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Characteristic frequencies in averaged description of step-down (BUCK) DC-DC power converter

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Abstract: In the description of small-signal transmittances of switch-mode power converters several characteristic frequencies are usually used, corresponding to poles and zeros of transmittances. The knowledge of these frequencies is important in the design of control circuits for converters and usually are assumed to be constant for a given power stage of a converter. The aim of the paper is to evaluate the influence of converter primary parameters and load conductance on characteristic frequencies. Analytical derivations and numerical calculations are performed for an ideal and non-ideal BUCK converter working in continuous or discontinuous conduction mode.

Key words: switch mode DC-DC converter; step-down converter, BUCK power converter; small-signal transmittances; sharacteristic frequencies

Selected symbols and abbreviations

C – capacitance; C_Z – equivalent capacitance; d_A, D_A – duty ratio and its steady-state value; f_S – switching frequency; *f (with other subscripts)* – characteristic frequencies; *G* – load conductance; G_A – equivalents conductance defined by Eq. 39; *H (with subscripts)* – small-signal transmittances of converter; *L* – inductance; M_I – current static transmittance in *DCM*; *Q* – coefficient in $H_t(s)$ dependence; *R* – load resistance; *R (with subscripts)* – parasitic resistances; R_Z – equivalent parasitic resistance; *SW1, SW2* – ideal semiconductor switches; T_S – switching period; θ – small-signal representations of duty ratio in s-domain; ω *(with subscripts)* – characteristic pulsations; *CCM* – continuous conduction mode; *DCM* – discontinuous conduction mode.

1. Introduction

Characteristics and parameters of switch-mode DC-DC power converters with PWM control are described in many sources including textbooks such as, for example [1] and [2]. A typical converter consists of the power stage and control circuit. The power stage of the converter together with the control circuit should be considered as nonlinear, dynamic system with feedback.

A standard description of the converter power stage, useful in the designing control circuit, is obtained in two steps. First, the averaged model is derived, describing mutual relations of currents and voltages averaged over a single switching period. Next, a small-signal model is obtained by linearization of the averaged model [1] – [4]. The small-signal model may be expressed in the form of a set of transmittances or an equivalent circuit [5]. The control circuit consists of the so-called compensator and PWM modulator and may be realized in an analog or digital form. The compensator in this paper is considered as a linear object and may be described by properly defined transmittance.

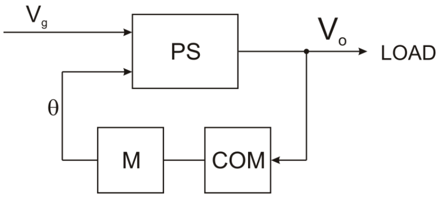

Fig. 1. Model of power converter with feedback

A model of the converter with feedback (Fig. 1) consists of the small-signal model of power stage (PS), compensator (COM) and PWM modulator (M) described by transmittances H_d, H_c and H_m respectively. The loop gain H loop of the converter is defined as:

$$H_{\text{loop}} = H_d \cdot H_c \cdot H_m. \tag{1}$$

The methods of converter design, in particular their control circuits are considered in textbooks and papers [1, 2, 6-10] as well as in manufacturer's technical reports and application notes, for example [11-15]. In a standard approach, an analog control circuit is used, based on an error amplifier. In particular, so called "type II" or "type III" compensator circuits are used [7, 12, 14]. The frequency characteristic of compensator transmittance H_c is determined by values of capacitances and resistances in a local feedback loop. The transmittance H_m is usually assumed to be real number independent of frequency, therefore the frequency characteristic of loop gain is determined by H_d and H_c transmittances [11-15].

The principal step of compensator design is to shape the frequency dependence of its transmittance in such a way that the loop gain of converter with feedback fulfils specified criteria concerning its magnitude and phase, to ensure system stability. It may be achieved if poles and zeros of compensator transmittance H_c are chosen properly to compensate poles and zeros of the control-to-output transmittance H_d of the power stage. Therefore, the knowledge of characteristic frequencies corresponding to poles and zeros of the control-to-output transmittance of the converter power stage is necessary for finding satisfactory characteristics of the control circuit. A typical design of the control circuit assume a constant position of the above poles and zeros however, the position of poles and zeros of transmittances depends on primary converter parameters. The main objective of the paper is the analysis of the influence of load conductance, as well as inductance L and capacitance C on characteristic frequencies of the BUCK converter. In particular, the situations are considered in which the dependence of characteristic frequencies on load conductance cannot be neglected.

The description of the small-signal transmittances of simple PWM converters and their characteristic frequencies is presented in Sec. 2 in general form for the case of complex or real poles. Sec. 3 is devoted to the presentation of the expressions for transmittances of an ideal BUCK converter for the continuous conduction mode (*CCM*) and discontinuous conduction mode (*DCM*) and exemplary results of numerical calculations of characteristic frequencies corresponding to these transmittances. Similar analysis and calculations for a non-ideal BUCK converter (with parasitic resistances) are presented in Sec. 4.

2. General form of power stage transmittances and characteristic frequencies

Several transmittances may be used in the small-signal, s-domain description of the converter power stage including control-to-output and input-to-output transmittances, output impedance and input admittance [1-5, 16]. Control-to-output and input-to-output transmittances H_d and H_g are defined as:

$$H_d(s) = \left.\frac{V_o}{\theta}\right|_{V_g=0}, \tag{2}$$

$$H_g(s) = \left.\frac{V_o}{V_g}\right|_{\theta=0}. \tag{3}$$

Symbols V_o, V_g and θ denote s-domain representations of small-signal terms of output and input voltage and duty ratio of a switching waveform respectively. The power stage of a simple converter such as a basic step-down (BUCK) converter contains two semiconductor switches and reactive components: the capacitor and inductor coil. For such converters the above transmittances are of the second order, i.e. have the following general form:

$$H_t = H_o \frac{1 + s/\omega_Z}{1 + s/(Q \cdot \omega_o) + s^2/\omega_o^2}. \tag{4}$$

In some cases, the form of transmittances is simpler one:

$$H_s = H_{so} \frac{1 + s/\omega_{ZS}}{1 + s/\omega_{PS}}. \tag{5}$$

Four characteristic frequencies correspond to parameters of expressions (4) and (5):

$$f_o = \frac{\omega_o}{2\pi}, \quad f_Z = \frac{\omega_Z}{2\pi}, \quad f_{ZS} = \frac{\omega_{ZS}}{2\pi}, \quad f_{PS} = \frac{\omega_{PS}}{2\pi}, \tag{6}$$

where f_Z is frequency of zero of H_t and f_{PS} – frequency of pole of H_s transmittance. Poles of the transmittance H_t, according to Eq. (4) are:

$$s_{1,2} = \omega_o \left(-\frac{1}{2Q} \pm \sqrt{\frac{1}{4Q^2} - 1} \right). \tag{7}$$

For the condition:

$$Q > \frac{1}{2}, \tag{8}$$

the above poles are complex:

$$s_{1,2} = \sigma \pm j \cdot \omega_R , \tag{9}$$

where:

$$\sigma = -\frac{\omega_o}{2Q} \tag{10}$$

and

$$\omega_R = 2\pi \cdot f_R = \omega_o \sqrt{1 - \frac{1}{4Q^2}} , \tag{11}$$

where f_R is the pole frequency (or resonant frequency) of H_t transmittance. In some situations the term proportional to s in numerator of expression (4) may be omitted and the magnitude of H_t is:

$$\left| H_t(\omega) \right| = \frac{H_{to}}{\sqrt{\left(1 - \omega^2 / \omega_o^2\right)^2 + \omega^2 / \left(Q \cdot \omega_o\right)^2}} . \tag{12}$$

Zero in transmittances describing the BUCK converter is a consequence of parasitic resistance (ESR) of the capacitor. In the idealized description of BUCK (as for example in [1]) this zero is neglected. The maximum value of $|H_t(\omega)|$corresponds to pulsation ω_M:

$$\omega_M = 2\pi \cdot f_M = \omega_o \cdot \sqrt{1 - \frac{1}{2Q^2}} \tag{13}$$

and is equal to:

$$H_{tM} = H_{to} \frac{2Q^2}{\sqrt{4Q^2 - 1}} \qquad \text{(for } Q > 1/2\text{)}. \tag{14}$$

For $Q > 3$ one obtains approximate expressions for ω_R, ω_M and H_{tM}:

$$\omega_R \cong \omega_M \cong \omega_o , \tag{11'}$$

$$H_{tM} \cong H_{to} \cdot Q . \tag{14'}$$

In the case of $Q < 1/2$, the transmittance H_t has two real poles given by:

$$s_{1,2} = -\omega_A \pm \Delta\omega , \tag{15}$$

where:

$$\omega_A = 2\pi \cdot f_A = \frac{\omega_o}{2Q} \tag{16}$$

and

$$\Delta\omega = 2\pi \cdot \Delta f = \omega_A \cdot \sqrt{1-4Q^2} \; . \tag{17}$$

The pulsation values corresponding to real poles for $Q < 1/2$ are:

$$\omega_1 = 2\pi \cdot f_1 = \omega_A \cdot \left(1+\sqrt{\frac{1}{4}-Q^2}\right), \tag{18}$$

$$\omega_2 = 2\pi \cdot f_2 = \omega_A \cdot \left(1-\sqrt{\frac{1}{4}-Q^2}\right). \tag{19}$$

Typical graphs of the frequency dependence of magnitude of H_t (for $1/\omega_Z = 0$) are presented in Fig. 2 for complex poles and Fig. 3 for real poles.

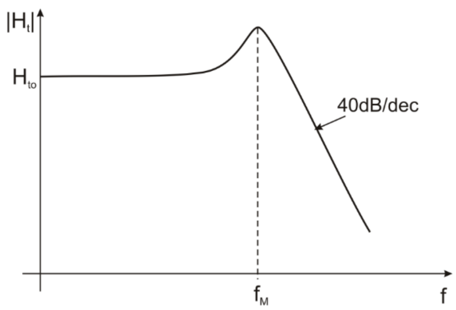

Fig. 2. Frequency characteristics of transmittance H_t magnitude in the case of complex poles

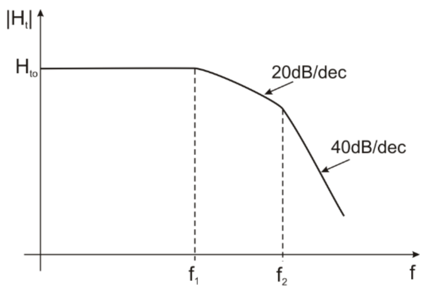

Fig. 3. Frequency characteristics of transmittance H_t magnitude in the case of real poles

The transmittance with two real poles is not typical for real converters and may be observed only in some specific situations, namely for relatively high values of load conductance, especially in a converter with parasitic resistances.

3. Characteristic frequencies of ideal BUCK converter

3.1. Continuous conduction mode (*CCM*)

The scheme of the power stage of an ideal BUCK converter is depicted in Fig. 4. The ideal converter consists of the linear loss-less capacitor and inductor and ideal inertia-less semiconductor switches *SW*1 and $SW2$. The main (or active) switch $SW1$, usually referred to as a transistor is controlled by an external switching signal with the duty ratio d_A. The second switch $SW2$ is treated as passive (diode) in this paper but in practice it may be an additional transistor as well. The load is represented by conductance G or resistance R.

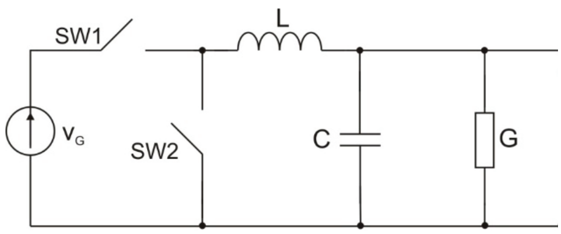

Fig. 4. Basic power stage of ideal BUCK converter

The small-signal transmittances of the ideal BUCK converter in *CCM* obtained from an averaged model are expressed by formulas [1]-[5]:

$$H_{g,i}(CCM) = \frac{D_A}{LC \cdot s^2 + G \cdot L \cdot s + 1}, \tag{20}$$

$$H_{d,i}(CCM) = \frac{V_O}{LC \cdot s^2 + G \cdot L \cdot s + 1}, \tag{21}$$

where additional index i refers to ideal converter. D_A and V_O denote *D.C.* components of duty ratio and output voltage.

By comparing expressions (20) or (21) with general formula (4) one obtains

$$\omega_{oi} = 2\pi \cdot f_{oi} = \frac{1}{\sqrt{L \cdot C}} \tag{22}$$

and

$$Q_i = R \cdot \sqrt{\frac{C}{L}} = (1/G) \cdot \sqrt{\frac{C}{L}}. \tag{23}$$

From the condition (8) for complex values of poles of transmittance one can obtain condition for load conductance or resistance:

$$G < G_{KC} \quad \text{or} \quad R > R_{KC}, \tag{24}$$

where, from (8) and (23):

$$G_{KC} = \frac{1}{R_{KC}} = 2 \cdot \sqrt{\frac{C}{L}}. \tag{25}$$

The values of poles are real numbers for load conductance greater than G_{KC}. Further increasing load conductance (above G_{KC}) results in decreasing Q_C below 1/2 and, according to Eqn. (17), increasing the difference $\Delta\omega_C$ between angular frequencies corresponding to real poles.

The decrease of load conductance below G_{KC} value results (for constant L and C) in increasing Q_C (above 1/2) according to Eqn. (23). As a consequence of decreasing load conductance, the converter may achieve the threshold of discontinuous conduction mode (*DCM*). The condition of working in *DCM* is:

$$G < G_D = \frac{1}{R_D}, \tag{26}$$

where the threshold value of load conductance is:

$$G_D = \frac{T_S}{2L} \cdot (1 - D_A), \tag{27}$$

where T_S is a switching period. The operation in *DCM* is discussed in the next subsection.

A typical design of th BUCK converter assures the operation in *CCM* and complex values of poles of its transmittance H_d that corresponds to the value of load conductance described by condition:

$$G_D < G < G_{KC}. \tag{28}$$

Quantities σ, ω_R, ω_M, ω_A, $\Delta\omega$, ω_1, ω_2 described in the previous section (and corresponding characteristic frequencies $f = \omega/(2\pi)$) may be expressed as functions of converter primary parameters: L, C and load conductance G. The results are presented below.

$$\sigma_i = -\frac{G}{2C}, \tag{29}$$

$$\omega_{Ri} = \frac{1}{2C} \cdot \sqrt{\frac{4C}{L} - G^2} \quad \text{for} \quad G^2 < \frac{4C}{L}, \tag{30}$$

$$\omega_{Mi} = \frac{1}{2C} \cdot \sqrt{\frac{4C}{L} - 2G^2} \quad \text{for} \quad G^2 < \frac{2C}{L}, \tag{31}$$

$$\omega_{Ai} = -\sigma = \frac{G}{2C}, \tag{32}$$

$$\Delta\omega i = \frac{1}{2C} \cdot \sqrt{G^2 - 4C/L} \quad \text{for} \quad G^2 > \frac{4C}{L}, \tag{33}$$

$$\omega_{1,2i} = \frac{1}{2C} \cdot \left(G \pm \sqrt{G^2 - \frac{4C}{L}} \right) \quad \text{for} \quad G^2 > \frac{4C}{L}. \tag{34}$$

Frequency f_{0i}, according to Eqn. (22), is independent of load conductance G and, according to Eqn. (11), characteristic frequencies f_R and f_M are nearly independent of load conductance for Q > 3.

3.2. Discontinuous Conduction mode (*DCM*)

Contrary to the situation for the continuous conduction mode, formulas describing small signal transmittances (for example H_d) of a converter working in *DCM* depend on the method of their derivation. It is shown in [16] that the expressions for transmittances H_g and H_d for *DCM* obtained by switch averaging technique [1], [2] and by the separation of variables approach [4, 5], are different but differences in numerical values calculated with both approaches for typical set of converter primary parameters are relatively small. In this paper the following expressions for transmittances obtained by separation of variables are used:

$$H_{g,D} = H_{gDO} \cdot \frac{1}{1 + s/\omega_{Di}}, \tag{35}$$

$$H_{d,D} = H_{dDO} \cdot \frac{1}{1 + s/\omega_{Di}}. \tag{36}$$

Additional subscript *D* refers to *DCM* mode. Eqs. (35) and (36) correspond to respective equations for H_{gD} and H_{dD} in paper [16].

The above expressions have a single real pole and correspond to description expressed in Sec. 2 by Eq. (5), without a term with zero pulsation. There is only one characteristic pulsation in the above transmittances of an ideal converter in *DCM* and, in accordance with [16] it is described by equation:

$$\omega_{Di} = 2 \cdot \pi \cdot f_{Di} = \frac{G + G_A \cdot M_I^2}{C}, \tag{37}$$

where:

$$M_I = \frac{1}{2} \cdot \left(\sqrt{1 + \frac{4G}{G_A}} + 1 \right) \tag{38}$$

and

$$G_A = D_A^2 \cdot \frac{T_S}{2L}. \tag{39}$$

3.3. Numerical examples

Some numerical experiments have been performed to illustrate the dependencies of characteristic frequencies of the converter power stage on its primary parameters (in particular – load conductance). A standard set of parameters is as follows: $L = 20$ µH, $C = 200$ µF. DC input voltage V_G, duty ratio D_A, and switching frequency f_S do not influence characteristic frequencies of an ideal converter in *CCM*. Characteristic frequency f_{Di} in *DCM* depends on values of D_A and f_S and in calculations for *DCM* (and determination of boundary between *CCM* and *DCM*) it is assumed $D_A = 0.5$, $f_S = 200$ kHz. The load current I_O corresponding to a given value of conductance *G* depends on V_G and D_A, namely it is $G\,V_O$ or (in *CCM*) $G\,D_A\,V_G$, therefore, if for example, $G = 1$ S, $V_G = 5$ V, the load current I_O equals 2.5 A. The load conductance G (or corresponding resistance R) is treated as an independent variable in the most of calculations. Part of calculations is performed for other values of L or C. As it is previously mentioned, in typical conditions the BUCK converter is operated in a *CCM* mode with complex values of the poles of H_d transmittance. The corresponding region of load conductance values is described by inequality (28), where values of G_D and G_{KC} are given by (25) and (27). For the numerical values of the converter parameters given above, we have: $G_D = 0.0625$ S and $G_{KC} = 6.32$ S that corresponds to $R_D = 16\ \Omega$, $R_{KC} = 0.158\ \Omega$ and $Q_D = 50.6$ and $Q_{KC} = 0.5$. For values of load conductance greater than 6.32 S, poles of transmittances are real numbers. For basic set of primary parameters the characteristic pulsation ω_{oi} and corresponding frequency are: $\omega_{oi} = 15.81$ krd/s; $f_{oi} = 2.51$ kHz.

The Q factor of an ideal BUCK converter as a function of load resistance is presented in Fig. 5: for standard set of parameters (curve (*a*) and other values of capacitance C (curves *b* and *c*). Load resistance (not conductance) is chosen as a variable, because, according to Eq. (23), the Q factor depends linearly on R.

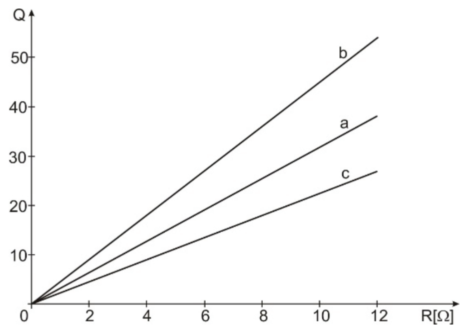

Fig. 5. Dependence of Q factor of ideal BUCK converter on load resistance: (*a*) for standard set, (*b)* for $C = 400\ \mu$F, (*c*) for $C = 100\ \mu$F

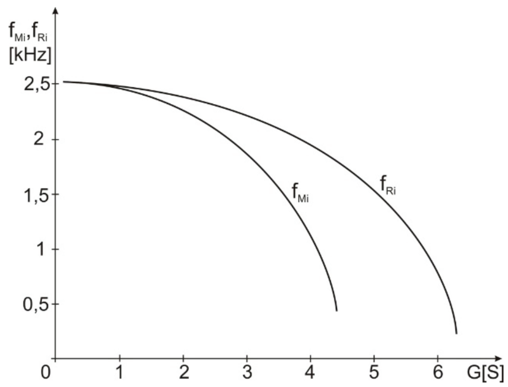

Fig. 6. Dependencies of resonant frequency f_{Ri} and frequency f_{Mi} corresponding to maximum value of $|H_d|$ of ideal BUCK converter on load conductance, for a standard set of parameters

Dependencies of characteristic frequencies f_{Ri} and f_{Mi}, corresponding to pulsations described by Eqs. (30) and (31) on load conductance G, for the case of complex poles are presented in Fig. 6 for basic set of parameters. In Fig. 7 the dependence of frequencies f_{Ri} and f_{Mi} on inductance L are presented for conductance $G = 4$ S and capacitance value as in standard set. The possible values of inductance L in the power stage are limited. For a *CCM* mode, inductance should be greater than threshold value, which may be calculated from Eq. (27) and for a given set of parameters equals 0.3125 μH. For too high inductance values, characteristic frequencies f_R and f_M may be undetermined as may be observed in Fig. 7.

Quantities σ (corresponding to the case of complex poles) and ω_A (for real poles), are described by similar Eqs. (10) and (16), and are presented in Fig. 8 for separated regions of load conductance. Fig. 9 presents dependencies of f_1 and f_2 (see Eqs. 18, and 19) on load conductance G for the case of real poles.

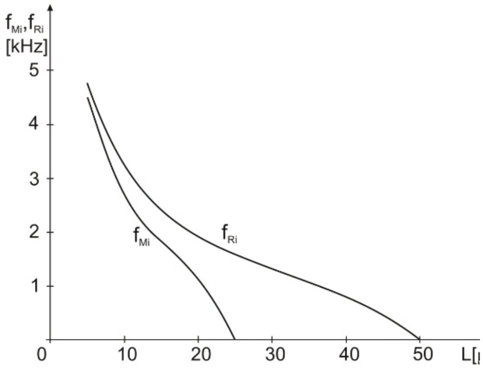

Fig. 7. Dependence of frequencies f_{Ri} and f_{Mi} on inductance L for G = 4 S

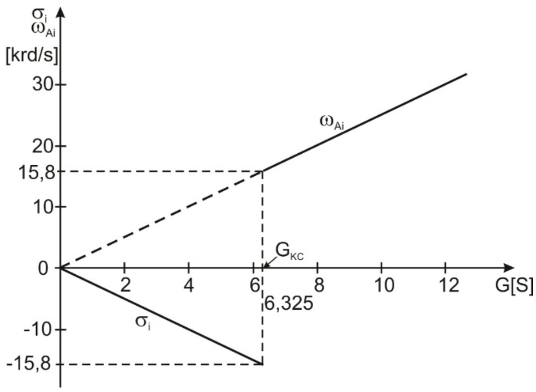

Fig. 8. Dependence of quantities σ and ω_A of ideal BUCK converter on load conductance

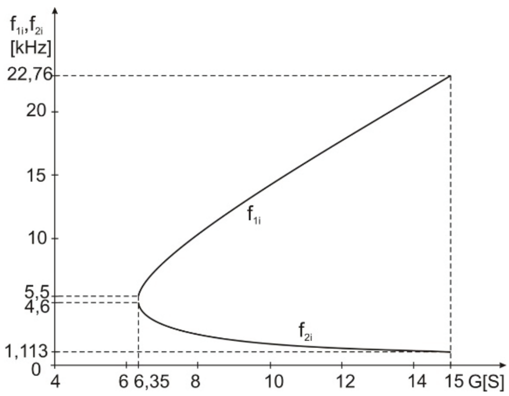

Fig. 9. Dependencies of quantities f_1 and f_2 of ideal BUCK converter on load conductance G for the case of real poles

In the description of frequency characteristics of an ideal BUCK converter in a discontinuous conduction mode (Eqs. 35 and 36), single characteristic frequency, namely f_{Di} is used. The dependence of frequency f_{Di} on load conductance in the region corresponding to *DCM* (i.e. $G < G_D$) for a standard set of converter parameters is presented in Fig. 10.

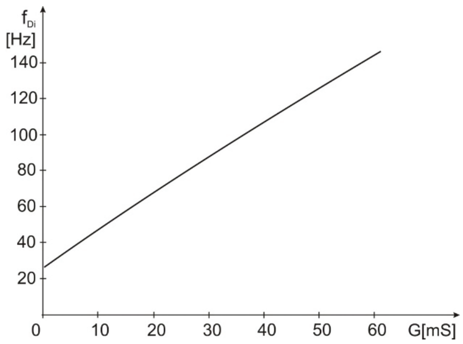

Fig. 10. Dependence of characteristic frequency f_{Di} on load conductance G for *DCM* (C = 200 µF, L = 20 µH, D_A = 0.5, T_S = 5 µs)

It may be observed in Figs. 7-9, that the dependence of characteristic frequencies defined for a continuous conduction mode (*CCM*), on load conductance is relatively weak in the region of low and medium conductance values. Only for higher values of load conductance (in given examples, for $G > 3$ S), corresponding to high values of load current, the changes of characteristic frequencies become more substantial. According to Fig. 10, the influence of load conductance on frequency f_{Di} is significant for the whole range corresponding to *DCM*.

4. Characteristic frequencies of nonideal BUCK converter

4.1. Continuous conduction mode (*CCM*)

Parasitic effects in components of converter power stage influence its characteristics, in particular, its small-signal transmittances and characteristic frequencies. In some sources discussing dynamic characteristics of switch-mode converters, the influence of parasitic effects is neglected [1], or considered only to a limited extent [11-15], for example – by including only parasitic resistances of capacitors and inductors. In papers [4, 5] and [16], the parasitic effects are represented by resistances in series with an ideal transistor, diode, capacitor and inductor. The same description is accepted in the present paper with notation R_T, R_D, R_C and R_L respectively (see Fig. 11).

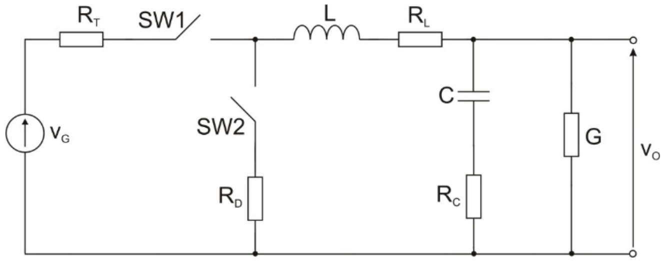

Fig. 11. Power stage of BUCK converter with parasitic resistances (symbols *SW*1, *SW*2, *L* and *C* denote ideal components)

Equations describing the dependence of small-signal transmittances of a non-ideal BUCK converter on s-variable for a *CCM* mode corresponds to general formula (4) but parameters Q, ω_0, ω_Z are different than for an ideal converter. Using the expression for transmittance H_d for the non-ideal converter given in [16] one obtains:

$$\omega_{0P} = \frac{1}{\sqrt{LC}} \cdot \sqrt{\frac{G \cdot R_Z + 1}{G \cdot R_C + 1}}\,, \tag{40}$$

$$Q_P = \frac{\sqrt{L \cdot C_Z \cdot \left(G \cdot R_Z + 1\right)}}{G \cdot L + C_Z \cdot R_Z + C \cdot R_C}\,, \tag{41}$$

$$\omega_Z = \frac{1}{C \cdot R_C}\,, \tag{42}$$

where:

$$R_Z = R_L + D_A \cdot R_T + \left(1 - D_A\right) \cdot R_D\,, \tag{43}$$

$$C_Z = C \cdot \left(1 + G \cdot R_C\right). \tag{44}$$

Additional subscript P concerns converter with parasitic resistances. Using expressions (40)-(42) for quantities ω_{0P}, Q_P and ω_Z together with general formulas (11), (13), (16), (18), (19), one may calculate characteristic frequencies of a nonideal BUCK converter in *CCM*. The examples of such calculations are presented in Sec. 4.3.

4.2. Discontinuous conduction mode (*DCM*)

The influence of parasitic resistances on small-signal transmittances of a converter depends on the value of term $G{\cdot}R_Z$. The converter may work in *DCM* for the sufficiently low value of load conductance G, expressed by inequality (26). In such a case, the term $G{\cdot}R_Z$ is relatively low and parasitic resistances have a weak influence on the converter transmittances (and corresponding characteristic frequencies). For the data assumed in calculations presented in Sec. 3.3, the maximum value of load conductance for a *DCM* mode is $G_D = 0.0625$ S. The real value of equivalent parasitic resistance R_Z (see Eq. 43) does not exceed 0.2 Ω. The resulting value of the product $G\,R_Z$ is below 0.0125, i.e. $G\,R_Z \ll 1$. As a consequence, the denominator of converter small-signal transmittances is not practically influenced by parasitic resistances. Parasitic resistance of capacitor R_C (known as equivalent series resistance) introduces additional zero to expressions for H_d transmittance in *DCM* as well as in *CCM* [16], therefore this transmittance for a non-ideal converter in *DCM* may be approximated by expression:

$$H_{d,DP} \cong H_{dDO} \cdot \frac{1 + s/\omega_Z}{1 + s/\omega_{Di}}\,. \tag{45}$$

The frequency of ESR-induced zero in *DCM* is the same as in *CCM* and is described by Eq. (42).

4.3. Numerical examples

Eqs. (40)-(44) are used in calculations of quantity Q_P and frequency f_{0P} for nonideal BUCK converter in *CCM*. Apart from a standard set of parameters (the same as in Sec. 3.3), parasitic resistances of components are taken into account. It is assumed $R_Z = 3\,R_C$ and calculations have been performed for two exemplary values of R_C: 20 mΩ and 40 mΩ. Dependencies of Q_P obtained for the region corresponding to complex poles on load resistance R are shown in Fig. 12. The influence of parasitic resistances on the dependence of Q on R may be evaluated by comparison of Figs. 5 and 12.

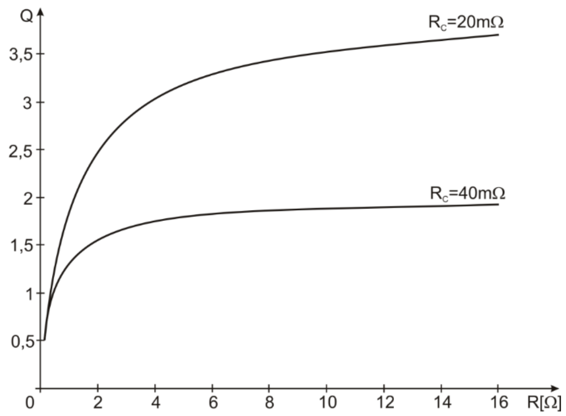

Fig. 12. Dependence of *Q*-factor on load conductance for nonideal converter

The frequency f_{0P} does not change itself substantially in the presence of parasitic resistances. For example, for $R_C = 20$ mΩ and $R_Z = 60$ mΩ, the values of f_{0P} corresponding to $G = 0.1$ S and 6.0 S equal 2.52 kHz and 2.77 kHz respectively (whereas in ideal case, $f_{0P} = 2.51$ kHz).

Once the values of ω_{0P} and Q_P are found, the Equations (11), (13), (16), (18) and (19) may be used for calculations of characteristic frequencies f_{RP} and f_{MP} for the case of complex poles and f_{1P} and f_{2P} for real poles. The exemplary results are shown in Figs. 13 and 14 respectively. By comparing Figs. 13 and 14 with characteristics for an ideal converter (Figs. 6 and 9) the influence of parasitic resistances on characteristic frequencies of the converter may be evaluated.

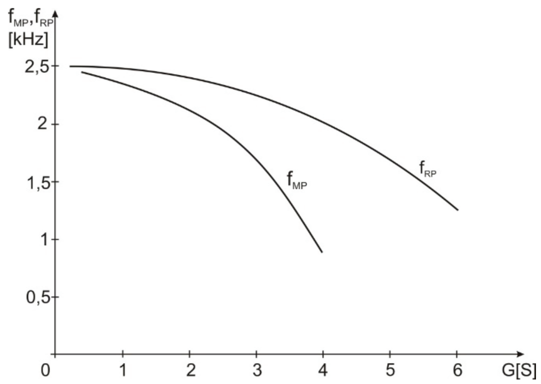

Fig. 13. Dependencies of characteristic frequencies f_{RP} and f_{MP} on load conductance G for $R_C = 20$ mΩ and $R_Z = 60$ mΩ

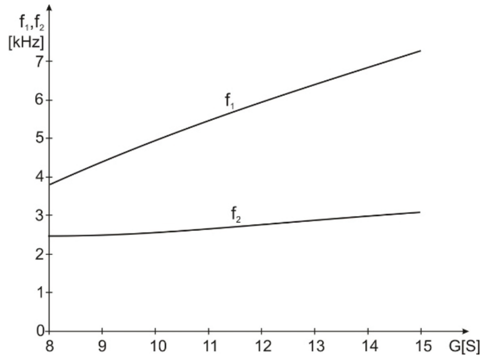

Fig. 14. Dependencies of characteristic frequencies f_{1P} and f_{2P} on load conductance G for R_C = 20 mΩ and R_Z = 60 mΩ

The value of frequency $f_Z = \omega_Z/(2\pi)$ corresponding to zero of transmittances of the non-ideal converter in *CCM* as well as in a *DCM* mode is described by Eq. (42). Taking C = 200 μF, R_C = 20 mΩ, one obtains $f_Z \approx$ 40 kHz. It should be pointed out, that the existence of additional zero in converter transmittance, introduced by parasitic resistance of capacitor (usually referred to as ESR) may be considered as useful because it facilitates the compensation of transmittance poles.

5. Summary and conclusions

Characteristic frequencies, corresponding to poles and zeros of BUCK converter small signal transmittances have been analyzed and calculated in this paper. The knowledge of these transmittances is necessary in the process of designing control circuit of a converter. In a typical approach, constant values of characteristic frequencies are assumed for given values of inductance and capacitance in the power stage. The purpose of this paper is to investigate the influence of converter primary parameters and operating condition on converter characteristic frequencies. The main original result of the paper is the observation, that the characteristic frequencies of the BUCK converter depend not only on inductance and capacitance values of the power stage, but also on the load current. The substantial changes of load current represented by conductance G in the converter model are often observed in converter operation. The resulting changes of characteristic frequencies should be taken into account in control circuit design. Equations derived in the paper concern the ideal converter and converter with parasitic resistances of components. Apart from an analytical description, the exemplary numerical calculations have been performed. The changes of characteristic frequencies corresponding to changes of load current (represented by changes of load conductance) are relatively high, particularly in the discontinuous conduction mode, from 30 Hz to 140 Hz in the example presented in Fig. 10. The dependence of characteristic frequencies on load conductance in a continuous conduction mode is not as significant as in *DCM* and reveals itself only in the region of relatively high values of load conductance (or corresponding load current). Another source of the changes of characteristic frequencies, not analyzed in the paper, may be the dependence of converter parameters (L, C and parasitic resistances) on temperature.

Additional observation resulting from the presented analysis and calculations is relatively strong influence of parasitic resistances of the converter on its characteristic frequencies in a *CCM* mode of operation. It is also worth noting that characteristic frequencies corresponding to poles of converter transmittances (f_R) and to the maximum value of their magnitude (f_M) differ (in some cases – significantly) from frequency f_0, defined by Eq. (22). These differences are neglected in many sources, see for example [11]-[15].

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