**archives
of thermodynamics**
Vol. **34**(2013), No. 3, 137–146
DOI: 10.2478/aoter-2013-0020

Optimization of idealized ORC in domestic combined heat and power generation

WITOLD RYBIŃSKI*
JAROSŁAW MIKIELEWICZ

The Szewalski Institute of Fluid-Flow Machinery, Polish Academy of Sciences, Fiszera 14, 80-231 Gdańsk, Poland

Abstract Organic Rankine cycle (ORC) is used, amongst the others, in geothermal facilities, in waste heat recovery or in domestic combined heat and power (CHP) generation. The paper presents optimization of an idealized ORC equivalent of the Carnot cycle with non-zero temperature difference in heat exchangers and with energy dissipation caused by the viscous fluid flow. In this analysis the amount of heat outgoing from the ORC is given. Such a case corresponds to the application of an ORC in domestic CHP. This assumption is different from the most of ORC models where the incoming amount of heat is given.

Keywords: ORC; CHP; Thermodynamic optimization; Endoreversible

Nomenclature

A	–	heat transfer area, m^2,
c	–	thermal conductance of the heat exchanger, W/K
k	–	overall heat transfer coefficient, W/(m^{2}K)
$\dot{Q}$	–	heat flow rate in the heat exchanger, W
S, s	–	entropy, J/K and specific entropy, J/(kgK)
T	–	temperature, K or °C
$\dot{W}$	–	work rate, W
x, y	–	ratio (ordinary and modified) of thermal conductances

*Corresponding Author. E-mail: witold.rybinski@imp.gda.pl

Greek symbols

α, β, γ	–	coefficients

Subscripts

H, L	–	high and low
HC, LC	–	high and low in the internal Carnot cycle
$L1$	–	unit conductance of the low heat exchanger
N	–	net output work
*	–	reduced (and dimensionless).

1 Introduction

Domestic combined heat and power (CHP) generation is a way of distributed electric energy production [1]. Own design of the prototype of domestic CHP is carried out at the Institute of Fluid-Flow Machinery. In this prototype the heat from the domestic boiler feeds the ORC installation where mechanical/electrical work is produced. Heat rejected from the ORC is used in the domestic heating system. To design this prototype many engineering or more theoretical and general models have been considered. Two of them are the models of an endoreversible heat engine. The first one is the well known Curzon-Ahlborn heat engine [2–4]. The second, presented here realistic model, includes also the energy dissipation. The presented direct optimization for maximum output work is equivalent to entropy generation minimization principle described in [5]. An example of the ORC working with ethanol is shown in Fig. 1. Our analysis differs from the similar one carried out by Bejan in [2], where the incoming amount of heat is fixed.

As described in [2] the ORC may be represented by the Carnot cycle with the temperatures T_{HC} and T_{LC} equal to the average high and low temperatures of the ORC:

$$T_{HC} = \frac{1}{S_2 - S_3}\left(\int_3^4 TdS + \int_4^5 TdS + \int_5^1 TdS + \int_1^2 TdS\right),$$

$$T_{LC} = \frac{1}{S_2 - S_3}\int_3^2 TdS \; . \qquad (1)$$

The temperatures T_H and T_L of higher and lower reservoirs are calculated in the same manner.

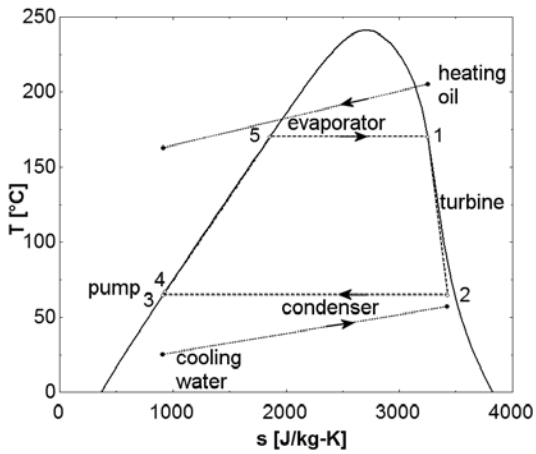

Figure 1. An example of the ORC working with ethanol.

It is assumed in further calculation that there is no heat leak to the ambient (an adiabatic case) and part of the work produced in the turbine feeds the pump. Because the ORC is a heat source for the domestic heating system, the outgoing amount of heat, Q_L, from the ORC is assumed to be given. This assumption is different from the most of ORC models where the incoming amount of heat, Q_H, is given.

2 Model 1 — an endoreversible Carnot cycle without energy dissipation

This model is the classical Curzon-Ahlborn heat engine with finite temperature differences in the hot and cold heat exchangers connected to the internally reversible (endoreversible) Carnot cycle (Fig. 2). However there is no energy dissipation in this model. The high and low temperatures, T_H, T_L, of the reservoirs and the heat rate, Q_L, are established and given. The further assumption of fixed total heat transfer area of both exchangers makes possible determination of the optimal ratio of the heat transfer areas A_H and A_L for maximum work production.

Heat flow rates in the heat exchangers are given by

$$\dot{Q}_H = c_H(T_H - T_{HC}), \quad \dot{Q}_L = c_L(T_{LC} - T_L) \ , \tag{2}$$

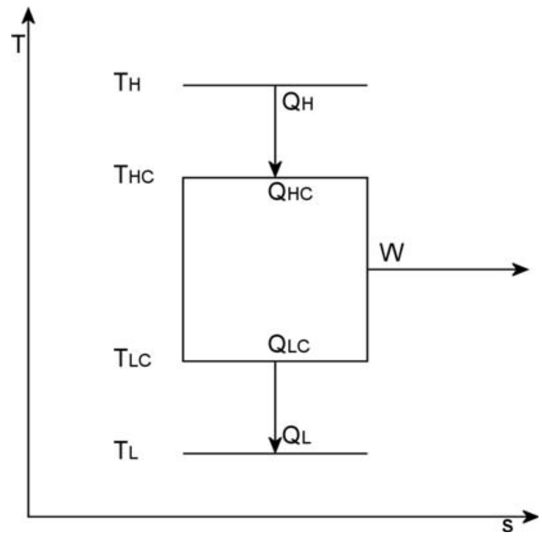

Figure 2. An endoreversible Carnot cycle with finite temperature differences in both heat exchangers without work dissipation.

with thermal conductances

$$c_H = k_H A_H, \quad c_L = k_L A_L \;, \tag{3}$$

where k_H and k_L are the overall heat transfer coefficients of the evaporator and condenser.

The reversible Carnot cycle is described by

$$\frac{T_{HC}}{T_{LC}} = \frac{\dot{Q}_H}{\dot{Q}_L} \text{ and } \dot{W} = \dot{Q}_H - \dot{Q}_L \text{ (since } \dot{Q}_{HC} = \dot{Q}_H \text{ and } \dot{Q}_{LC} = \dot{Q}_L) \;. \tag{4}$$

Introduction of the thermal conductance ratio, x, unit and reduced thermal conductances c_{L1}, c_L^* of the cold heat exchanger, maximum output work W_{MAX} (for $T_{HC} = T_H$ and $T_{LC} = T_L$) defined as

$$x = \frac{c_H}{c_L}, \quad c_{L1} = \frac{\dot{Q}_L}{T_H - T_L}, \quad c_L^* = \frac{c_L}{c_{L1}}, \quad \dot{W}_{MAX} = \left(\frac{T_H}{T_L} - 1\right)\dot{Q}_L \;, \tag{5}$$

gives the formula for the output work as a function of two dimensionless variables x and c_L^*

$$\dot{W} = \frac{x \cdot (c_L^* - 1) - 1}{1 + x\left(1 + c_L^* \frac{\dot{Q}_L}{\dot{W}_{MAX}}\right)}\,\dot{Q}_L \;. \tag{6}$$

Since $\dot{W} \geq 0$ there are two conditions for x and c_L^*:

$$x \geq \frac{1}{c_L^* - 1}, \quad c_L^* > 1 . \tag{7}$$

The second condition is always satisfied because is equivalent to $T_H > T_{LC}$ and $T_{LC} > T_L$.

In a real situation the constraint of limited total heat transfer area should be introduced. It is assumed that the sum of heat transfer areas of both heat exchangers is established and given [2], and both heat transfer coefficients are equal:

$$A_H + A_L = A \, , \quad k_H = k_L = k \, . \tag{8}$$

These construction assumptions give

$$kA_H + kA_L = kA \, , \tag{9}$$

what is written as

$$c_H + c_L = c \, . \tag{10}$$

Addition of thermal conductances c_H and c_L does not make physical sense because the heat flows Q_H and Q_L are not parallel. It represents the constraint of total heat transfer area only. These construction assumptions lead to the interpretation of x and c_L^* as the ratios of the heat transfer areas and the reduced heat transfer area of the cold heat exchanger respectively:

$$x = \frac{A_H}{A_L} \, , \quad c_L^* = \frac{c^*}{x+1} \, , \tag{11}$$

where $c^* = \frac{c}{c_{L1}}$ is the reduced, dimensionless sum of thermal conductances.

The domain of positive work is shown in Fig. 3 with the hyperbolas of zero work and of an exemplary constraint $c^* = 5$. These lines are described by

$$x_{W=0} = \frac{1}{c_L^* - 1} \, , \quad x_{c*=const} = \frac{c^*}{c_L^*} - 1 \, . \tag{12}$$

Combination of (6) and (11) gives the final formula for the output work

$$\dot{W} = \frac{x\left(\frac{c^*}{x+1} - 1\right) - 1}{1 + x\left(1 + \frac{c^*}{x+1}\frac{\dot{Q}_L}{\dot{W}_{MAX}}\right)}\dot{Q}_L \, . \tag{13}$$

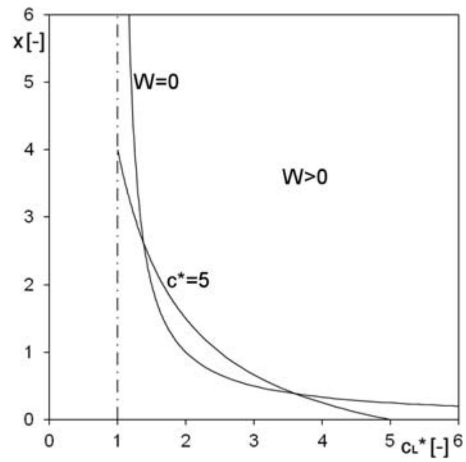

Figure 3. The domain of positive output work in the $x - c_L^*$ plane and the line of an exemplary constraint $c^* = 5$.

The optimal value of x for the maximum output work comes from the condition

$$\frac{d\dot{W}}{dx} = \frac{c^*\left(1+\frac{\dot{Q}_L}{\dot{W}_{MAX}}\right)(1-x)}{\left[1+x\left(1+\frac{c^*}{x+1}\frac{\dot{Q}_L}{\dot{W}_{MAX}}\right)\right]^2(1+x)}\dot{Q}_L = 0\ . \tag{14}$$

It gives the optimal ratio of heat transfer areas

$$x = 1 \tag{15}$$

what means $c_H = c_L$ and then $A_H = A_L$ (both heat transfer areas are equal). This result is identical to the result obtained in [2] on the assumption that $Q_H = const$.

3 Model 2 — endoreversible Carnot cycle with energy dissipation

The second model is an extension of the first one. It is assumed that there are two sources of dissipated work W_{HC} and W_{LC}. The first source comes from work loss in the pump, in the turbine and from pressure drop in the hot heat exchanger (evaporator). The second one comes from analogous pressure drop in the cold heat exchanger (condenser) (Fig. 4). Both dissipated work flow rates W_{HC} and W_{LC} are assumed to be known.

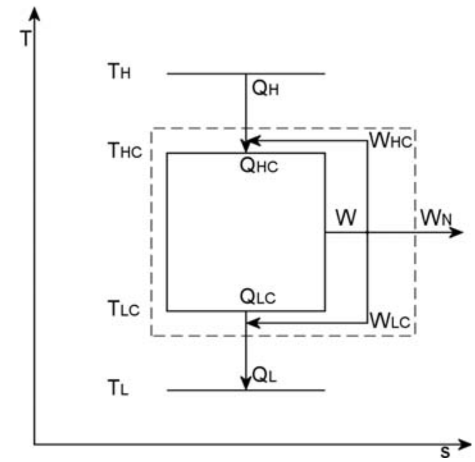

Figure 4. An endoreversible Carnot cycle with finite temperature differences in both heat exchangers with two ways of work dissipation.

The net output work is less than the output work from the endoreversible Carnot cycle by the amount of dissipated work:

$$\dot{W}_N = \dot{W} - \dot{W}_{HC} - \dot{W}_{LC} \; . \tag{16}$$

Dissipated work W_{HC} supplies the cycle so the heat flow Q_{HC} is bigger than the flow Q_H,

$$\dot{Q}_{HC} = \dot{Q}_H + \dot{W}_{HC} \; . \tag{17}$$

On the other hand, dissipated work W_{LC} does not supply the cycle but is rejected through the cold heat exchanger (condenser),

$$\dot{Q}_L = \dot{Q}_{LC} + \dot{W}_{LC} \; . \tag{18}$$

Combination of these three above formulae and the second one from (20) gives the result satisfying the law of conservation of energy

$$\dot{W}_N = \dot{Q}_H - \dot{Q}_L \; . \tag{19}$$

The reversible Carnot cycle is described by

$$\frac{T_{HC}}{T_{LC}} = \frac{\dot{Q}_{HC}}{\dot{Q}_{LC}} \quad \text{and} \quad \dot{W} = \dot{Q}_{HC} - \dot{Q}_{LC} \; . \tag{20}$$

The formulae for c_H, c_L, their ratio x, and reduced thermal conductivity $c_L{}^*$ are the same as in model 1. The formula for unit thermal conductivity c_{L1} is slightly different:

$$c_{L1} = \frac{\dot{Q}_L}{T_H \frac{1}{\alpha} - T_L}, \text{ where } \alpha = \left(1 + \frac{\dot{W}_{HC}}{\dot{Q}_L}\right) / \left(1 - \frac{\dot{W}_{LC}}{\dot{Q}_L}\right). \quad (21)$$

Coefficient $\alpha = 1$ where no dissipation exists ($W_{HC} = 0$ and $W_{LC} = 0$) and $\alpha > 1$ where any kind of dissipation exists ($W_{HC} > 0$ or $W_{LC} > 0$).

Maximum net output work is similar to the maximum work in the previous model. It is assumed again that $T_{HC} = T_H$ and $T_{LC} = T_L$, but work dissipation still takes place:

$$\dot{W}_{NMAX} = \left(\frac{T_H}{T_L} - 1\right) \dot{Q}_L - \frac{T_H}{T_L} \dot{W}_{LC} - \dot{W}_{HC}. \quad (22)$$

The variable $y = x\alpha$ may be called the modified ratio x.

The formula for the output net work is a function of two dimensionless variables y and c_L^*

$$\dot{W}_N = \frac{y(c_L^* - 1) - 1}{1 + y\left(\beta + c_L^* \frac{\dot{Q}_L}{\dot{W}_{NMAX}}\right)} \dot{Q}_L \text{ where } \beta = \frac{1}{1 + \frac{\dot{W}_{HC}}{\dot{Q}_L}} \quad (0 < \beta \leq 1). \quad (23)$$

Since $\dot{W}_N \geq 0$ there are two conditions for y and c_L^* again

$$y \geq \frac{1}{c_L^* - 1}, \quad c_L^* > 1. \quad (24)$$

The conditions (23) for y and c_L^* have the same form as the conditions (6) for x and c_L^* shown in Fig. 3. The same constraint of limited total heat transfer area is assumed again. The reduced heat transfer area of the cold heat exchanger is then equal to

$$c_L^* = \frac{c^*}{x + 1} = \frac{\alpha c^*}{y + \alpha}. \quad (25)$$

Combination of (23) and (24) gives the final formula for net output work

$$\dot{W}_N = \frac{y\left(\frac{\alpha c^*}{y+\alpha} - 1\right) - 1}{1 + y\left(\beta + \frac{\alpha c^*}{y+\alpha} \frac{\dot{Q}_L}{\dot{W}_{NMAX}}\right)} \dot{Q}_L. \quad (26)$$

The optimal value of y for the maximum net output work comes from

$$\frac{d\dot{W}_N}{dy} = 0 \; . \tag{27}$$

This condition leads to the quadratic equation for y

$$[c^*\alpha(\beta+\gamma) + (1-\beta)]y^2 + 2\alpha(1-\beta)y + \alpha^2[(1-\beta) - c^*(1+\gamma)] = 0 \; , \tag{28}$$

where $\gamma = \frac{\dot{Q}_L}{\dot{W}_{NMAX}}$. The positive solution for y gives the optimal value of ratio x

$$x = \frac{-(1-\beta) + \sqrt{c^*[c^*\alpha(\beta+\gamma) + (1-\beta)](1+\gamma) - c^*\alpha(\beta+\gamma)(1-\beta)}}{c^*\alpha(\beta+\gamma) + (1-\beta)} \; . \tag{29}$$

4 Results

There are two simple cases of the formula (28) for the optimal heat transfer area ratio x:

1. Lack of work dissipation; $W_{HC} = 0$ and $W_{LC} = 0$.
   The condition $W_{HC} = 0$ means $\beta = 1$ and together with the condition $W_{LC} = 0$ they give $\alpha = 1$. Introducing these values in (28) gives

   $$x = 1 \; . \tag{30}$$

   The result is the same as obtained in the model 1 (without dissipation) as expected.
2. Lack of dissipation; W_{HC}=0 and nonzero dissipation $W_{LC} > 0$.
   The condition $W_{HC} = 0$ gives $\beta = 1$ but the condition $W_{LC} > 0$ gives $\alpha > 1$. Introducing $\beta = 1$ in (28) gives the optimal ratio x

   $$x = \frac{1}{\sqrt{\alpha}} \; . \tag{31}$$

It means that $0 < x < 1$ if $\alpha > 1$. In other words $c_H < c_L$ and then $A_H < A_L$. The heat transfer area of the hot heat exchanger is smaller then the area of the cold one.

Comparison of the model with energy dissipation to direct simulations of the corresponding ORC acknowledges correctness of the endoreversible approach with energy dissipation.

5 Conclusions

- The results of presented endoreversible models are in good agreement with the results of direct simulations of the corresponding ORC's.
- The second endoreversible model with energy dissipation is more realistic and reflects main features of the ORC. It makes this model useful for theoretical investigation applied to domestic CHP.
- Introduction of energy dissipation sources allows for more realistic determination of power generation and efficiency of the ORC.

Acknowledgements This work has partially been funded from the National Project POIG.01.01.02-00-016/08 "Model agroenergy complexes as an example of distributed cogeneration based on local renewable energy sources."

Received 8 July 2013

References

[1] Mikielewicz J.: *Micro heat and power plants working in organic Rankine cycle.* Polish Journal of Environmental Studies **19**(2010), 3, 499–505.

[2] Bejan A.: *Advanced Engineering Thermodynamics.* Wiley, New York 1988, 404–426.

[3] Hoffmann K.H.: *An introduction to endoreversible thermodynamics.* Atti dell'Accademia Peloritana dei Pericolanti Classe di Scienze Fisiche, Matematiche e Naturali **86**(2008), Supplement 1.

[4] Bejan A.: *Thermodynamic optimization of inanimate and animate flow systems.* In: Thermodynamic Optimization of Complex Energy Systems (A. Bejan and E. Mamut, Eds.). Kluwer Academic Publishers, Dordrecht 1999, 45–60.

[5] Mikielewicz J.: *Role of thermodynamics in science and engineering.* Arch. Thermodyn. **25**(2004), 2, 3–19.