21}^1(1)]^{(b)} = 1.20, [\rho_{21}^1(2)]^{(b)} = 1.20, b = 3, 4, 5, 6,$$

- for asset  $A_{22}$

$$[\rho_{22}^1(1)]^{(b)} = 1.00, [\rho_{22}^1(2)]^{(b)} = 1.00,$$

$$b = 1, 2, 7,$$

$$[\rho_{22}^1(1)]^{(b)} = 1.20, [\rho_{22}^1(2)]^{(b)} = 1.20, b = 3, 4, 5, 6,$$

- for asset  $A_{31}$

$$[\rho_{31}^1(1)]^{(b)} = 1.00, [\rho_{31}^1(2)]^{(b)} = 1.00, b = 3, 5,$$

$$[\rho_{31}^1(1)]^{(b)} = 1.20, [\rho_{31}^1(2)]^{(b)} = 1.20, b = 1, 2, 4, 6, 7,$$

- for asset  $A_{32}$

$$[\rho_{32}^1(1)]^{(b)} = 1.00, [\rho_{32}^1(2)]^{(b)} = 1.00, b = 3, 5,$$

$$[\rho_{32}^1(1)]^{(b)} = 1.20, [\rho_{32}^1(2)]^{(b)} = 1.20, b = 1, 2, 4, 6, 7,$$

- for asset  $A_{33}$

$$[\rho_{33}^1(1)]^{(b)} = 1.00, [\rho_{33}^1(2)]^{(b)} = 1.00, b = 3, 5,$$

$$[\rho_{33}^1(1)]^{(b)} = 1.20, [\rho_{33}^1(2)]^{(b)} = 1.20, b = 1, 2, 4, 6, 7.$$

The coefficients of the weather change processes  $C1(t)$ ,  $C2(t)$ ,  $t \geq 0$ , impact on the piping assets intensities of ageing at the weather change processes states are as follows:

- for asset  $A_{11}$

$$[\rho_{11}^2(1)]^{(\beta 1)} = 1.00, [\rho_{11}^2(2)]^{(\beta 1)} = 1.00, \beta 1 = 1, 2,$$

$$[\rho_{11}^2(1)]^{(\beta 1)} = 1.036, [\rho_{11}^2(2)]^{(\beta 1)} = 1.048, \beta 1 = 3, 5, 6,$$

$$[\rho_{11}^2(1)]^{(\beta 2)} = 1.00, [\rho_{11}^2(2)]^{(\beta 2)} = 1.00, \beta 2 = 2, 6, 7, 10, 11, 15, 16,$$

- for asset  $A_{12}$

$$[\rho_{12}^2(1)]^{(\beta 1)} = 1.00, [\rho_{12}^2(2)]^{(\beta 1)} = 1.00, \beta 1 = 1, 2,$$

$$[\rho_{12}^2(1)]^{(\beta 1)} = 1.036, [\rho_{12}^2(2)]^{(\beta 1)} = 1.048, \beta 1 = 3, 5, 6,$$

$$[\rho 2_{12}^2(1)]^{(\beta 2)} = 1.00, [\rho 2_{12}^2(2)]^{(\beta 2)} = 1.00, \beta 2 = 2,6,7,10,11,15,16,$$

- for asset  $A_{21}$

$$[\rho 1_{21}^2(1)]^{(\beta 1)} = 1.00, [\rho 1_{21}^2(2)]^{(\beta 1)} = 1.00, \beta 1 = 1,2,3,5,6,$$

$$[\rho 2_{21}^2(1)]^{(\beta 2)} = 1.00, [\rho 2_{21}^2(2)]^{(\beta 2)} = 1.00, \beta 2 = 6,7,10,11,$$

$$[\rho 2_{21}^2(1)]^{(\beta 2)} = 1.004, [\rho 2_{21}^2(2)]^{(\beta 2)} = 1.007, \beta 2 = 2,15,16;$$

- for asset  $A_{22}$

$$[\rho 1_{22}^2(1)]^{(\beta 1)} = 1.00, [\rho 1_{22}^2(2)]^{(\beta 1)} = 1.00, \beta 1 = 1,2,3,5,6,$$

$$[\rho 2_{22}^2(1)]^{(\beta 2)} = 1.00, [\rho 2_{22}^2(2)]^{(\beta 2)} = 1.00, \beta 2 = 6,7,10,11,$$

$$[\rho 2_{22}^2(1)]^{(\beta 2)} = 1.004, [\rho 2_{22}^2(2)]^{(\beta 2)} = 1.007, \beta 2 = 2,15,16,$$

- for asset  $A_{31}$

$$[\rho 1_{31}^2(1)]^{(\beta 1)} = 1.00, [\rho 1_{31}^2(2)]^{(\beta 1)} = 1.00, \beta 1 = 1,2,3,5,6,$$

$$[\rho 2_{31}^2(1)]^{(\beta 2)} = 1.00, [\rho 2_{31}^2(2)]^{(\beta 2)} = 1.00, \beta 2 = 2,6,7,10,11,15,16,$$

- for asset  $A_{32}$

$$[\rho 1_{32}^2(1)]^{(\beta 1)} = 1.00, [\rho 1_{32}^2(2)]^{(\beta 1)} = 1.00, \beta 1 = 1,2,3,5,6,$$

$$[\rho 2_{32}^2(1)]^{(\beta 2)} = 1.00, [\rho 2_{32}^2(2)]^{(\beta 2)} = 1.00, \beta 2 = 2,6,7,10,11,15,16,$$

- for asset  $A_{33}$

$$[\rho 1_{33}^2(1)]^{(\beta 1)} = 1.00, [\rho 1_{33}^2(2)]^{(\beta 1)} = 1.00, \beta 1 = 1,2,3,5,6,$$

$$[\rho 2_{33}^2(1)]^{(\beta 2)} = 1.00, [\rho 2_{33}^2(2)]^{(\beta 2)} = 1.00, \beta 2 = 2,6,7,10,11,15,16.$$

The exemplary piping free of any outside impacts safety indicators, necessary to determine the pip

ing impacted by operation and weather changes resilience indicators, are:

- the mean value  $\mu^0(r)$  of the piping free of any outside impacts lifetime  $T^0(r)$  up to exceeding the critical safety state  $r = 1$ , given by:

$$\mu^0(1) = 62.5692 \text{ years}, \quad (22)$$

- the limit values of the piping free of any outside impacts degradation intensities given by

$$\lambda^0(1) = \lim_{t \rightarrow +\infty} \lambda^0(t, 1) = 0.03271 \left[ \frac{1}{\text{year}} \right],$$

$$\lambda^0(2) = \lim_{t \rightarrow +\infty} \lambda^0(t, 2) = 0.04533 \left[ \frac{1}{\text{year}} \right]. \quad (23)$$

### 3.2. Piping safety indicators

We can determine the piping safety and resilience indicators using the introduced in previous section safety model. The first safety indicator (**SafI1**) of the piping critical infrastructure impacted by its operation process related to weather changes is given by the vector

$$\mathbf{S}^3(t, \cdot) = [\mathbf{1}, \mathbf{S}^3(t, 1), \mathbf{S}^3(t, 2)], t \geq 0, \quad (24)$$

with the coordinates evaluated by

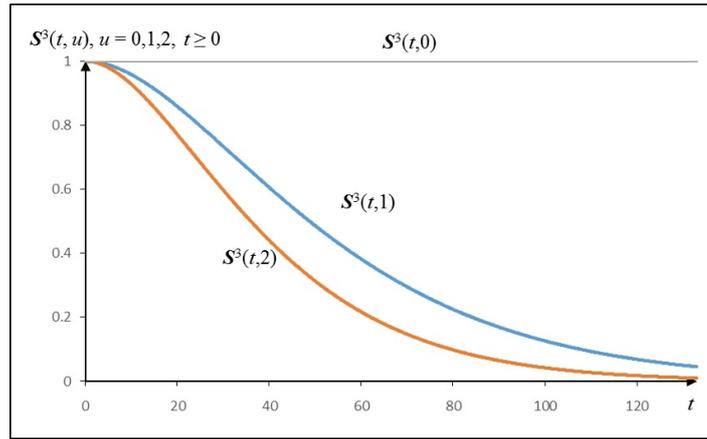
$$\mathbf{S}^3(t, u) \cong$$

$$\sum_{b=1}^7 \sum_{\substack{\beta 1=1,2,\dots,6 \\ \beta 2=1,2,\dots,16}} p_b q_{1\beta 1} q_{2\beta 2} [\mathbf{S}^3(t, u)]^{b, (\beta 1, \beta 2)} [\text{year}], \quad (25)$$

$$t \geq 0, u = 1, 2, \dots, z,$$

where  $[\mathbf{S}^3(t, u)]^{b, (l1, l2)}$ ,  $t \geq 0$ ,  $u = 1, 2$ ,  $b = 1, 2, \dots, 7$ ,  $\beta 1 = 1, 2, \dots, 6$ ,  $\beta 2 = 1, 2, \dots, 16$ , are the coordinates of the piping critical infrastructure impacted by its operation process related to weather changes  $\bar{Z}C(t)$ ,  $t \geq 0$ , conditional safety function,  $p_b$ ,  $b = 1, 2, \dots, 7$ , are the piping operation process  $Z(t)$ ,  $t \geq 0$ , limit transient probabilities at the operation states  $z_b$ ,  $b = 1, 2, \dots, 7$ ,  $q_{1\beta 1}$ ,  $\beta 1 = 1, 2, \dots, 6$ , are limit transient probabilities of the weather change process  $C1(t)$ ,  $t \geq 0$ , and  $q_{2\beta 2}$ ,  $\beta 2 = 1, 2, \dots, 16$ , are limit transient probabilities of the weather change process  $C2(t)$ ,  $t \geq 0$ .

The graphs of the piping unconditional safety function coordinates are shown in Figure 4.

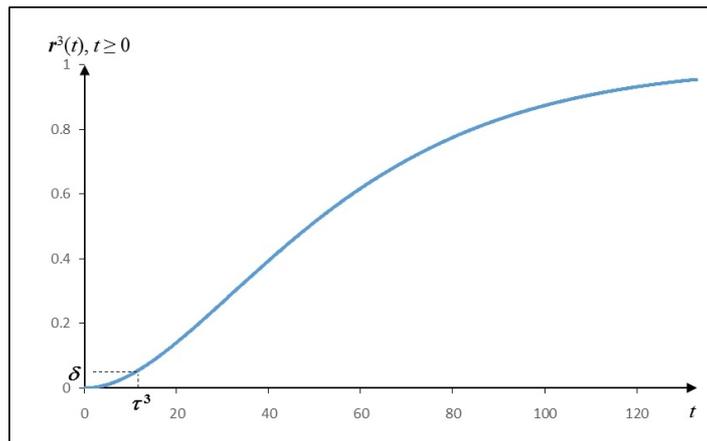


**Figure 4.** The graphs of the piping impacted by its operation process related to weather changes safety function coordinates.

According to (9), the second piping safety indicator (**SafI2**) is its risk function given by

$$r^3(t) = 1 - S^3(t, 1) \text{ [year]}, t \geq 0, \quad (26)$$

where  $S^3(t, 1), t \geq 0$ , is evaluated by (25). The third piping safety indicator (**SafI3**), the fragility curve, is presented in Figure 5.



**Figure 5.** The fragility curve of the piping impacted by its operation process related to weather changes.

The mean values and standard deviations of the piping unconditional lifetimes (**SafI4**)–(**SafI5**) are:

- in the safety state subset {1, 2}:

$$\begin{aligned} \mu^3(1) &\cong 56.7439 \text{ years,} \\ \sigma^3(1) &\cong 38.0292 \text{ years,} \end{aligned} \quad (27)$$

- in the safety state subset {2}:

$$\begin{aligned} \mu^3(2) &\cong 41.8663 \text{ years,} \\ \sigma^3(2) &\cong 28.0922 \text{ years.} \end{aligned} \quad (28)$$

The mean lifetimes  $\bar{\mu}^3(u), u = 1, 2$ , of the piping in the particular safety states using (13) to (27)–(28) are:

$$\begin{aligned} \bar{\mu}^3(1) &= 14.8776 \text{ years,} \\ \bar{\mu}^3(2) &= 41.8663 \text{ years.} \end{aligned} \quad (29)$$

The moment  $\tau^3$  of exceeding an acceptable value of the critical infrastructure risk function level  $d = 0.05$ , according to (14) is given by

$$\tau^3 = (r^3)^{-1}(0.05) = 10.9838 \text{ years.} \quad (30)$$

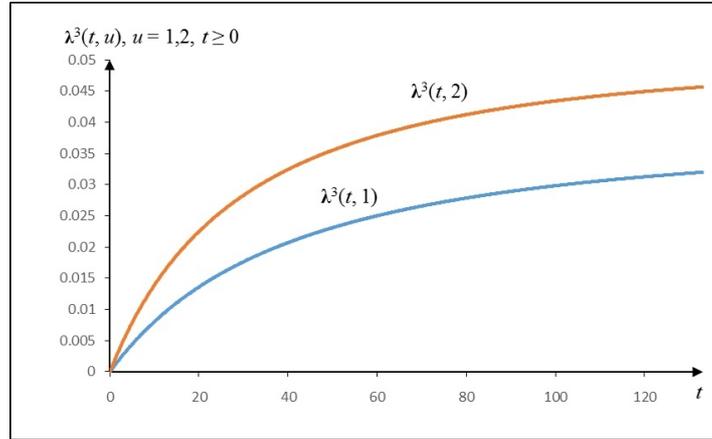
The limit values of the piping degradation intensities are as follows

$$\lambda^3(1) = \lim_{t \rightarrow +\infty} \lambda^3(t, 1) = 0.03563 \left[ \frac{1}{\text{year}} \right],$$

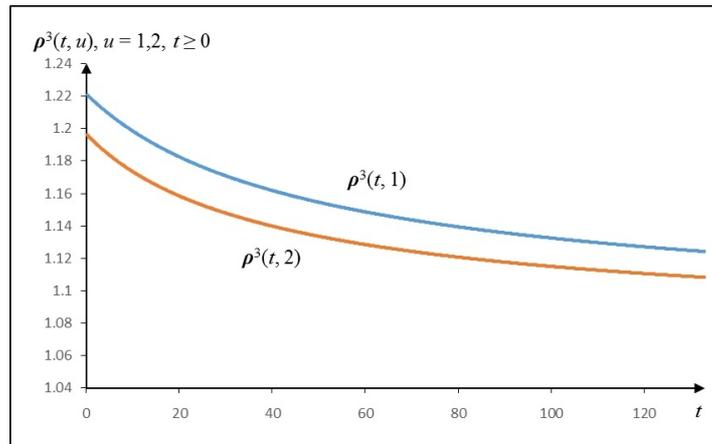
$$\lambda^3(2) = \lim_{t \rightarrow +\infty} \lambda^3(t, 2) = 0.048966 \left[ \frac{1}{\text{year}} \right]. \quad (31)$$

The graphs of the piping degradation intensities are shown in Figure 6.

The graphs of coefficients of the operation process related to weather changes impact on the piping degradation intensities of ageing, are presented in Figure 7.



**Figure 6.** The graphs of the ageing intensities of the piping impacted by the operation process related to weather changes.



**Figure 7.** The graphs of coefficients of the operation process related to weather changes impact on the piping degradation.

### 3.3. Piping resilience indicators

The piping second resilience indicator (**ResI2**) using (20) to (23) and (31) is given by

$$RI2^3 = \lim_{t \rightarrow +\infty} \frac{\lambda^0(t,1)}{\lambda^3(t,1)} = \frac{0.03271}{0.035630} \cong 0.918. \quad (32)$$

The third resilience indicator (**ResI3**) of the piping to the operation process related to weather changes impact using (21) to (22) and (27) is

$$RI3^3 = \frac{56.7439}{62.5692} \cong 0.9069. \quad (33)$$

### 3.4. Comparison of results for three safety models

If we replace all values of coefficients of the weather impact on critical infrastructure asset intensities of ageing fixed with experts and operators by the value 1 in the formula (1), then we receive a safety model of the critical infrastructure impacted by only its operation process (the safety model 1). Similarly, we can construct a safety model of the critical infrastructure impacted by only weather changes (the safety model 2). Using

these models, we can evaluate the safety and resilience indicators of the considered piping critical infrastructure in the case when the piping is impacted by only its operation process and in the

case when piping is impacted by only weather change processes at its operating area. The comparison of received values of indicators is presented in Table 1 [Torbicki, 2019].

**Table 1.** Piping critical infrastructure safety and resilience indicators

The safety model ID $i, i = 0,1,2,3$	Piping critical infrastructure safety and resilience indicators						
	$\mu^i(1)$ in years	$\mu^i(2)$ in years	$\sigma^i(1)$ in years	$\sigma^i(2)$ in years	$\tau^i$ in years	$RI2^i$	$RI3^i$
0	62.5692	45.8198	41.8793	30.7346	12.1328	–	–
1	56.7545	41.8811	38.0357	28.1014	10.9868	0.918	0.9070
2	62.5574	45.8030	41.8715	30.7239	12.1304	1	0.9998
3	56.7439	41.8663	38.0292	28.0922	10.9838	0.918	0.9069

The results in the above table are showing that the operation process has the undoubted impact on the piping safety and resilience, while the weather changes impact on the piping safety and resilience is very small. This occur due to fact that in the opposite to the operation process, the probability of occurring dangerous states of the weather changes processes are very small and coefficients of weather change processes impacts on the piping assets safety parameters are nearly equal 1 even for most dangerous weather states. The operation process related to weather changes impact on the piping safety and resilience is noticeable because of the influence of the operation process.

**4. Conclusion**

The values of evaluated safety and resilience indicators of the port oil piping transportation critical infrastructure impacted by changing operating and weather conditions are compared with values of its safety and resilience indicators without of the operating and/or weather process impact. The differences between them justify that the presented approach to the safety and resilience analysis is correct and it has a practical value for users and operators of real critical infrastructures.

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