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MINIMIZATION OF THE WOOD WASTES FOR AN INDUSTRY OF FURNISHING: A TWO DIMENSIONAL CUTTING STOCK PROBLEM

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Abstract

This paper presents the modeling and the resolution of a two dimensional cutting stock problem for a wooden industry. It is about a real problem of minimization of the wood wastes for an industry of furnishing. The raw material to be cut is a set of beams of various sizes. The purpose of the cut is to supply a list of orders characterized by a set of articles of various sizes. The problem is converted into an integer linear program where the decision variables are the numbers of beams to cut according to a set of feasible "patterns".
The designed solution is a heuristic in two stages:
– Generation of the feasible patterns by various classic heuristics of the Bin-packing Problem.
– Resolution of the integer linear program with the generated patterns as input variables.
Moreover, based on this approach, the "Application Cutting Optimization" is developed to allow the immediate resolution of the problem and widening the stock management horizon. To end, a real case is studied to confirm the effectiveness of this approach.

Keywords

two-dimensional cutting stock problem, industry of furnishing, heuristic in two stages, bin packing problem, generating patterns.

Introduction

SPA is a company which has operated in the field of traditional and modern furnishings for three decades. The main activity of this company is the production of modern and traditional furniture products. The SPA unit covers an area of 15 000 m^2 and includes several workshops providing various activities as carpentry, varnishing, tapestry, wood warehouses, consumables and tissues. This unit is designed for producing different kind of furniture. It's equipped with high-tech equipment "CNC machines" for the preparation, assembly, finishing, varnishing wood and upholstery. In addition, the flexibility of its workshops allows the realization of any specific request. SPA stands today as one of the leaders in its field. Strong competition from several national and foreign companies has forced the Furniture Production Company called SPA (Société de Production Ameublement) to improve its performance and to include policy of reducing costs of non-quality. In this context, SPA company aims to optimize its wood consumption. The stock contains several sets of wooden beams, every set includes a quantity of beams, and each beam is characterized by its rectangular shape. The Figs. 1 to 3 show examples of a set of wooden beams in the warehouse of SPA.

The noble wood such as ash, beech, Chain, and Mahogany, are the most expensive and have different sizes: the length and the width differ from one beam to another.

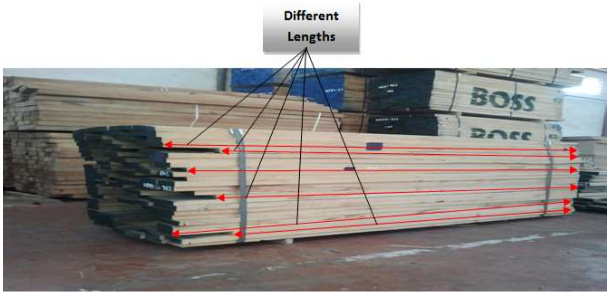


Fig. 1. Various lengths of wooden beams in the same set.

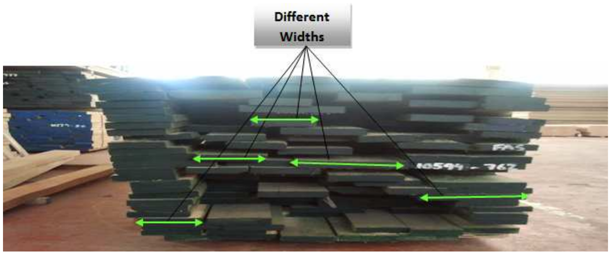


Fig. 2. Various widths of wooden beams in the same set.

Fig. 3. Identical sizes for the less expensive wood.

The Fig. 1 shows various lengths of beams in the same set. The width of the noble wood varies remarkably as shown in the Fig. 2. Other types of wood, less expensive, such as white wood and red wood have identical sizes as shown in Fig. 3, and whether the wood is noble or not, the thickness remains identical in each set of beams. The fact that the beams are of different sizes makes the cutting task more difficult. The operator must program more cutting models and the possible combinations are obviously more numerous. Then, the problem becomes more complex. Competition from foreign and national companies is an obstacle to the increase of revenues of the company. Wood wastes are part of the non-quality costs that significantly increases the cost of the product. The latter is a crucial and decisive factor in determining the sale price. Subsequently, minimization of the wastes is a necessity to survive in a competitive world. Wooden beams sizes are imposed by suppliers. Moreover, articles (items) sizes are also imposed by clients. In this context, the current study aims to minimize wooden wastes and automation of the cutting assignment process. Wood losses are inevitable and assigning items to adequate beams is a major concern for the company. Allocation methods based on intuition and common sense are no longer appropriate, and all rational methods and techniques of oriented research are the best way to make choices and lead to the desired solution or the best possible solution. In view of all this specificities, the problem identified is a cutting stock problem: but the question is its dimension. The wooden beams and the orders are obviously characterized with three-dimensions. But the thickness does not allow more than one article so the third dimension can be ignored. A brief literature review of this problem is presented bellow.

Literature review

P. Gilmore and R. Gomory solved a typical problem of the one-dimensional cutting stock by formulating it as a Linear Program [1, 2]. Next, they addressed the two-dimensional rectangular cutting stock, and solved this problem in two stages [3]. After that, P. Gilmore and R. Gomory gave an algorithm for computing a two-dimensional knapsack function [4].

As far as surveys on cutting and packing problems and their industrial applications, A. Lodi et al. surveyed concerning two-dimensional packing problems and discussed related mathematical models, lower bounds, classical approximation algorithms, heuristic methods, and exact enumerative approaches [5]. C.H. Cheng presented a survey related to the cutting stock and discussed specially the methodologies and the practical aspects related to bin packing problem, the assortment problem, pallet loading problem [6]. Due to the significant growth of papers in cutting and packing, G. Wäscher et al. presented an improved typology based on categorization of the cutting and packing literature from the years between 1995 and 2004 [7].

R. Morabito and M. Arenales addressed the cutting of stock plates in a furniture company [8], their proposed model is based on the classical approach [3], which combines a linear program and a column generation procedure. A.C. Cherri et al. considered a one-dimensional cutting stock problem in which the non-used material in the cutting patterns may be used in the future, if large enough [9]. Some desirable characteristics of solutions are defined and classical heuristic methods are modified, so that cutting patterns with undesirable leftover (not large enough to be used, nor too small to be acceptable waste) are redesigned. J. Erjavec et al. focused on the connection of the CS process with other processes in a company to get improved results [10]. Three years later, they proposed a method for assessing the optimal stock size for the expected order size for a single-period one-dimensional cutting stock problem and the stock size is optimal when the expected total costs of trim loss, warehousing, and non-fulfillment are minimum [11].

In recent years, B. Vacharapoom and B. Sdhabhon developed solution procedures which result in the cutting plans with minimum trim loss and number of stocks used [12]. The procedures consist of three steps namely the generation of a set of cutting patterns, the optimization of one-dimensional cutting stock problem, and the cut of undersupplied items with the best fit decreasing algorithm. C. Gracia presented a hybrid approach based on the use of genetic algorithms to solve the problem of cutting structural beams arising in a local metalwork company [13]. This latter belongs to the class of one-dimensional multiple stock sizes cutting stock problem. K. Li addressed the two-dimensional bin-packing (2BP) problem with variable conflict penalties, which incur if conflicting items are loaded into the same bin [14]. They proposed a heuristic method based on the IMA algorithm and adapted it to solve this problem. A. Toscano et al. addressed the two-dimensional cutting stock problem considering two potentially conflicting objectives: the minimization of the total number of objects and the minimization of the total number of saw cycles [15]. This work added to the literature a heuristic method to solve the problem considering the conflict between these two objectives, and treating it in the context of the furniture factories. A. Dems et al. developed a model which provides decision support with respect to how to harvest the different cut-blocks according to the bucking priority list used, and in what quantities harvested logs should be transported to sawmills [16]. It aims to minimize the nonlinear harvesting cost, the transportation cost, and the inventory cost and to maximize the product value. In fact, our work is a logical continuation of the research [16], in other words, the output of this latter can be considered as an input of our research, because within company SPA the planks are already ready to be cut in order to get items of different forms according to demands of clients. Further, unlike A. Toscano et al. we do not aim to minimize sawing neither in time nor in price [15]. Moreover, the selling price of wooden wastes is not taken into consideration because it's negligible compared to purchase price of beams. Lately and in related field, A. Kujawińska et al. presented a methodology of assessment of new technology variants in terms of minimization of the input material waste [17].

In this study, the problem described above is solved based on two stages heuristic. The first step consists of generation patterns by solving two-dimensional bin-packing problem; the second step aims to choose optimal patterns by solving mathematical integer programming (MIP). In the next section, we present mathematical modeling of the two-dimensional cutting stock problem concerning the problematic presented above. In detail, the first subsection addresses a model where the bins sizes are identical, and more generally the problem related to different sizes bins will be investigated in the second subsection. In Sec. 4, adequate heuristics are presented for generating patterns and realizing good quality solutions within a reasonable time. The mathematical approach and the proposed heuristics are then programmed and implemented by "Application Cutting Optimization" in the Subsec. 4.2. Finally, to evaluate the efficiency of our approach, a real case from the company SPA is investigated.

Mathematical modeling of a real two-dimensional cutting stock problem

The aim of this section is to provide improved mathematical models for the problem described above.

Model with identical bins

Throughout this study a beam will be called: **bin**.

This subsection deals with the case of bins with identical sizes, the width and the length can not change from one bin to another. Moreover, the bins are rectangular.

- Notations

m is the total quantity of bins;
L is the length of identical bins;
W is the width of identical bins;
$A = L * W$ is the area of identical bins;
The items a_i are also considered rectangular.
$i \in [\![1; n]\!]$ is the index related to items a_i which have the same sizes $(l_i; w_i)$;
r_i is the quantity of items a_i;
l_i is the length of items a_i;
w_i is the width of items a_i;
$S_i = l_i * w_i$ is the area of items a_i;
$r = \sum_{i=1}^{n} r_i$ is total quantity of requested items.

To address this problem, we list the possible patterns to cut a bin. For example, a pattern k consists of taking $3a_1, 6a_2, ..., 1a_i, ..., 7a_n$ from a bin, and a pattern k' consists of taking $5a_1, 0a_2, ..., 2a_i, ..., 3a_n$ from a bin. For the same bin, we can have several patterns. Let K be the number of proposed patterns, then k and k' belong to $[\![1; K]\!]$. c_i^k stands for the number of items a_i obtained according to the pattern k. For the pattern k we have $c_1^k = 3, c_2^k = 6, ..., c_i^k = 1, ..., c_n^k = 7$, whereas concerning the pattern k' we have $c_1^{k'} = 5, c_2^{k'} = 0, ..., c_i^{k'} = 2, ..., c_n^{k'} = 3$. f^k is the area of wastes due to choosing the pattern k, and is given by

$$f^k = A - \sum_{i=1}^{n} c_i^k * S_i \qquad \forall k \in [\![1; K]\!]. \tag{1}$$

- Mathematical formulation

The goal is to minimize wastes with respect to the constraints of the problem. The number of bins is assumed to be sufficient for getting all items.

Decision variables

x^k is the number of bins to cut within the k-th pattern.

Objective function

$$\min \sum_{k=1}^{K} f^k * x^k. \tag{2}$$

Constraints

$$\sum_{k=1}^{K} c_i^k * x^k \geq r_i \qquad \forall i \in [\![1; n]\!]. \tag{3}$$

The objective function (2) minimizes the area wastes, by searching the suitable x^k, number of times for the use of each pattern k. There are K decision variables $x^1, x^2, ..., x^k, ..., x^K$. Thus, the model is K decision variables. The constraint (3) is added to ensure sufficient quantity of each item a_i. The relation (1) calculates the area of wooden losses related to pattern k.

The optimum number of bins to cut is $m^* = \sum_{k=1}^{K} x^{k*}$, where x^{k*} are the solutions.

The minimum wooden wastes F^* is given by

$$\begin{aligned} F^* &= \sum_{k=1}^{K} x^{k*} * f^k \\ &= \sum_{k=1}^{K} x^{k*} * \left(A - \sum_{i=1}^{n} c_i^k * S_i \right) \\ &= m^* * A - \sum_{k=1}^{K} x^{k*} \sum_{i=1}^{n} c_i^k S_i \\ &= m^* * A - \sum_{k=1}^{K} \sum_{i=1}^{n} x^{k*} * c_i^k * S_i. \end{aligned} \tag{4}$$

Model with various bins

This subsection deals with the case of various sizes, width and length can change from one bin to another. This fact increases the complexity of the problem.

- Notations

Bins which have the same size are indexed by j, where $j \in [\![1; J]\!]$.
J is the number of types of bins with the same size $(L_j; W_j)$;
m_j is the quantity of bins having the same size $(L_j; W_j)$;
L_j is the length of bin j;
W_j is the width of bin j;
$A_j = L_j * W_j$ is the area of bin j (the thickness is overlooked as explained above);
$m = \sum_{j=1}^{J} m_j$ is the total quantity of bins;
K_j is the number of proposed pattern to cut bin j, where $K = \sum_{j=1}^{J} K_j$;
c_{ij}^k is the quantity of items a_i obtained by cutting bin j according to the corresponding pattern k, where $k \in [\![1; K_j]\!]$;
f_j^k is the area of wooden wastes for a bin j if it is cut according to the pattern k, and is given by

$$f_j^k = A_j - \sum_{i=1}^{n} c_{ij}^k * S_i \quad \forall k \in [\![1; K_j]\!] \quad \forall j \in [\![1; J]\!]. \tag{5}$$

- Mathematical formulation

Different sizes of beams will raise the number of decision variables and will also increase the size of the problem and its complexity.

Decision variables

x_j^k is the number of bins from the type j to cut according to the pattern k.

Objective function

$$\min \sum_{j=1}^{J} \left(\sum_{k=1}^{K_j} x_j^k * f_j^k \right) \tag{6}$$

Constraints

$$\sum_{j=1}^{J} \left(\sum_{k=1}^{K_j} x^k * c_{ij}^k \right) \geq r_i \quad \forall i \in [\![1; n]\!], \tag{7}$$

$$\sum_{k=1}^{K_j} x_j^k \leq m_j \quad \forall j \in [\![1; J]\!]. \tag{8}$$

The objective function (6) is for minimizing the wooden wastes by seeking adequate x_j^k and respecting the constraints. The unknown variables are $\{x_1^1, x_1^2, ..., x_1^{K_1}, x_2^1, x_2^2, ..., x_2^{K_2}, ..., x_J^1, x_J^2, ..., x_J^{K_J}\}$, K is the number of decision variables and

$\sum_{j=1}^{J} K_j = K$. Then, this model is K decision variables. The constraint (7) is for providing at least the request quantity of each item a_i. The constraint (8) reflects the impossibility of exceeding the available beams m_j. The relation (5) calculates the area losses after cutting a bin j according to the pattern k.

The optimum number of bins to cut is $m^* = \sum_{j=1}^{J} m_j^* = \sum_{j=1}^{J}\sum_{k=1}^{K_j} x_j^{k*}$, where x_j^{k*} are the solutions.

In this case, the minimum wooden wastes F^* is given by

$$F^* = \sum_{j=1}^{J}\left(\sum_{k=1}^{K_j} x_j^{k*} * f_j^k\right) = \sum_{j=1}^{J}\left(\sum_{k=1}^{K_j} x_j^{k*} * \left(A_j - \sum_{i=1}^{n} c_{ij}^k * S_i\right)\right) = m^* * A_j - \sum_{j=1}^{J}\sum_{k=1}^{K_j} x_j^{k*}\left(\sum_{i=1}^{n} c_{ij}^k * S_i\right). \quad (9)$$

To achieve efficient results through these mathematical models, automatic generation of possible patterns is required. This need to generate the patterns suggests the use of heuristics as detailed in the next section.

The solution designed: a two stage heuristic

To improve the results quality and to converge toward the optimal solution, automatic generation of patterns are proposed via improved heuristics to get the coefficients c_{ij}^k as output of the first stage and input of the second stage. One the one hand, the first stage consists of adopted heuristics to provide the best possible patterns and to generate the coefficients c_{ij}^k. On the other hand, the second stage consists of proposed mathematical integer programming models to get the solutions x_j^{k*}.

To solve the problem presented in the previous section, heuristic methods will be proposed in order to obtain good quality solutions within a reasonable time especially for the pattern generating stage.

Heuristics proposed to resolve two bin-packing problem (2BP)

Given a set of items in rectangular shapes with known sizes and given a larger bins and rectangular with known sizes, the bin-packing problem in two dimensions (2BP) is to determine the minimum number of bins needed for storing items without overlapping them (items do not cross bins and do not overlap). The items are stored in a way their edges are parallel to those of their assigned bins. The (2BP) is a generalization of the bin packing problem in one dimension (1BP), which is known to be an NP-hard problem. More formally, the bin packing problem in two dimensions (2BP) is defined as follows: given a set of n rectangular objects (items or articles) $A = \{a_1, ..., a_n\}$ and an unlimited number of identical rectangles (bins) of a larger size than those of the items.

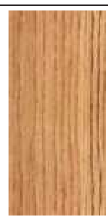
This subsection presents the main heuristics that are compatible to our problem, see [18–20]. These heuristics used are explained and illustrated by simulations. They can be divided into two different families [18]: algorithms in a single phase & algorithms in two phases.

- **Algorithms in two phases:**

The first phase concerns Strip Packing (SP). In most of the proposed approaches, the first step of Strip Packing (SP) is to sort items according to their Decreasing Height (DH) and to put them in succession in a bin of infinite height, forming level by level, each one is defined by the height of the biggest item. An example of this first step is illustrated as follows.

Illustration (SP) problem: see Table 1 and Fig. 4.

Table 1
Items used for illustrating (SP) in (DH).

Items	a_6	a_7	a_8	a_9
Quantities	3	2	3	6
Dimensions and forms	$(100, 40)$	$(80, 60)$	$(60, 40)$	$(40, 20)$
				

The building of levels can be shown clearly in a bin of infinite height in the Fig. 4. In the second phase, each level is considered as an item and these new items (levels) are stored in a Next-Fit Strategy, First-Fit or Best-Fit, etc. To summarize, this model is converted to (1BP) problem in the second phase, see [18, 19].

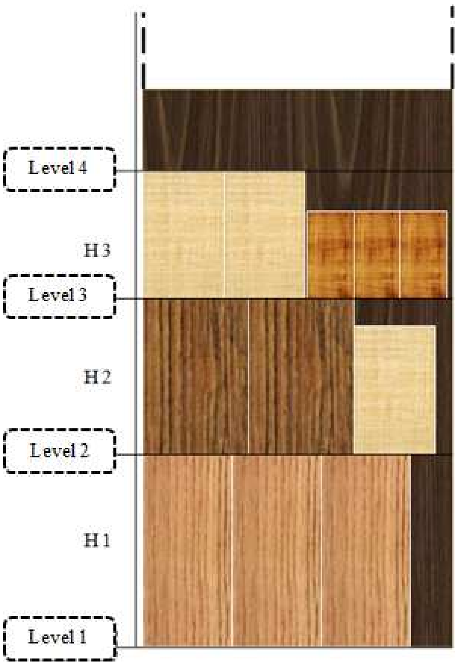


Fig. 4. Illustration of (SP).

- **Algorithms in single phase:**

The Finite First Fit algorithm (FFF) is one of the algorithms that proceed in a single phase. It puts items by levels and is to sort the objects in descending order of height. Each item is put in the lowest level of the first bin that can contain it. If no level can contain this item, a new level is created in the first appropriate bin or by setting a new bin. For better results, the Finite Best Fit algorithm (FBF) was adopted. In this algorithm, each item is put in the level that presents the least gap among the levels of open bins that can contain it. If no level can contain the this item, a new level is created in the first appropriate bin or by setting a new bin. For more clarifications, see illustration of the Finite Best Fit algorithm (FBF) in Fig. 5.

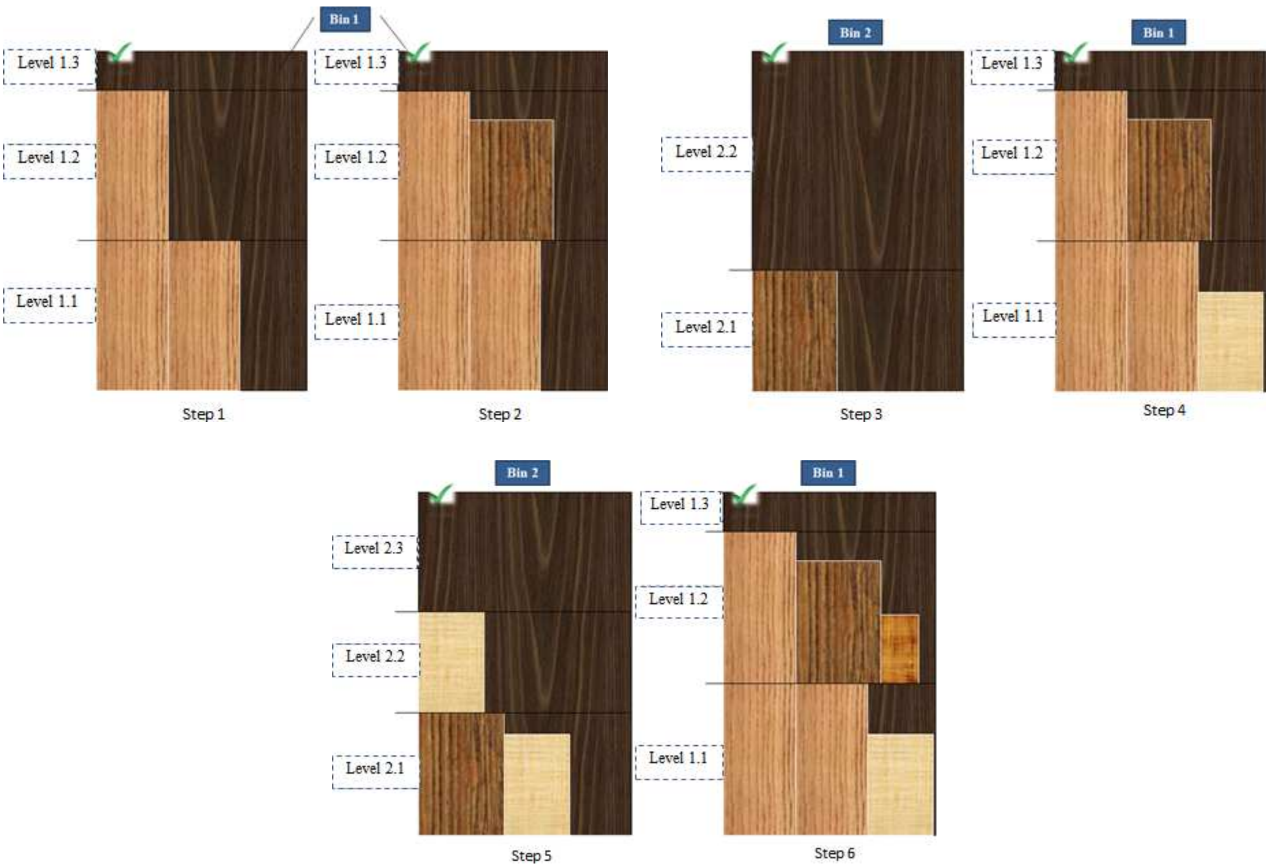


Fig. 5. Simulation of FBF steps 1-6.

Developing “Application Cutting Optimization”

To cope with the problematic described in the first section, an application called “Application Cutting Optimization” or “ACO” is developed, see Fig. 6. To implement the designed solution, the chosen heuristics are programmed in Visual Basic.Net. The solution consists of two different approaches. Firstly, we make use of adopted heuristics in order to generate the possible pattern k for each bin j. Each pattern must respect the ceiling, the maximum allowed wastes rate, in other words, the wastes generated by each pattern k must be under a predetermined rate called the ceiling. Secondly, the outputs of those heuristics are c_{ij}^k which present the input of the mathematical model. This latter is numerically solved to get x_j^{k*}, the optimal number of times the bin j should be cut according to the pattern k. For more clarification, the Fig. 7 describes the operating principle of “ACO”.

Fig. 6. Login interface of “ACO”: type the password and valid it.

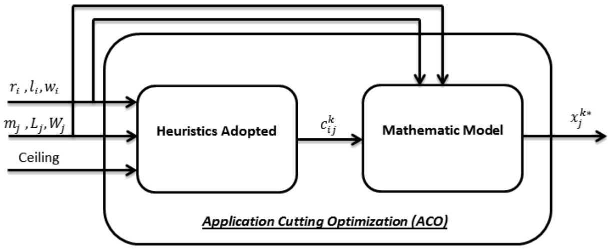


Fig. 7. Block diagram of “ACO”.

A real case study results

The main goal of the company SPA is to minimize the wooden wastes deduced of cutting the noble wood. Not only this latter is expensive compared to the white wood and the red one but also its rate wastes is very high due to its beams of various sizes. So, we have focused on assessing the benefit achieved by cutting the noble wood. To be clear and brief, a real case study will be presented in detail in this section.

In this real case, we have $J = 10$ types of bins, and 12 types of items a_i where $i \in [\![1; 12]\!]$. Characteristics of each bin are described in Table 2.

On the other hand, characteristics of each item are described in Table 3.

Table 2
Characteristics of each bin.

Bins	1	2	3	4	5	6	7	8	9	10
Length [cm]	490	490	490	490	490	490	490	490	490	490
Width [cm]	14.5	16.5	19	20	22	24	26	28.5	31	37
Thickness [cm]	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2	5.2
Quantity m_j	7	17	7	11	4	5	11	1	3	5

Table 3
Characteristics of each item.

Items	Quantity requested r_i	Length cm	Width cm	Thickness
a_1	78	230	11	4
a_2	30	83	11	4
a_3	12	63	11	4
a_4	3	100	11	4
a_5	12	63	16	4
a_6	18	83	16	4
a_7	3	100	16	4
a_8	24	63	8.5	4
a_9	36	83	8.5	4
a_{10}	6	100	8.5	4
a_{11}	6	125	8	4
a_{12}	93	230	8	4

There is a little difference observed between the thickness of items and that of the beams, and this difference does not allow more than one article so this fact justifies the conversion of the problem from the three-dimensional cutting stock problem into a two-dimensional cutting stock problem. Via heuristics adopted in "ACO", all patterns possible are generated and we get the coefficient c_{ij}^k as soon as the wooden wastes do not exceed the Ceiling 40%. For instance, patterns related to the bin 8 are detailed in the Table 4.

Each column $c_{i8}^1, c_{i8}^2, ..., c_{i8}^8$ are the patterns proposed for cutting the bin 8, those coefficients are the input of the mathematical model. The optimal quantity of bins used to resolve this real case is presented in Table 7. For more details concerning x_j^{k*}, the optimal patterns chosen, see the corresponding Table 5. To explain the Table 5, for the bin $j = 2$, the quantity available in the stock is $m_j = 17$, according to our approach we propose using just 10 bins. 8 bins will be cut according to this pattern $(0,0,0,0,0,0,0,0,0,0,0,4)$, whereas 2 bins will be cut according to this pattern $(0,0,0,0,0,0,0,0,0,0,1,3)$ which propose cutting 1 item a_{11} and 3 items a_{12}. To prove the effectiveness of the current research and to assess the economic gain, this proposed approach has to be compared with the method used by the company SPA, see Table 6. To clarify, for the beam 2 (bin j=2), the company has used all the 17 available beams, and the rest is 0. So, the company got 12 items a_5, 18 items a_6, 6 items a_7, and 44 items a_{12}.

Next, the volume of wood proposed is compared with the volume of wood used by SPA to resolve the handled set of data as described above.

We exploit the Table 7 which describes bins used by each method in order to evaluate V_{bc}, the volume of bins used by the company and V_{ba}, the volume of bins proposed by the optimal approach.

Table 4
Patterns generated for bin 8.

Items/Patterns	Pattern 1	Pattern 2	Pattern 3	Pattern 4	Pattern 5	Pattern 6	Pattern 7	Pattern 8
c_{i8}	c_{i8}^1	c_{i8}^2	c_{i8}^3	c_{i8}^4	c_{i8}^5	c_{i8}^6	c_{i8}^7	c_{i8}^8
Item a_1	0	0	0	0	0	2	4	0
Item a_2	6	4	0	0	6	0	0	0
Item a_3	1	0	2	0	0	0	0	0
Item a_4	0	1	0	0	0	0	0	0
Item a_5	1	0	3	0	0	0	0	0
Item a_6	3	3	0	0	3	0	0	0
Item a_7	0	1	0	0	0	0	0	0
Item a_8	0	0	0	0	0	0	0	0
Item a_9	2	0	10	0	2	0	0	0
Item a_{10}	0	2	0	0	0	0	0	0
Item a_{11}	0	0	0	0	0	0	0	2
Item a_{12}	0	0	0	06	0	3	0	4
wastes [%]	8.34	12.27	16.39	19.53	20.75	22.88	26.23	31.77

Table 5
Optimal patterns.

Bin j	1	2		3	4			5	6			7			8	9		10
Quantity available m_j	7	17		7	11			4	5			11			1	3		5
Quantity proposed	0	10		7	11			4	5			6			1	3		4
x_j^{k*}	0	8	2	7	2	7	2	4	2	1	2	3	1	2	1	2	1	4
Item a_1	0	0	0	2	0	2	0	4	0	2	0	0	0	0	0	4	0	6
Item a_2	5	0	0	0	5	0	0	0	0	0	0	4	6	0	4	0	0	0
Item a_3	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	4	0
Item a_4	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
Item a_5	0	0	0	0	1	0	1	0	0	0	0	1	1	0	0	0	4	0
Item a_6	0	0	0	0	0	0	0	0	0	0	0	3	0	3	3	0	0	0
Item a_7	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
Item a_8	0	0	0	0	0	0	8	0	0	0	0	1	1	0	0	0	4	0
Item a_9	0	0	0	0	5	0	0	0	0	0	0	3	6	3	0	0	5	0
Item a_{10}	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	0	0	0
Item a_{11}	0	0	1	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0
Item a_{12}	0	4	3	2	0	2	0	0	6	3	4	0	0	0	0	2	0	0

Table 6
Extract of the excel file used by SPA to resolve cutting problem.

Items			1	2	3	4	5	6	7	8	9	10	11	12	
Beam	Used	Rest													Wastes %
1	3	4	0	15	3	0								0	0.505
2	17	0					12	18	6					44	0.368
3	7	0	10	10	2	0				2	10	0	0	10	0.358
4	11	0	22	0	0					0	0			22	0.397
5	4	0	16												0.354
6	4	1	16					0	0	0	0	0	0	0	0.408
7	3	8	0	0	0	0	0	0	0	18	27	9	0	0	0.385
8	1	0	0	5	1	0	1	5	0	0		0		0	0.345
9	3	0	12											6	0.382
10	5	0	15	0	0	0				20	20	20		0	0.414
Items obtained			91	30	6	0	13	23	6	40	57	29	0	82	

Table 7
Quantity of bins used for each method.

bin j	1	2	3	4	5	6	7	8	9	10
Quantity available m_j	7	17	7	11	4	5	11	1	3	5
Quantity proposed	0	10	7	11	4	5	6	1	3	4
Quantity used by SPA	3	17	7	11	4	4	3	1	3	5

Table 8
Unsatisfied items for each method.

Methods	SPA method		Proposed method	
Items	Items obtained	Unsatisfied items	Items obtained	Unsatisfied items
Item a_1	91	0	78	0
Item a_2	30	0	32	0
Item a_3	6	6	12	0
Item a_4	0	3	3	0
Item a_5	13	0	12	0
Item a_6	23	0	18	0
Item a_7	6	0	3	0
Item a_8	40	0	24	0
Item a_9	57	0	36	0
Item a_{10}	29	0	6	0
Item a_{11}	0	6	6	0
Item a_{12}	82	11	93	0

Table 9
Annual consumption of noble wood.

Noble wood	Ash	Beech	Mahogany	Chain
Annual consumption [m^3]	504	48	120	6
Purchasing price [euro/m^3]	1.375	733	917	1.466
Annual Cost [euro]	639.000	35.184	110.040	8.796
Total cost [euro]	793.020			

In addition to this, unlike our approach, the company does not satisfy all items requested as detailed in Table 8. SPA needs another wood to get all items needed and not forget to take into consideration the waste wooden declared by the company.

The volume of wood saved G is written by the next expressions.

$$G = V_{bc} + V_n - V_{ba}, \quad (10)$$

$$V_n = V_{ns}/(1 - r), \quad (11)$$

V_n stands for the volume needed to get of unsatisfied items, V_{ns} stands for the volume of unsatisfied items, and r is the rate of the wood wastes declared by SPA.

The rate of the wood wastes saved p_G is written by

$$p_G = (100 * G)/(V_{bc} + V_n). \quad (12)$$

Thus, $G = 0.52$ m^3 and $p_G = 15.2\%$.

The annual consumption of noble wood is detailed in Table 9. So, the annual saving is about 120.5 thousand euros.

Conclusion

The contribution of this work is embodied in several levels. First, wood wastes are optimized and the production cost is minimized. In the second hand, by combining the proposed heuristics with the developed mathematical models, the two dimensional CSP is solved, especially by developing a useful application. Furthermore, this research investigates the application of the proposed approach using a real case study to show its efficiency and applicability.

This two-stage heuristic can be used by a decision maker as a decision support tool to minimize wood wastes. Finally, this resolution approach can be applied in many other sectors such as the tissue, steel or glass industries. Note that this model can be developed to include the thickness and become three-dimensional cutting stock problem in order to resolve other suitable industrial problems. Moreover, as future research, a column generation model can be explored in order to improve the efficiency of the pattern generation stage and thus the quality solution.

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