Influence of the slipe effect on simply supported composite beam

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The work concerns the slip effect which may occur in simply supported composite beams. The occurrence of the slip effect may lead to additional increase in beam deflection. The main reason for the increase is susceptibility of the anchor bolts that join the beam. The analysis of the slip effect occurrence is based on the following criteria: steel and concrete are linearly elastic, junction goes along the whole length of the considered beam, concrete slab and steel rib move by the same value along the whole length of the beam, simply supported beams are analysed. In order to present the slip effect in composite beams and its results, a numerical example is used. The example shows three kinds of the load capacity of anchor bolts as well as their impact on the slip effect and deflection of steel-concrete composite beams.

Keywords and phrases: slip effect, composite beams, simply supported beams.

Introduction

Popularity of composite structures has increased considerably thanks to their properties, which are tensile strength of steel materials and compressive strength of concrete. Composite steel-concrete load-bearing structures are shaped by appropriate steel structural elements with concrete (reinforced or compressed). In bending elements, floor slabs connected with beams and binding joists appear most frequently.

Thanks to such shape, steel consumption decreases by using compressive strength of concrete, so in span beams. The appropriate steel-concrete composition, thanks to the properties of steel and concrete, gives designers a wide range of possibilities. In addition, it provides increased rigidity and resistance to loss of stability of the structure. Basically, composite beams and binding joists have T-section profiles. This system consists of steel ribs and reinforced concrete slab.

In order to explain the mechanism of design of the above-mentioned beam construction, according to [1], it is essential to indicate how the slab behaves in positive and negative torque. In the first case, the slab is compressed and tensile force is taken over by the steel rib, in particular by its lower part. In turn, in negative torque, the slab will be used only when concrete tensile forces are balanced. This can be done by compression. However, this solution is hardly ever applied. Slab compression is justified only in case of heavily loaded, wide-span constructions, such as composite bridges. In conventional structures composite beams usually take the form of simply supported beams.

Terms of cooperation between steel and concrete while shaping simply supported composite beams

While designing composite components it should be taken into consideration that in bending elements steel and concrete can interact only if they are protected against delamination. The beam consisting of two unlinked materials will deform under load as a result of independent work of each of the two beams.

In order to design the beam optimally, both layers need to cooperate. A lot of information about the composite beams can be found in work [3], where two variants of beams have been distinguished. Variant I presents steady combination of elements and variant II illustrates free displacements.

This figure clearly indicates that in variant I, normal stress is twice smaller than in variant II.

The deflection is almost four times smaller, while the maximum tangential stresses τ do not change. supported beams.

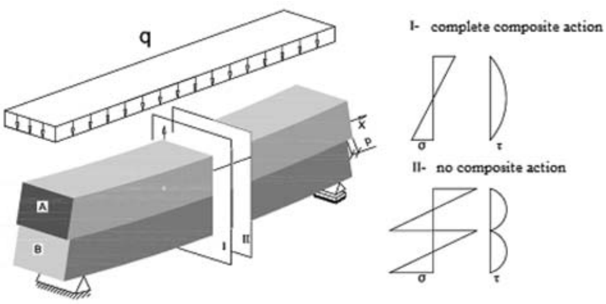

Fig. 1. Variants of simply supported beam.

The phenomenon of slip effect at the joint of the composite cross section

According to the theory of elasticity, the beam composed of two layers unlinked with each other in a solid way, with unit longitudinal deformation ε_s in the direction of the x-axis of the lower fibres of the top layer, equals deformation of top fibres of the lower layer. The deformations have the same values, but they are opposite directed. As a result there is a slip deformation at the value of 2 ε_p. Similar to deformation, which for the SGU is the ratio of change of displacements, such slip deformation is the ratio of changes in slip along the beam axis. Assuming the elastic range of the beam, it can be concluded that the shear stress at the joint point of two materials is proportional to the slip.

The slip effect in a non-composite beam with no deformations is well interpreted in Fig. 2.

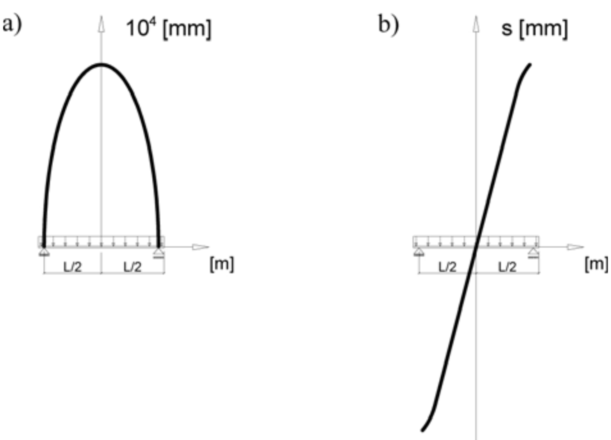

Fig. 2. a — graph of slip deformation, b — graph of slip in a non-composite beam.

The slip effect at the joint of the ceiling slab and top shelf

The most optimal solution, which is now widely used, is a composite action with bolt anchors. This method allows for quick realization and effective cooperation of cross sections at the same time. Experimental studies have shown that this composite action is very practical, although full cooperation of both materials is not always achieved, which depends on susceptibility of the bolts.

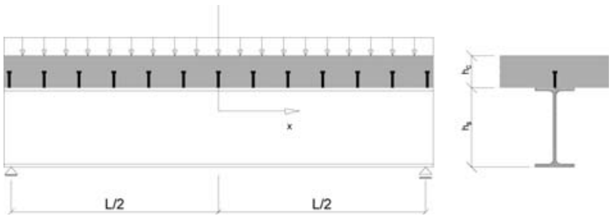

Fig. 3. Typical composite beam charged with continuous load.

The composite action of the ceiling slab to the steel beam occurs by bolt anchors welded to the top shelf of the steel beam. Thus, the equation (1) that satisfies the condition of rigidity in the direction of the x-axis of the composite ceiling beam (according to Fig. 1, variant I) has been formulated.

$$l\tau = KP \tag{1}$$

where: l — distance between anchor bolts,
τ — shear stress vector,
K — load shear capacity of bolts (composite cross section),
P — slip.

Slip deformation in a steel cross section and in a slab is presented by a graph in Fig. 2 according to [2].

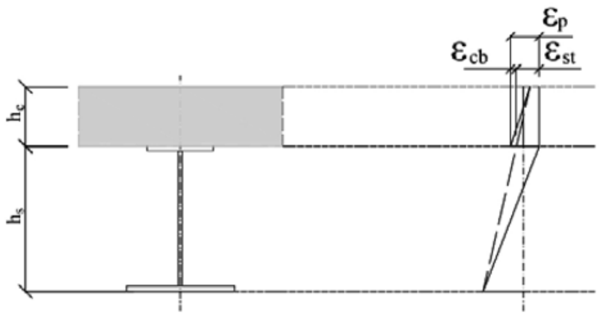

Fig. 4. Graph of slip deformation of a beam composed of two layers.

A condition of simply supported beam equilibrium (Fig. 4) on finite length dx in the horizontal direction can be formulated by cutting part of the beam in (Fig. 1) of a length dx.

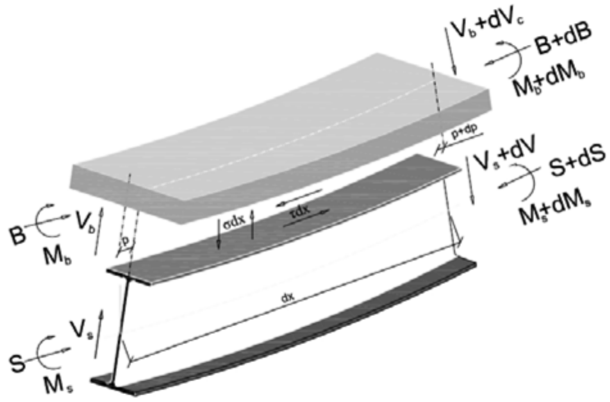

Fig. 5. Graph of slip deformation of the analyzed beam.

The example discussed in this work was based on the following assumptions:

- steel and concrete are linearly elastic for stretching and compression;
- composite action is continuous on the length dx;
- slip on the length dx of the deformed element is directly proportional to the load of bolt anchors;
- reinforced concrete slab and secondary beam are displacing of the same values on the entire length of the beam;
- the adopted analysis applies only to simply supported beam.

The condition of longitudinal force equilibrium of a section of composite beam loaded as shown in the Fig. 3 on the length dx can be written in the form of formulas 2 and 3, as shown in the Fig. 3:

$$\frac{\mathrm{dB}}{\mathrm{dx}} = \frac{\mathrm{dS}}{\mathrm{dx}} = -\tau \tag{2}$$

where: B — compression in the concrete slab,
S — stretching in the steel cross section.

The condition of equilibrium in vertical direction:

$$V_b + V_S = 0{,}5Q \tag{3}$$

where: V_b — shear strength in the slab,
V_s — shear strength in the steel profile,
Q — continuous load on a length dx.

According to the equilibrium of bending moments, if we treat treating reinforced concrete slabs and steel cross section separately as the two centres, we obtain as follows:

$$\frac{dM_B}{dx} = -V_c + \frac{\tau \cdot y_{cb}}{2} - \sigma dx \tag{4}$$

$$\frac{dM_S}{dx} = -V_s + \frac{\tau \cdot y_{st}}{2} - \sigma dx \tag{5}$$

where: M_B — moment in reinforced concrete slab;
M_S — moment in steel cross section, load;
Σ — normal stress between the slab and the steel beam;
y_{cb} — distance from the lower surface of the slab to the neutral axis of the concrete profile;
y_{st} — distance from the joint of the surface of the slab with the steel cross section to neutral axis of steel cross section.

The compound binding the curvature of the slab and steel beam with bending moment in these two centres can be derived while analyzing lateral bending aspect, (6).

$$\varphi = \frac{M_S}{E_S I_S} = \frac{M_B}{E_B I_B} \tag{6}$$

where: E_B — modulus of elasticity of concrete,
E_S — modulus of elasticity of steel,
I_S — moment of inertia of steel beam,
I_B — moment of inertia of a slab profile.

$$d\varphi = \frac{-0{,}5Q + \tau \cdot d_b}{E_S I_S + E_B I_B} \tag{7}$$

where: $d_b = y_{cb} + y_{st}$

Considering the surface of the joint point of concrete and steel, strain equations in these two centres, i.e., in reinforced concrete slab and in steel beam, can be written, as well as in (ε_{st}):

— lower fibres of the slab (ε_{cb})

$$\varepsilon_{cb} = -\frac{M_B y_{cb}}{E_S I_S} - \frac{B}{E_B A_B} = \varphi \cdot y_{cb} - \frac{B}{E_B A_B} \tag{8}$$

— top fibres of the steel section (ε_{st}):

$$\varepsilon_{st} = -\frac{M_S y_{st}}{E_S I_S} + \frac{S}{E_S A_S} = -\varphi \cdot y_{cb} - \frac{B}{E_B A_B} \tag{9}$$

Slip deformation is the same as the difference in deformations of the concrete slab and steel beam and it was included in the formula (10) according to [2].

$$\frac{dp}{dx} = \varepsilon_p = \varepsilon_{cb} - \varepsilon_{st} = \varphi \cdot d_b - \frac{S}{E_s A_s} - \frac{B}{E_B A_B} \tag{10}$$

Having transformed formulas (4–10) appropriately, we obtain a second order differential equation. In order to obtain the solution of the equation (11), appropriate boundary conditions need to be asked.

$$\frac{d^2 p}{dx} - \alpha^2 p + \frac{\alpha^2 \beta Q}{2} = 0 \tag{11}$$

The equation 12 includes the coefficients α, β, which are the extensions after the transformation of the equations (1), (2) and (7) on the slip boundary according to [2].

where: $\alpha = \dfrac{K}{A_1 E_s I_O p}$, $\beta = \dfrac{A_1 d_b p}{K}$, $A_1 = \dfrac{A_0}{(I_0 + A_0 d_b^2)}$,

The boundary conditions for the steel and concrete simply supported beam from Fig. 3 are as follows:

P = 0 i.e. for *x = 0* (support) and *dp/dx*
for *x = L/2 (where L — effective length of a beam).*

Thanks to characteristic points of simply supported beam, we obtain a slip between $0 \le x \le L/2$.

The solution of the differential equation (11) in the above interval is the value of the slip. While determining the graph of the slip, Mathcad package was used. Using the interface of the program, the values of the slip in successive points of the interval were calculated on the length $0 \le x \le L/2$. The graphs and equations were compared to the works [2, 3]. The value of the slip in following points of the above interval is as follows:

$$P = \frac{\beta q x (1 + e^{-\alpha L} - e^{\alpha x - \alpha L} - e^{-\alpha x})}{2(1 + e^{-\alpha L})} \tag{12}$$

In aid of the equation (10) the slip deformation in the same section, i.e. in the interval $0 \le x \le L/2$, can be obtained.

$$\varepsilon_p = \frac{\alpha \beta q x (e^{-\alpha x} - e^{\alpha x - \alpha L})}{2(1 + e^{-\alpha L})} \tag{13}$$

Assuming that the concrete slab and secondary beam are moving with the same values for the entire length of the beam, the angle of deflection for the steel plate and steel section is identical, as it was illustrated in Fig. 5.

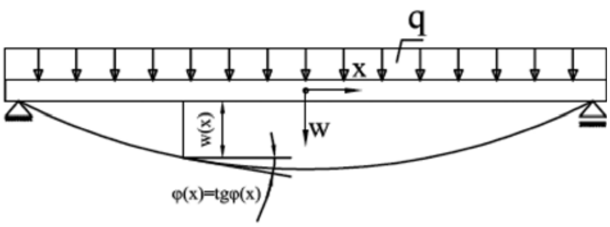

Fig. 6. The curvature of deflection of deformed beam due to applied load.

The elements that determine the curvature of slip are deformations in concrete and steel. They are proportional to their height of the cross section:

$$\Delta\varphi = \frac{\varepsilon_{cs}}{h_c} = \frac{\varepsilon_{st}}{h_s} = \frac{\varepsilon_p}{h} \tag{14}$$

where: h_c — height of the concrete slab,
h_s — height of the steel section,
h — height of the composite section.

According to the National Annex A1.4 [3] to the standard EN 1990, there are methods of measuring limited displacements for simply supported beams. In turn, in the National Annex PN-EN 1993-1-1, deflection limits are specified.

In a situation when the increase in assembly was not used in the assembly, then w_{max} should not exceed L/250, where L-span beam. In most of the constructions, simply supported beams are subjected to continuous loads.

In the above example, deflection increment to an existing arrow w_{max} was applied, which was caused by the slip effect on the joint point of the composite elements. Moreover, numerical part of the analysis of simply supported beam was included, as shown in Fig. 6. For this purpose, mathematical packages of Mathcad and Matlab were used. Slip and influence of its effect on the arrow of composite beam deflection from Fig. 7 for different degrees of using the capacity of the beam were analyzed.

Numerical example of simply supported beam

In numerical example of simply supported beam, numerical part of the beam analysis with a length of 7 m according to Fig. 3 was included. For this purpose, mathematical packages of Mathcad were used. Slip and

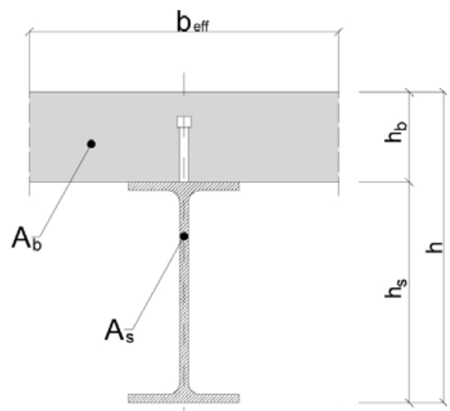

Fig. 7. Cross-section of the analyzed beam.

Table 1. Geometric characteristics of the I-beam IPE 270:

A_a cm^2	I_x cm^4	I_y cm^4	W_x cm^3	W_y cm^3	i_x cm	i_y cm	m kg/m
45.9	5790	420	429	62.2	11.2	3.02	36.1

Table 2. Geometric characteristics of the analyzed composite beam.

Concrete slab	Steel beam	Composition	Composite beam length and load
h_b = 100 mm b_{eff} = 1740 mm E_c = 30 GPa $E_{c,eff}$ = 15 GPa	h_s = 270 mm E_s = 205 GPa	l = 7cm $P_{Rd(min)}$ = 65kN n_f — 50 szt	L = 7 m 1. q_{k1} = 12 kN/m; (q_1 = 15,8 kN/m); [0,5] 2. q_{k2} = 18 kN/m; (q_2 = 23,8 kN/m); [0,75] 3. q_{k3} = 24 kN/m (q_3 = 31,7 kN/m); [1,0]

where: l — spacing od anchor bolts,
P_{Rdmin} — minimum bolt shear strength,
n_f — the required number of bolts on the shear length $L_s = 3.5m$
q_k — characteristic load,
q_k — effective load,
numbers in brackets indicate the degree of utilization of cross-section (capacity of composite beam).

influence of its effect on the arrow of composite beam deflection from Fig. 7 were analyzed. The coordinate system x, y according to Fig. 3 was chosen.

According to the differential equation (12), the coefficients α and β are required for the analysis. They form geometric dependences of two cross-sections (centres), i.e. concrete slab and steel rib IPE 270. In order to compose the beam, anchor bolts that are welded directly to the I-beam shelf, were applied. The bolts with a diameter d = 19 mm and height h = 80 mm.

According to [5–7] minimum bolt shear strength was matched. Stratification force for the width b_{eff} was calculated.

The solve block was based on a second order differential equation (11) with the following initial conditions:

$$p''(x) - \alpha^2 p(x) = -\left(\beta \cdot g \cdot x \cdot \frac{\alpha^2}{2}\right) \qquad p(0) = 0 \qquad p'(3.5) = 0$$

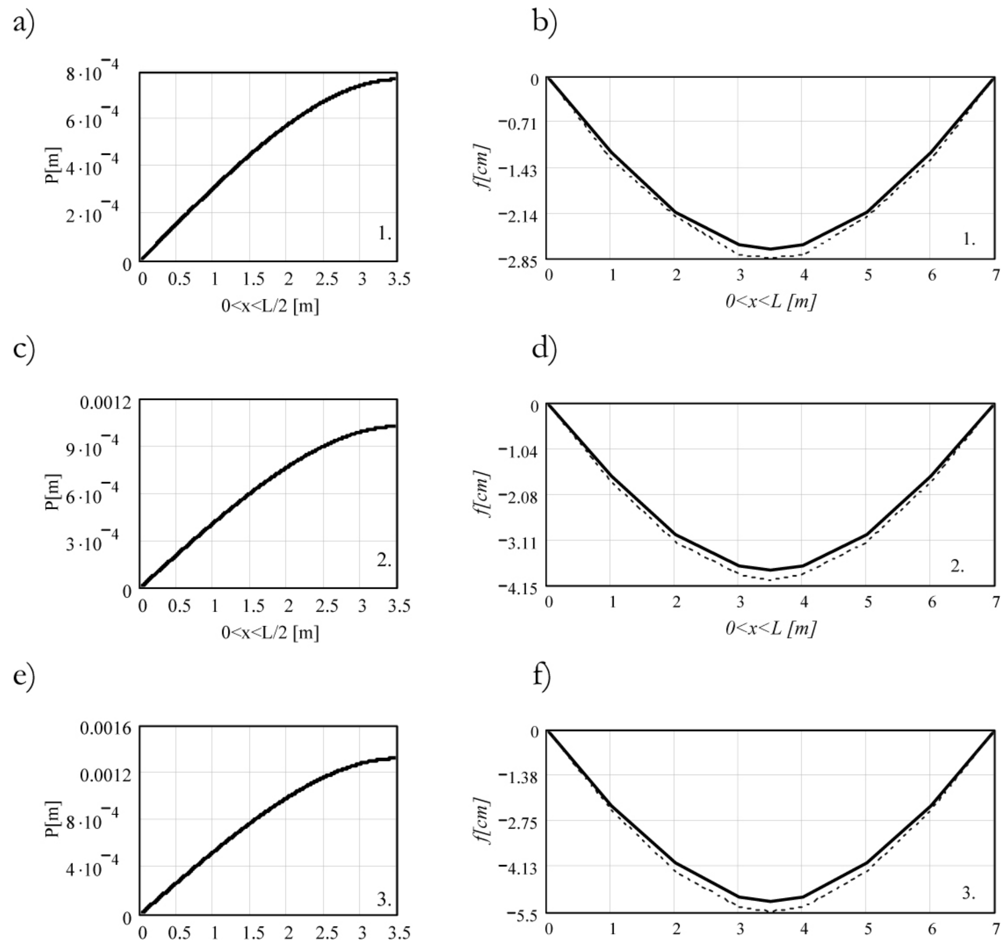

Fig. 8. Graph of slip for the load from the table 2.: 1(a), 2(c), 3(e), arrows of beam deflection for the load from the table 2: 1(b), 2(d), 3(f). Line (---) determines the increase of deflection due to the slip.

In the present example, the beam is dimensioned only in SGN. Only the deflection arrow from the Fig. 8 b) fulfills the SGU requirements. The dashed line on graphs Fig. 8. (b, d, f) indicates the increase in deflection caused by the slip. This increase can be calculated as in [8, 9]. In the case of the analyzed beam, the shape of the graph (dashed line) corresponds to the relation (16) by [2], which is associated with the increase in angle of deflection according to the formula (15).

$$\Delta f = \beta g L^2 \left[\frac{1}{8h} + \frac{2e^{\frac{(\alpha L)}{2}} - e^{\alpha L}}{\alpha^2 h L^2 \left[e^{\alpha L} + 1 \right]} \right] \qquad (15)$$

Given that the form of the exponential function $e^{-\alpha L}$ tends to zero with increasing L, therefore the formula may take the form (16).

$$\Delta f = \beta g L^2 \left[\frac{1}{8h} + \frac{2}{\alpha^2 L^2 h} \right] \qquad (16)$$

Increase in deflection arrows of different types of loads are presented in works [2, 3, 8, 9]. Numerical example that was considered proves that as far as the capacity of the interval 0.5–1 exhausts (the result of increased workload g), the additional arrow of deflection caused by the slip grows almost twice (Δf for 50% of capacity of the cross-section — 0,151 cm to 0,301 cm for 100% use of bending capacity of the cross-section).

Conclusion

This article presents the slip phenomenon occurring at the joint point of the concrete slab and top shelf of steel section. The composition, taking the form of anchor bolts welded the top shelf of steel section may lead to a slip because of susceptibility of the bolts.

In the numerical analysis based on differential equations, deflection increment caused by increase in slip was examined. Additional deflection arrow does not exceed 3 mm, even when the beam capacity reaches 100% of use of the cross-section. Therefore, it can be assumed that the bolts of this type are optimal.

Analysing the slide graph, certain dependence can be noticed: stiffness namely stiffness of simply supported composite beams increases, when the anchor bolts are thickened near the supports. The additional deflection arrows caused by the slip will be far greater if the beams are subjected to greater loads (greater span). This situation causes the risk of exceeding the SGU for simply supported beams, though.

The disadvantage of the adopted analysis is a multitude of variables and an effort at transformation for other types of loads. Therefore, the optimal solution in this case seems to be a dimensional analysis. It would be a certain generalization of the issue, giving the conclusions in the form of directives for design. Paper [10] presents the possibility of setting anchor bolts in various configurations of intensity. It is very important because correct anchor bolts layout (e.g. by the supports) significantly reduces deflection compared to equal distribution. Then, the functions require transformations of the differential equation (11) for various bolts distributions, such as equal, linear or parabolic. So far, the greatest flexural rigidity have girders of parabolic density of composite construction.

In order to speak of an optimal steel-concrete composite, less quantity of bolts than required for full composite action [11] should be used. The composite action should be sufficiently susceptible to estimate the controlled slide in the joint surface of the steel beam and concrete slab. The standards [5, 6, 12] allow, so-called. partial composite action (slip). However, for the practice of engineering, calculation methods should be specified to avoid negative effects posed by the slip in the joint area.

References

[1] Nie, J., J. Fan, and C.S. Cai. "Stiffness and Deflection of Steel-Concrete Composite Beams under Negative Bending". *J. Struct. Eng.* 130, 1842 (2004).

[2] Johnson, R.P. *Composite Structures of Steel and Concrete*. Vol. 1: *Beams, Slabs, Columns, and Frames for Buildings*. Blackwell Scientific Publications 1994.

[3] Nie, J. and C.S. Cai. "Steel-Concrete Composite Beams Considering Shear Slip Effects". *Journal of Structural Engineering* 129 (4), 2003.

[4] PN-EN 1993-1-1 Eurokod 3: *Projektowanie konstrukcji stalowych*. Part 1-1: *Reguły ogólne i reguły dla budynków* [in Polish].

[5] pr EN 1994-1-1 FINAL DRAFT: *Design of Composite Steel and Concrete Structures*. Part 1.1: *General Rules and Rules for Buildings*, January 2004.

[6] PN-B-03300 *Konstrukcje zespolone stalowo-betonowe. Obliczenia statyczne i projektowanie* [in Polish].

[7] Kucharczuk, W. and S Labocha. *Konstrukcje zespolone stalowo-betonowe budynków*. Warszawa: Arkady, 2007 [in Polish].

[8] Jasim, N.A. *The effect of partial interaction on behaviour of composite beams*. Ph.D. thesis submitted to University of Basrah, Iraq, 1994: 201.

[9] Jasim, N.A, and A.A. Mohamad Ali. „Deflections of composite beams with partial shear connection". *The Structural Engineer* 75(4), 1997: 58–61.

[10] Adrjan, P., W. Lorenc, and E Kubica, *Zagadnienie sztywności giętnej dźwigarów zespolonych z częściowym zespoleniem o różnej intensywności.* 2004 [in Polish].

[11] Johnson, R.P, and I.M. May. "Partial interaction design of composite beams." *The Structural Engineer* 53(8), 1975: 305–311.

[12] British Standards Institution. Draft Standard for the use of structural steel in buildings. Part 3: Composite construction. London: British Standards Institution, 1976: 147.