Is Time Inhomogeneous ?

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ABSTRACT

In this article, we discuss probability of inhomogeneous time in high or low energy scale of physics. Consequently, the possibility was investigated of using theories such as varying speed of light (VSL) and fractal mathematics to build a framework within which answers can be found to some of standard cosmological problems and physics theories on the basis of time non-homogeneity.

Keywords: Quantum Gravity; Lorentz Invariance; Symmetry Breaking; Varying Speed of Light; Fractal Mathematics

PACS: 04.60.-m; 11.30.Cp

1. INTRODUCTION

The possibility of breaking physical symmetry in certain cases in the quantum gravity formalism has opened a new window onto this realm of physics. This study concentrates on phenomenological concepts such as Lorentz invariance, in an effort to further investigate the breaking of fundamental symmetries in Einstein's theory of relativity. As we know, violation of Lorentz invariance in high-energy quantum fields theory is predictable [1-4]. In other words, Lorentz invariance cannot represent a perfect symmetry at all energy scales. Our main argument in this research is that perhaps the effects of CPT/Lorentz invariance violation can be observable in nature.

In this case, a new fundamental theory in physics should be sought which takes into account the effects of such symmetry breaking. On the other hand, emergence of new theories in physics, e.g., varying speed of light theory [5] in physics or fractals theories [6] in mathematics, has prompted researchers to consider the matter more seriously. In recent research, the possibility of the existence of fractal (non-integer) dimensions is emphasized. For example, it is possible that spatial dimensions are fractal, e.g., volume might be 3D whereas area may be considered to have 2.1 rather than 2 dimensions. It should be re-emphasized here that when talking about fractal dimensions or mass distributions, we are concerned only with very large or very small scales.

In other view point, fractal cosmology is a set of minority cosmological theories which state that the distribution of matter in the Universe, or the structure of the universe itself, is a fractal. More generally, it relates to the usage or appearance of fractals in the study of the universe and matter. A central issue in this field is the fractal dimension of the Universe or of matter distribution within it, when measured at very large or very small scales. One study of the SDSS data in 2004 found "The power spectrum is not well-characterized by a single power law but unambiguously shows curvature", "thereby driving yet another nail into the coffin of the fractal universe hypothesis and any other models predicting a power-law power spectrum" [2].

Their analysis shows the fractal dimension of the arrangement of galaxies in the universe (up to the range of 30 Mpc/h) to be about 2.1 (plus or minus 0.1). In this regard, theories like Causal dynamical triangulation [3] and Quantum Einstein gravity [3] are fractal at the opposite extreme, in the realm of the ultra-small near the Planck scale. In July 2008, Scientific American featured an article on Causal dynamical triangulation, [4] written by the three scientists who propounded the theory, which again suggests that the universe may have the characteristics of a fractal. However, the underlying fractal of space-time is most noticeable in the quantum world.

2. TIME NON-HOMOGENEITY ?

The main idea in this research is that although the non-homogeneity of time can be directly extracted from the theory of relativity, we should instead, with due regard to astronomical observation data, the suggested models, and the immense density of the universe at the time of the Big Bang, we can use fractal calculations for cosmological evolution. Moreover, fractal calculations can also be utilized to explain other physical phenomena including black holes. In all, fractal mathematics can be used for describing those physical phenomena that cannot be expressed through the use of non-fractal functions or analytical formulas. In following , this article is divided into four main parts: 1) fractal calculus, 2) the varying speed of light theory, 3) fractal physics and 4) conclusion.

2. 1. Fractal mathematics

Non-linear fractal equations and, in particular, differential fractal equations have already been implemented in many fields of physics and engineering. For example, fractal derivatives are used in Newtonian fluids as well as dynamic systems equations. The interacting variation method was first used successfully by HE [7] for ordinary differential equations in control engineering as well as other fields [7]. The mathematical definition of fractal calculations has been considered at different approximation levels. The best known case is called the Riemann-Liouville's Fractal Integration defined as:

$$\frac{d^{-q}f(x)}{dx^{-q}} = \frac{1}{\Gamma(q)} \int_0^x \frac{f(t)dt}{(x-t)^{1-q}}, \quad x > 0 \tag{1}$$

where $q > 0$. On the other hand, the Riemann-Liouville's Fractal derivative is obtained from the following relation:

$$\frac{d^q f(x)}{dx^q} = \frac{d^n}{dx^n}\left(\frac{d^{-(n-q)} f(x)}{dx^{-(n-q)}}\right) = \frac{1}{\Gamma(n-q)}\frac{d^n}{dx^n}\int_0^x \frac{f(t)dt}{(x-t)^{1-n+q}} \qquad (2)$$

where n is an integer that satisfies $n-1 \le q < n$. In other formalism we have

$$D_{a+,t}^d f(t) = \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(t')dt'}{\Gamma(n-d(t'))(t-t')^{d(t')-n+1}}, \qquad (3)$$

$$D_{b-,t}^d f(t) = (-1)^n\left(\frac{d}{dt}\right)^n \int_t^b \frac{f(t')dt'}{\Gamma(n-d(t'))(t'-t)^{d(t')-n+1}} \qquad (4)$$

where $\Gamma(x)$ is Euler's gamma function, and a and b are some constants from $[0,\infty)$. In these definitions, as usually, $n = d+1$, where d is the integer part of d if $d \ge 0$ (i.e. $n-1 \le d < n$) and $n = 0$ for $d < 0$.

In the following sections, we refer briefly to the varying speed of light theory, and consequently use in Section 3 the previously discussed subjects for calculation and analysis purposes.

2. 2. Varying Speed of Light theory

In recent years, a new theory, VSL, has been suggested which, through assuming varying speed for light, can be used for correction of the standard cosmological problems. A series of existing problems in the standard cosmological model, such as the flatness problem, the horizon, and the cosmological constant, can be solved through the implementation of this theory [8]. In the most general definition of this theory, it could be claimed that within the framework of this theory, laws of physics may be obtained by substituting c (speed of light) in the standard action (which is subject to Lorentz invariance) with the field $c = c(x^\mu)$. As can be seen in this equation, the speed of light is no longer constant, but depends on a specific time and place in space. In some models, only the time dependence is considered for the speed of light. However, by further taking into account the spatial dependence of the speed of light, we would pave the way for fractal calculus since in so doing, we actually divide space into fractals with unique properties (for more information, see [8]). Usually, the temporal dependence of the speed of light is coupled with an exponent of the cosmological scale factor, i.e., the transformation of the cosmological model in turn determines the way the speed of light varies with time. For example in some models [5,8] speed of light relation given as

$$c = c_0[a(t)]^n \qquad (5)$$

where a is scale factor and n is an arbitrary constant. We note that n in above equation determine by observational data fine tuning [8]. In any case, by using this theory and the fractal calculus, we can, in the following section, revisit some of the problems in physics and cosmology. This means by using fractal mathematics for determination of n in equation(5) can solve some physics problems in fractal differential framework.

2. 3. Fractal Physics and Cosmology

In this section, we first present a brief description of the fractal dimensions method for time and space, where we investigate the extraction of generalized equations for theories of physics. Consequently, we consider the particular case of this problem in cosmological relations. As an example, we begin with the basic governing equations in dynamics of particles, namely, Newton's Law of Motion. It can be shown that upon application of fractal calculus, the classic form of Newton's equation for gravitational forces is expressed as [9]:

$$D_{-,t}^{d_t(r,t)} D_{+,t}^{d_t(r,t)} r(t) = D_{+,r}^{d_r} \Phi_g(r(t)) \tag{6}$$

here, the formerly integer time and space derivatives are replaced by non-integer fractions that depend on the value of parameter d. In a more generalized case, the Euler equations in classical mechanics can be written in fractal form as:

$$\frac{\partial}{\partial L}(a(L)\frac{\partial d_{t,\alpha}}{\partial L}) + b(L)(L-L_0)d_\alpha + c(L)d_\alpha^2 = 0 \tag{7}$$

Thus, it can be shown that physical fields may be reconstructed into their fractal form in which temporal and spatial dimensions are non-integers.

In other hand, impact fractal calculations and Cosmology, we can point out providing a satisfactory answer to anomalies reported in the detection of ultrahigh energy cosmic rays (UHECR) and TeV-photon spectra [9]. For example, one has suggested a possible solution that complies with relativity and is consistent with the Cantorian geometry of space–time at high-energy scales or equivalent to fractal calculation [9]. There, the author assumed a new dispersion relation for relativistic energy in fractal formalism $E^{2\beta} =| p |^{2\alpha} +(m_0^0)^2$ and shown a attractive solution for (UHECR) anomalies. Also fractal calculations offer a reliable tool for study the deep ultraviolet region of field theory, action principle, gauge-free theories, origin of spin [6,9].

In context of fractal cosmology, previous work[10] have been obtained the equation for the relative density of dark matter and dark energy and the deceleration parameter. This model demonstrates new types of evolution, which are not common to cosmological models with this type of interaction. In this regards, the continuity equation in fractal cosmology takes the form

$$\dot\rho + \left[(D-1)H + \frac{\dot v}{v}\right](\rho + p) = 0, \tag{8}$$

when $v = 1$ and $D = 4$ (where $D = 2 + \varepsilon$), we recover the standard Friedmann equations in four dimensions.

3. CONCLUSIONS

At the start of this article, we asked fundamental questions such as "Are time and space continuous?", "Why is time non-invertible?", and "How could time and space dimensions be

described?". This research presents a theory on the nature of time and space based on fractal geometry. It is obvious that the fractal dimensions of space lead to the production of a new density for space-time. Interestingly, sometimes it is not possible to separate the changes of space and time form each other. In this discussion, we have assumed that space and time are material fields specified according to the contents of the universe. Based on this, the presented model would be the very first theory to include all forces within a fractal framework. On the other hand, the principle of adaptation also holds, e.g., by ignoring the temporal fractal corrections, we can easily obtain the special theory of relativity. It should of course be noted that determination of the non-integer parts through fractal calculations is to be adjusted through comparison with experimental data. For example, through the use of varying speed of light theory, its dependence on space and time fractals can be specified and further used to describe the non-integer (fractional) order of the differential equations (of course, upon the setting of this condition, no more limitations can be imposed in any comprehensive manner on the speed of the frame and particle at all times). In this way, many thus far unanswered questions in cosmology can be addressed through application of fractal physical fields. The most significant result of this research is that fractal calculations can actually be used to correct all the theories of modern physics. The alternative way would be to convert the fractal derivatives into integer derivatives without introducing any corrections.

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