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OPTIMIZATION AND DESIGN OF FOUR GRIPPER PNEUMATICAL RUBIK'S CUBE SOLVER

Abstract

Robotic Rubik's solver is an excellent example of science popularizing mechatronic device, since it combines knowledge of several technological fields including computer vision techniques, advanced numerical algorithms and control of industrial pneumatic components with popularity of Rubik's cube in one resulting device. First version of solver constructed by our department was equipped only by 2 lower grippers, resulting in approximately four minutes for single cube solution. Proposed paper describes not only experience gained by development of upgraded 4 gripper version but includes statistical analysis of Thistlethwait's 45 algorithm required for solving process optimization. Mechanical design, electronics, system overview, performance and limitations of upgraded 4 gripper version are explained in detail as well.

1. INTRODUCTION

Rubik's cube is famous puzzle invented by Erno Rubik in 1974. 57mm wide cube consisting of 6 center, 8 edge and 12 corner pieces, allows more than $43x10^{18}$ mutual combinations. Consequently, solving the Rubik's cube like many other problems is the NP-hard problem (see: eg. [1-5]). Since Rubik's cube introduction, its popularity has inspired many students and researchers to build robots capable of solving this 3D puzzle. Several solving mechanisms have been proposed already [6], most of them using NXT components, but some pneumatical robots have been constructed as well [7]. Considering speed, David's Gilday's Cubestormer III is the current Guinness world record holder in category Fastest Rubik's cube solving robot - time 3.253s [8].

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No matter what mechanics is used, the principle of real cube solving process remains the same including three phases: Detection of cube state, Generating solution and Realizing particular moves. Proposed paper describes mechanical construction in first section, followed by solving algorithm description, system overview, electronics specification and statistical analysis of Thistlethwait's 45 algorithm. Last section provides information from real testing including solving times and personal experience gained during several public presentations of this showpiece. The main drawback of the whole system identified during presentation phase was vision based cube detection algorithm. It's weakness is mainly caused by mechanical construction, because in order to keep clear industrial design, USB webcam is not only opposite-visitor oriented, but moreover mounted on gooseneck platform, which prevents long-term fixed position of camera with respect to handled cube. Every time the showpiece is set up, the position of the camera slightly changes. This drawback in combination with various lighting conditions during several day-long presentations requires more robust algorithm for cube detection than the one previously used. The development of robust computer vision algorithm for Rubik's Cube state detection can be found in [9].

2. MECHANICAL CONSTRUCTION

As mentioned in the beginning, our motivation was to by develop fully pneumatical Rubik's cube solver to demonstrate possibilities of pneumatical approach in precise manipulation tasks. 3D CAD design shown in Fig. 1 gives an overview of whole construction.
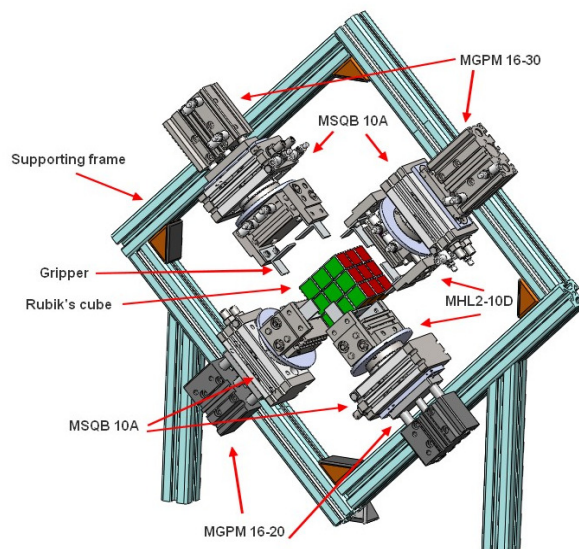
**Fig. 1. Mechanical design of Rubik's solver [source: own study]**

Supporting frame is assembled of aluminum mounting profiles 30x30mm and attached to wooden base providing cover for electronic part of a system. Four manipulation units consist of three air modules – compact guide cylinder, rotary table and a gripper module. For our particular construction, SMC modules were used, since they provided best price/performance ratio.

The very first component that needed to be selected in Rubik's solver mechanical design was gripper unit. SMC provides several types of components including angle, parallel or three point grippers. Considering all possible options, MHL parallel units were selected as best suitable for our task. MHL in general are double piston parallel style grippers with large holding force, which are available in several sizes.

In order to select particular type of MHL gripper module, it was necessary to compute holding force and determine holding point distance. If Rubik's cube weight is $m_{rc} = 0{,}12 \ kg$, safety factor k = 10 , then required holding force is computed as follows:

$$F_u = m_{rk} \times g \times k = 11{,}8 N \qquad (1)$$

Considering holding force F_u , distance from grasping point R = 30 mm and Rubik's cube width L = 57 mm , MHL2-10D unit, providing opening/closing stroke range 56-76 mm was selected. As shown in diagram in Fig. 2 required holding force can be achieved already at 0.4 MPa.
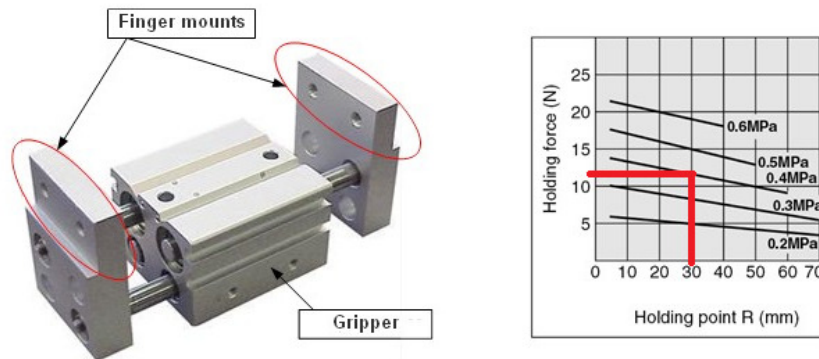

**Fig. 2. MHL2-10D gripper unit and appropriate
holding force graph [source: own study]**

Rotation of gripper unit is handled by SMC MSQ units. MSQ is compact rotary table including load bearings, mounting face, with a rack-and-pinion style rotary acutator. Achievable rotating range of MSQ tables is 190 degrees, in case of Rubik's cube manipulation; adjustments are set to 0-90°. In order to select

optimal rotary table type, two important variables - moment of inertia and time of 90° rotation must be determined. Considering weight of Rubik's cube m_{rc} = 0,12 kg , weight of gripper unit m_{GU} = 0,28 kg as well as dimensions of gripper unit, moment of inertia can be computed as follows:

$$I = (m_{rc} + m_{GU}) \times \frac{a^2 + b^2}{12} = 1,51 \times 10^{-4} \, kgm^2 \qquad (2)$$

If duration of 90 degree rotation is set to 0.3s, MSQB-10A unit meets computed requirements as shown in diagram in Fig. 3.
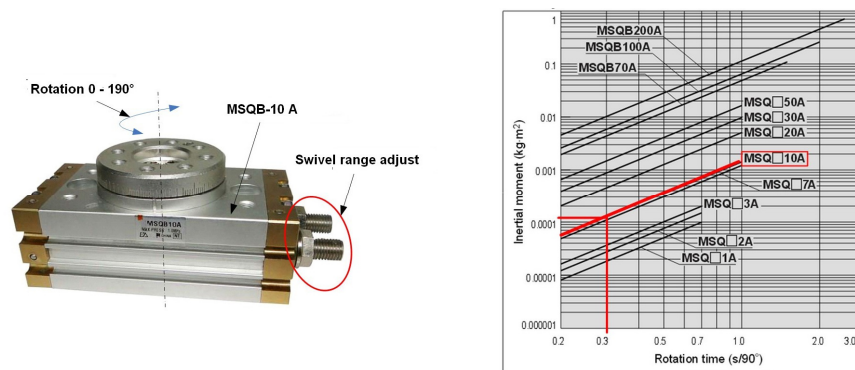
**Fig. 3. MSQB-10A unit and appropriate rotation time –
inertial moment selection graph [source: own study]**

As mentioned above, current mechanism allows manipulating with four sides of cube directly. But solving process require clockwise and counterclockwise manipulation with all six sides of the cube, therefore it is necessary to turn the cube every time the front or back side of the cube needs to be rotated. However, turning the cube without stroking of particular units is not possible because of mutual gripper collisions. Application of compact guide cylinders eliminates this problem, by providing stroking capability for each manipulation unit. When rotating the cube, only gripper holding the cube can be ejected, remaining three must be shut to avoid collision. From all available cylinder units, MGPM class was most suitable for our construction mainly because of its flat design and rectangular mount allowing simple attachment of rotary unit.
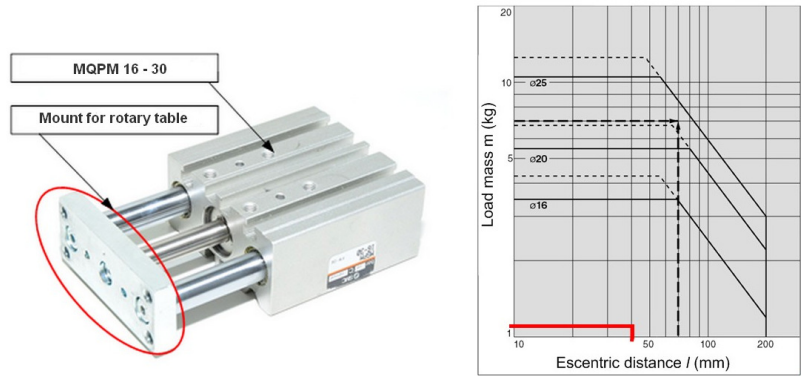
**Fig. 4. MPQM 16-30 and appropriate load mass –
eccentric distance selection graph [source: own study]**

MGPM in general is compact guide cylinder designed for high side load applications with available strokes within range 20 – 400 mm. Since MGPM load of single unit in our case was approximately 1kg, model size with cylinder diameter D=16 mm according to graph in Fig. 4 was selected. Lower two gripper units are equipped by 20 mm stroke, while upper grippers units use 30 mm strokes. This configuration in combination with dimensions of supporting frame results in appropriate gripper position with respect to the cube.

Comparison of prior and current version of Rubik's solver is shown in Fig. 5.
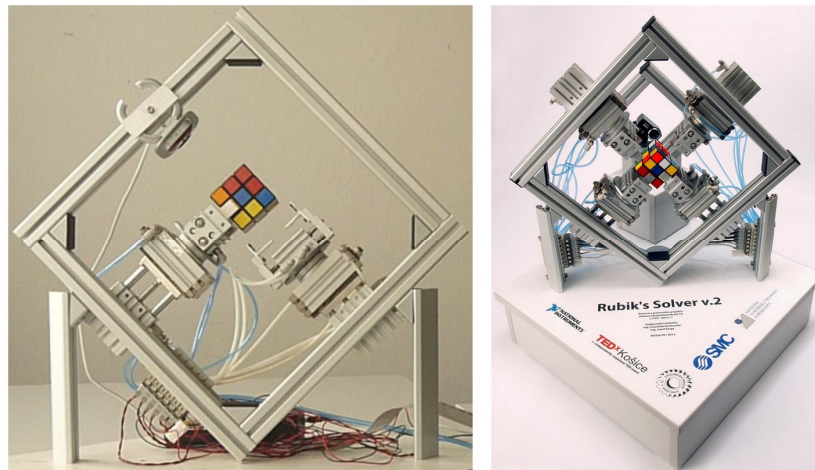
Fig. 5. Comparison of previous and current solver version [source: own study]

2. SOLVING ALGORITHM

Rubik's cube solving algorithms can be in general divided into two categories: Human algorithms & Computer algorithms.

Human algorithms are quite simply memorable but not as fast as computer's. Computer algorithms require extensive computer searching, which is not possible in human approach. First computer algorithm introduced in 1981 by Morwen Thistletwait [9] provided maximum 52 move solution of arbitrary scrambled cube. By exhaustively searching, it was later found, that the best possible number of moves is 45, resulting in so called Thistletwait45 algorithm [10]. Our solver uses Joren Heit's implementation of T45 algorithm, written in Matlab under BSD license, open source and free for public [11]. Median number of moves for T45 method is 31 – quarter of average moves required for basic human algorithm – Layer by Layer.

Thistlewait algorithm was improved by Herbert Kociemba in 1992. Using this algorithm, solutions are typically found of fewer than 21 moves, though there is no proof that it will always do so. Implementation of Kociemba algorithm will be probably the next step in future development.

3. SOFTWARE DEVELOPMENT

The whole application was programmed in Matlab, not only because the solving algorithm was written in this language, but moreover because the fact, that Matlab provides all necessary features, including vision algorithms, tools for designing graphical user interface and support of hardware input output devices via Data acquisition toolbox.
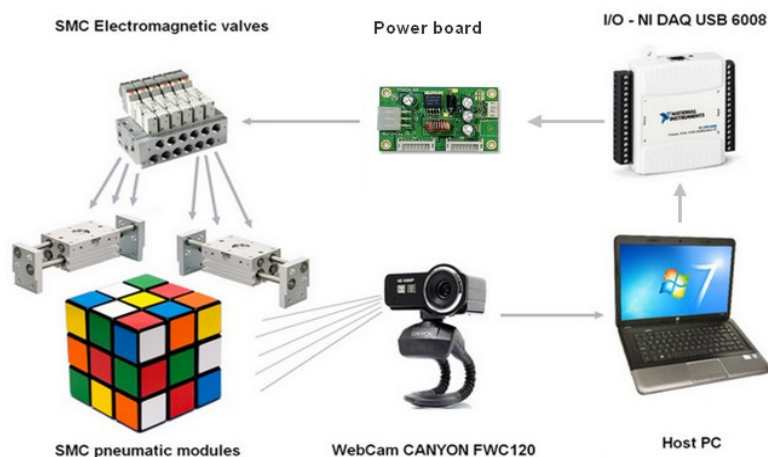

Fig. 6. System Overview [source: own study]

4. ELECTRONICS

Overview of solver control is shown in Fig. 6. The most important part of whole system is physical link between host PC and SMC electromagnetic valves. It was realized by low-cost USB DAQ 6008 from National instruments, supported by Data acquisition toolbox. This combination allows direct control of 12I/O by single line of code. The only complication was the fact, that SMC electromagnetic valves as most of industrial components require 24V for switching, but USB6008 provides only 0-5V I/O voltage levels. Hence simple power board including Darlington NPN transistors TIP122 were used to raise the voltage level up to 24V. Outputs of USB6008 are not connected to transistor base directly but via optocouplers providing galvanic isolation between control and power part of the system.

5. OPTIMIZATION

There are three ways to manipulate with single cube side:
- 90 degrees clockwise (L,R,F,B,U,D)
- 90 degrees counterclockwise (L', R', F', B', U', D')
- 180 degrees (L2, R2, F2, B2, U2, D2

Rubik's cube solution is always a combination of these 18 types of moves, what differs is its implementation in particular solvers.

Our implementation uses principle "*Basic cube orientation --- Move --- Basic cube orientation*", so cube returns to basic orientation after every single move. Since our mechanism allows us to manipulate only with 4 cube sides directly, time consuming re-grasping is required in case of manipulating with remaining two cube sides. Therefore the main question in initial stage of implementation was selecting basic cube orientation to minimize inevitable re-grasping moves.
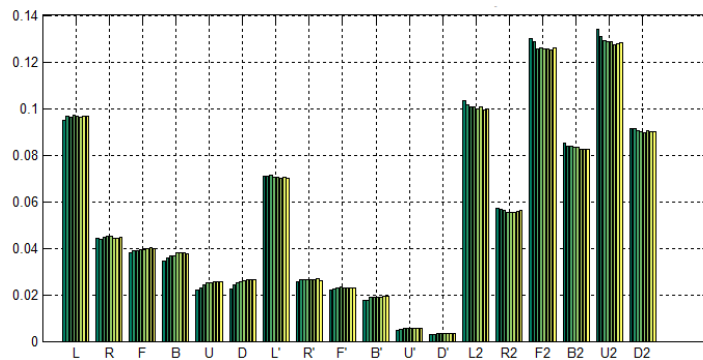

**Fig. 7. Relative abundance of all possible moves for 8 different mixing values
[source: own study]**

In order to find pair of cube sides (L-R, U-D, F-B) least frequently manipulated in Thistlethwait's 45 algorithm, we have conducted statistical analysis on 80 000 of randomly mixed cubes. Function generating random cubes takes a number of moves used for mixing the cube as an input argument. 8 different mixing values (15, 20, 25, 30, 35, 40, 45, 50) were used, each for 10 000 cubes resulting in relative abundance of particular moves shown in Fig. 7. As one can see, U2 and F2 are most frequently used moves, while U', D' are almost not used at all in T45 algorithm. Summarization of first chart with respect only to cube sides is shown in Fig. 8.
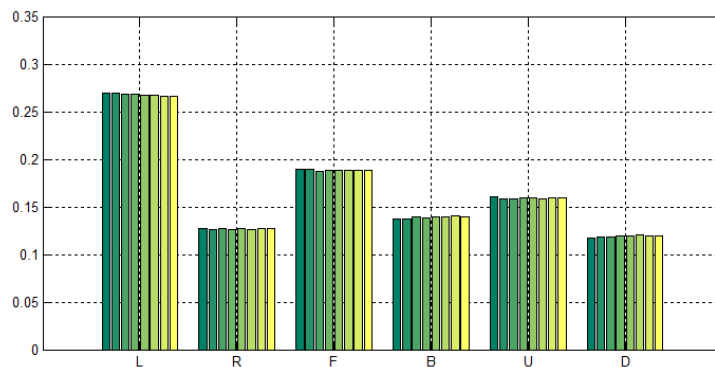
Fig.8. Relative abundance of manipulated cube sides [source: own study]

As shown in Fig. 8, left side is most frequently manipulated, in contrary with least manipulated, right cube side. It is also worth noticing, that changing number of mixing moves does not influence relative abundance markedly, values are almost the same for different settings. Combining results from Fig. 8 into opposite cube pairs yields to most important chart for optimizing of solving process shown in Fig. 9.
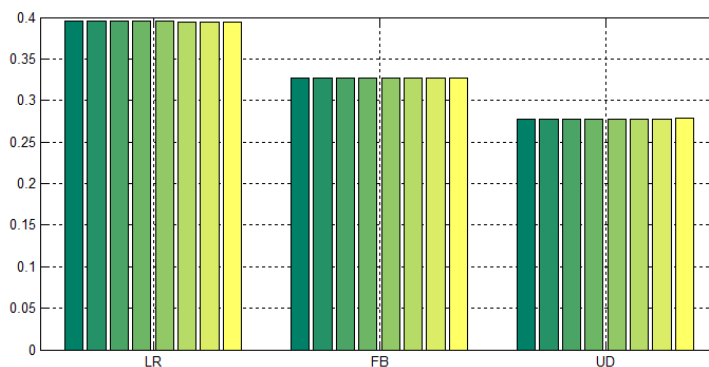
Fig. 9 Relative abundance of manipulated cube side pairs [source: own study]

It is obvious that "Up-Down" pair is statistically least manipulated, so when following general Rubik's cube notation:

Up side – White center square   Down side – Yellow center square
Left side– Green center square   Right side – Blue center square
Front side – Red center square   Back side – Orange center square

It means that in order to minimize time-consuming re-grasping, white and yellow center sides needs to be front/back oriented with respect to the solver, while Green, Blue, Red and Orange cube sides are directly reachable by particular grippers.

Besides cube side manipulation, statistical analysis of number of solution moves has been conducted as well. Solving 80 000 sample cubes has resulted in normal Gaussian probability distribution with median value 31 as shown in Fig. 10. Maximum number of moves in our sample set was 40, but as the name of algorithm indicates, cube states requiring exactly 45 moves are known as well.
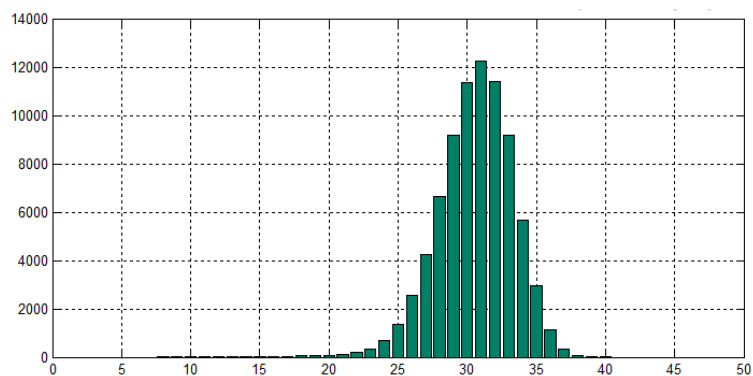
Fig. 10. Thistlethwait's 45 - Number of solution moves (80 000 samples)
[source: own study]

6. CONCLUSION

Construction of Rubik's solver can provide a lot of useful experience, especially when studying mechatronics, since it requires combining several different subsystems into one functional mechanism. Our main motivation was to build Rubik's cube solver using standard industrial components to demon-strate pneumatical approach for solving precise manipulation tasks. First version has proven the basic functionality, adding 2 more grippers in second version have reduced average solving time under 2 minutes.

Rotary table allowing movement only between two endpoints is the main constraint of current mechanism. In case of Rubik's solver, endpoints are mechanically set to 0 and 90 degrees. It is possible to solve the cube with such a limitation but frequent time-consuming re-grasping is required. Optimal endpoints for clockwise, counterclockwise and double moves would be -90, 0, 90 and 180 degrees. Replacing current rotary units for stepper motors could rapidly reduce solving time to less than one minute, but for the price of simple pneumatical design.

Video of proposed Rubik's cube solver developed by our department is available on http://www.youtube.com/watch?v=-VhLoyQBhj8.

7. FUTURE WORK

Rubik's cube solver is the main presentation showpiece used by Faculty of Mechanical engineering during several science popularizing actions. Authors of this paper have spent hours and hours presenting this mechanism in public, while solving hundreds of cubes randomly mixed by visitors.

Mechanical construction as well as electronic part of system has proven its reliability during this period, though no further upgrades of these parts are foreseen. What is worth considering is the robot control software since new solving algorithms have been released recently. Implementation of more advanced algorithms could reduce average number of moves and thus reduce overall solving time even more.

Rubik's solver is also a great experimental platform for testing different vision algorithms. In future, we would like to try different approach to move from static image recognition to video tracking of the cube. Moreover existing dataset of positive and negative scans could be also used to train a support vector machines algorithm for cube position detection. Such an approach has the potential to bring more sophisticated solution and reduce the scan time significantly.

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