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ON STATISTICAL ESTIMATIONS OF VEHICLE SPEED MEASUREMENTS

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Abstract

The accuracy of vehicle speed measured by a speedometer is analysed. The stress on the application of skew normal distribution is laid. The accuracy of measured vehicle speed depends on many error sources: construction of speedometer, measurement method, model inadequacy to real physical process, transferring information signal, external conditions, production process technology *etc*. The errors of speedometer are analysed in a complex relation to errors of the speed control gauges, whose functionality is based on the Doppler effect. Parameters of the normal distribution and skew normal distribution were applied in the errors analysis. It is shown that the application of maximum permissible errors to control the measuring results of vehicle speed gives paradoxical results when, in the case of skew normal distribution, the standard deviations of higher vehicle speeds are smaller than the standard deviations of lower speeds. In the case of normal distribution a higher speed has a greater standard deviation. For the speed measurements by Doppler speed gauges it is suggested to calculate the vehicle weighted average speed instead of the arithmetic average speed, what will correspond to most real dynamic changes of the vehicle speed parameters.

Keywords: vehicle speed, speedometer, maximum permissible error, skew normal distribution, errors.

1. Introduction

Various methods have been developed to determine vehicle speed. Nowadays, big efforts of scientists and engineers are directed to estimate real-time vehicle speed using the image processing techniques to analyse data obtained from traffic control cameras [1–12] single loops [13–21] and license plate detection and tracking [22–25]. Some verification principles and problems of vehicle speedometers are observed in [26–29]. We would like to emphasize the influence of maximum permissible errors and parameters of the skew normal distribution on the vehicle speed estimation.

In the process of technical measurements the quality of the results is determined by the accuracy of these results. To estimate the accuracy of measurements a lot of parameters can be applied. It is important to differentiate the interval and time point estimations. When the estimation is based on a statistically small number of measurements and they are less reliable, the interval estimation is preferable. To control the measurement errors the maximum permissible errors are employed. They define the confidence intervals of measurement errors. In this case the measurement errors are distributed according to the skew normal distribution [30, 31].

We will analyse the application of the maximum permissible errors in the process of measurements of vehicle instantaneous speed v, carried out by speedometers. In fact, there are well known two types of speedometers: Eddy-current and electronic vehicle ones. For our research the most important feature is that speedometers of both types are measuring the number of wheel rotations, so any changes of wheel diameter are influencing the readings of speedometer. Mostly the vehicle speedometers measure vehicle speed with a relative error equal to $\delta_v = 5\%$. This is the measurement instrument error. By this value the speedometer errors are defined in the case of normal distribution. Also we could note, that due to inevitable measurement errors the task of measuring the vehicle speed can give ambiguous results.

2. Theoretical assumptions

The general principle of defining and calculation of the maximum permissible errors is the assumption that the occurrence of events of small probability is practically impossible. The usage of the maximum permissible errors changes the distribution of measurement errors. It becomes a so called skew normal distribution, having slightly different values of average and dispersion [30, 31]. Also, if the maximum permissible errors are applied to control the measurement errors, a correlation occurs between independently measured parameters [32, 33].

The maximum permissible error d of a random error Δ_φ of function φ can be defined from the formula:

$$d = t_\alpha \sigma_\varphi , \tag{1}$$

where t_α – a value of the parameter of distribution (usually set between 2 and 3); σ_φ – standard deviation based on random errors of function φ. The parameter t_α values are calculated according to an accepted probability α (in the case of normal distribution):

$$\frac{1}{\sqrt{2\pi}} \int_{-t_\alpha}^{t_\alpha} \exp - \frac{t^2}{2} dt = 2\Phi(t_\alpha) = \alpha, \tag{2}$$

where Φ – the symbol of Laplace function; $t = \dfrac{\Delta_\varphi}{\sigma_\varphi}$, $\Delta_\varphi = \varphi - M\varphi$, $M\varphi$ – average of function φ.

The maximum permissible error d is calculated in some range of confidence values according to a chosen probability α.

The choice of a probability has a significant practical importance, since it determines the range of confidence values. For example, in some works the "rule of 3σ" is recommended, what corresponds to the probability $\alpha = 0.9973$. For precise measurements the parameter $t_\alpha = 2$ is suitable, which corresponds to a probability $\alpha = 0.9545$. A decrease of the parameter t_α value guarantees the elimination of "big" errors, so the final results of measurements could be better, too. Therefore, tightening of the maximum permissible errors increases probabilities of the occurrence of the first and second order errors.

As we can see from (1), the maximum permissible error d is not a random parameter. Since in practice the standard deviation of function σ_φ is not known, it should be substituted by mean root error m_φ which is calculated from a big enough number of measurement results.

The usage of the maximum permissible errors to control the errors of functions of measurement values changes the parameters of distribution functions. Let us to define the central moments of a symmetrically skewed normal distribution.

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When the maximum permissible error is applied to control a single measurement value, an s-order central moment can be calculated according to the formula:

$$\mu'_s = \int\limits_{-d}^{d} (v - Mv)^s f'(v) dv, \tag{3}$$

where v – speed of a vehicle: $f'(v)$ – symmetrically skewed probability density function.

The central moment of normal distribution, which is cut symmetrically from the mathematical expectancy Mv, is expressed: $\mu'_s = \frac{1}{\alpha\sigma_v\sqrt{2\pi}} \int\limits_{Mv-d}^{Mv+d} (v - Mv)^s \exp -\frac{(v - Mv)^2}{2\sigma_v^2} dv$.

After some mathematical transformations we obtain:

$$\mu'_s = (s-1)\sigma_v^2 \mu'_{s-2} - \frac{2\sigma_v^2}{\alpha\sqrt{2\pi}} t_\alpha^{s-1} \exp -\frac{t_\alpha^2}{2}, \tag{4}$$

where $t = (v - Mv)/\sigma_v$.

By using the coefficient k_1 equal to:

$$k_1 = \frac{1}{\alpha\sqrt{2\pi}} \int\limits_{-t_\alpha}^{t_\alpha} t^2 \exp -\frac{t^2}{2} dt = 1 - \frac{2t_\alpha}{\alpha\sqrt{2\pi}} \exp -\frac{t_\alpha^2}{2}, \tag{5}$$

we can transform the formula to the recurrent form:

$$\mu'_s = (s-1)\sigma_v^2 \mu'_{s-2} + \sigma_v^s t_\alpha^{s-2}(k_1 - 1). \tag{6}$$

The values of coefficient k_1 and corresponding values of parameter t_α are presented in Table 1.

Table 1. Values of parameter t_α and coefficient k_1.

Parameters	**Values**					
α	0.3829	0.6827	0.8664	0.9545	0.9876	0.9973
t_α	0.50	1.00	1.50	2.00	2.50	3.00
k_1	0.081	0.290	0.552	0.774	0.911	0.972

Some expressions of central moments, derived from (6), are listed in (7):

$$\left.\begin{aligned} &\mu'_1 = 0 \\ &\mu'_2 = \sigma_v^2 k_1 \\ &\mu'_4 = \sigma_v^4 \left\{3k_1 + t_\alpha^2 (k_1 - 1)\right\} \\ &\mu'_6 = \sigma_v^6 \left\{15k_1 + \left(5t_\alpha^2 + t_\alpha^4\right)(k_1 - 1)\right\} \end{aligned}\right\}. \tag{7}$$

The values of central moments calculated from (7) are presented in Table 2.

The a priori relative error of the speedometer readings is a ratio of standard dispersion estimation and measurement speed. Because the measurement error values are random, the

Table 2. Values of some central moments.

t_α	μ'_2		μ'_4		μ'_6	
1.0	$0.29\sigma_v^2$	$\sigma_v'^2$	$0.16\sigma_v^4$	$1.90\sigma_v'^4$	$0.09\sigma_v^6$	$3.69\sigma_v'^6$
2.0	$0.77\sigma_v^2$	$\sigma_v'^2$	$1.42\sigma_v^4$	$2.37\sigma_v'^4$	$2.47\sigma_v^6$	$5.32\sigma_v'^6$
3.0	$0.97\sigma_v^2$	$\sigma_v'^2$	$2.66\sigma_v^4$	$2.82\sigma_v'^4$	$11.05\sigma_v^6$	$12.04\sigma_v'^6$

maximum permissible error d of their change depends on a chosen probability α and can be expressed by the formula [32, 33]:

$$|\Delta_v| \leq d = t_\alpha \sigma_v = t_\alpha \cdot \delta_v \cdot v, \tag{8}$$

where Δ_v – random error of speed measurement; t_α – parameter of normal distribution, which depends on the probability α; σ_v – standard deviation of speed measurement; δ_v – relative error of speed determination.

The usage of the maximum permissible errors to control measurement errors is related to a problem, when one parameter of the normal distribution – standard deviation – is changing under the influence of maximum permissible errors. This change can be calculated according to the formula [32, 33]:

$$\sigma_v'^2 = \sigma_v^2 k_1 , \tag{9}$$

where σ'_v – standard deviation of skew normal distribution; k_1 – coefficient, the value of which depends on the probability α (see Table 1).

In the case of a multidimensional value, when maximum permissible errors are employed to control measurement errors of linear functions (for example, weighted averages of speed):

$$\bar{v} = c_1 v_1 + c_2 v_2 + \ldots + c_n v_n \tag{10}$$

we obtain these formulae to calculate the standard deviation:

$$\sigma_{v_i}'^2 = \sigma_{v_i}^2 \left\{1 + c_i^2 \sigma_{v_i}^2 \sigma_{\bar{v}}^{-2} (k_1 - 1)\right\} \tag{11}$$

or

$$\sigma_{v_i}'^2 = \sigma_0^2 \left\{p_{v_i}^{-1} + c_i^2 p_{v_i}^{-2} p_{\bar{v}} (k_1 - 1)\right\}, \tag{12}$$

where p_{v_i} – weight of the i-th component of speed; σ_0 – standard deviation of the assumed measurement result, weight of which is equal to one.

When the function $\bar{v}$ is nonlinear, the values of coefficients c_i can be calculated as partial fluxions, applying linearization to the function $\bar{v}$: $c_i = \dfrac{\partial \bar{v}}{\partial v_i}$.

So, the standard deviation of skew normal distribution σ'_v is always smaller than the standard deviation of normal distribution σ_v, *i.e.* $\sigma'_v < \sigma_v$. When probability $\alpha = 0.9973$ and $t_\alpha = 3.0$, standard deviations of normal distribution and skew normal distribution are practically equal, *i.e.* $\sigma'_v \approx \sigma_v$.

3. Results and discussion

According to (8) we can calculate the values of t_α for some fixed vehicle speeds, assuming $\delta_v = 0.05 : t_\alpha = d/\sigma_v = 2/2.5 = 0.80$ (for $v_1 = 50$ km/h) and $\sigma_{v_1} = 2.5$ km/h and $t_\alpha = 2/5 = 0.40$ (for $v_2 = 100$ km/h and $\sigma_{v_2} = 5$ km/h).

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The corresponding values of parameter k_1 and probability α are equal to: $k_1 = 0.21$ and $\alpha = 0.5629$ when $t_\alpha = 0.80$; $k_1 = 0.04$ and $\alpha = 0.3108$ when $t_\alpha = 0.40$. We can notice, that the probabilities of different vehicle speed errors are different, although the maximum permissible errors applied are equal. That is related to nonlinear intervals of speedometer.

Applying (9) we can calculate standard deviations of skew normal distribution for the presented example: $\sigma'_{v_1} = 1.2$ km/h (when $v_1 = 50$ km/h) and $\sigma'_{v_2} = 1.0$ km/h (when $v_2 = 100$ km/h). These calculations give slightly paradoxical results; when applying the same maximum permissible error to different speeds we obtain that the standard deviations of higher speeds are greater.

The maximum permissible errors of vehicle speed measurement errors should be very reliable and determined with a great probability, which should be close to one. For the normal distribution we can accept $\alpha = 0.9973$ and $t_\alpha = 3.0$. For the example, we have $d_1 = 7.5$ km/h when $v_1 = 50$ km/h and $d_2 = 15$ km/h when $v_2 = 100$ km/h. However, these maximum permissible errors will be slightly greater due to additional errors of the measurement model, measurement method etc.

Modern electronic Doppler speed measurement devices define the average speed measurement in one- or few-second intervals. Therefore, the average speed $\bar{v}$ is not an estimation of average speed. The time point speed values which are traced by a vehicle speedometer are not equivalent from a point of view of precision. According to the statistics laws the weighted average speed should be calculated as:

$$\widetilde{v} = \frac{\sum\limits_i p_{v_i} v_i}{\sum\limits_i p_{v_i}}, \tag{13}$$

where p_i – weight of the i-th component of speed.

The weight of the time point speed v_i can be calculated from the formula:

$$p_{v_i} = \frac{\sigma_0^2}{\sigma_{v_i}^2}, \tag{14}$$

where σ_0 – standard deviation of the assumed measurement result, which weight is equal to one; σ_{v_i} – standard deviation of the i-th measurement result.

Standard deviation of the vehicle speed, which is a nonlinear function of random arguments – distance S and time t, can be defined from the formula [30, 31]:

$$\sigma_v^2 = \left(\frac{\partial v}{\partial \mathrm{S}}\right)_0^2 \sigma_\mathrm{S}^2 + \left(\frac{\partial v}{\partial \tau}\right)_0^2 \sigma_\tau^2 = \frac{1}{\tau^2}\left(\sigma_\mathrm{S}^2 + \mathrm{v}^2\sigma_\tau^2\right). \tag{15}$$

Accepting $\sigma_0 = \sigma_\tau$ from (14) we obtain:

$$p_v = \frac{\tau^2}{v^2 + \sigma_v^2} \approx \frac{\tau^2}{v^2}, \tag{16}$$

where $\sigma_\mathrm{S}/\sigma_\tau \to \sigma_v, \sigma_v \ll v$.

For example, when during a short interval (10 s or 1 s) the vehicle is moving with a speed $v_1 = 50$ km/h and further during the same time interval it is moving with a speed $v_2 = 90$ km/h, when it is doing, for example, an outrun manoeuvre, the average speed is $\bar{v} = 70$ km/h, however the weighted speed is equal to $\bar{v} = 58.8$ km/h. Differences are obviously seen in Fig. 1.

On the other hand, the vehicle speed is regulated by the driver, knowing the vehicle speed measured by the internal vehicle speedometer. In this case some error sources could be also

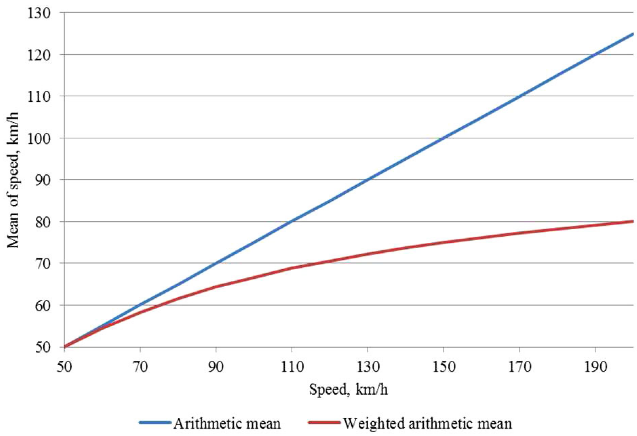

Fig. 1. Average speed against weighted average speed.

defined. For example, a significant influence on the estimation of vehicle speed have the dimensions of tires of the leading wheels. The usage of different tires causes different errors of vehicle speed measurements. For example, when tires of dimensions 185/70R15 are substituted by tires of dimensions 205/50R15 and a diameter of the wheel rim is 15 inches, the length of the external circle of the tire becomes shorter by 8.4%. Accordingly, the readings of vehicle speedometer will be 8.4% higher when the tires of dimensions 185/70R15 are in use, although in this case the real speed of vehicle remains the same. And a cumulative relative error of vehicle speed estimation will be $\delta_v = 5 + 8.4 = 13.4\%$.

When tires with shallower protector are substituted by tires with deeper protector (for example, summer tires are substituted by winter tires) we will receive smaller vehicle speedometer readings at the same vehicle speed.

A similar situation occurs due changes of air pressure p in a tire of the leading wheel. In Table 3 the changes of wheel radius r are presented due to changes of air pressure in a tire (two types: 205/55/R16; 185/65/R14), and, in consequence, changes in speedometer readings.

Table 3. Changes of speedometer readings due to changes of air pressure in a tire of the leading wheel.

***P*, bar**	**Wheel radius *r*, *m***		**Calculated speed, km/h**	
	205/55/R16	**185/65/R14**	**205/55/R16**	**185/65/R14**
1.0	0.295	0.265	96.7	96.4
1.2	0.301	0.268	98.7	97.4
1.4	0.302	0.270	99.0	98.2
1.6	0.303	0.272	99.3	98.9
1.8	0.304	0.274	99.7	99.6
2.0	0.305	0.275	100.0	100.0
2.2	0.306	0.276	100.3	100.4
2.4	0.307	0.277	100.7	100.7
2.6	0.308	0.278	101.0	101.1
2.8	0.308	0.279	101.0	101.4
3.0	0.309	0.280	101.3	101.8

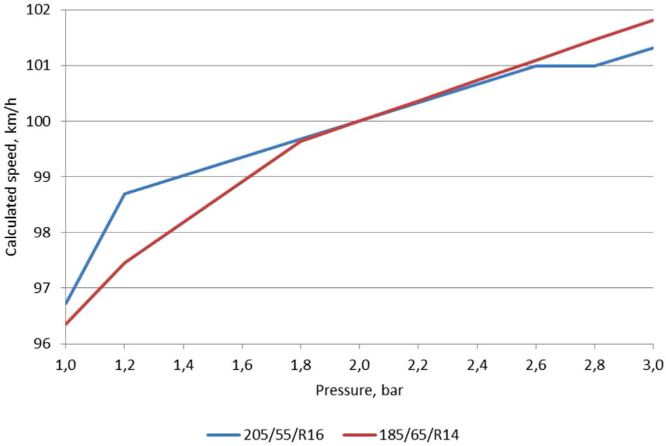

Fig. 2. Changes of speedometer readings due to changes of air pressure in a tire of the leading wheel.

Vehicle speedometers are not related to the usage of tires of corresponding dimensions by any instructions or normative documents. So errors, which occur due to random using tires of different dimensions can be treated as the random errors.

Also, it should be noted that the displaying devices of speedometers are differently graduated. Some displaying devices have equal intervals, and some are divided into two parts: in the first part the intervals have one value, and in the second part the value of intervals is usually greater. In other words, the displaying devices of speedometers are nonlinear. For example, in the first part of a displaying device (from 0 km/h to 80 km/h) the value of interval could be 5 km/h, whereas in the second part (from 80 km/h to 280 km/h) – 10 km/h. That means that the maximum permissible errors of vehicle speed are different for different speeds. In our example the maximum permissible errors at a speed between 80 km/h and 280 km/h will be two times greater than the maximum permissible errors at a speed between 0 km/h and 80 km/h. Taking into account that the car driver could obtain a reading from the speedometer displaying device with an accuracy not better than half of a single step of the displaying device, we have the accuracy of a single reading equal to about 5 km/h. Of course, the driver will achieve a better reading accuracy if the vehicle is equipped with a digital speedometer display showing vehicle speed in the numerical form.

4. Conclusions

1. The theoretical values of maximum permissible errors, which are declared in the instructions, are understandable as equal to zero, however practically they are treated according to the limits of standard deviation of the control radars. It is contrary to the mathematical statistics laws. The maximum permissible errors of speedometer are defined theoretically (when probability $\alpha = 0.9973$) by the formula:
   $|\Delta_v| \leq d = t_\alpha \sigma_v = t_\alpha \cdot \delta_v \cdot v = 3.0 \cdot 0.05v = 0.15v.$
2. Taking into account errors of the whole system, errors of the applied method, influence of the dimensions of tires, the maximum permissible errors can be calculated by the formula:
   $|\Delta_v| \leq d = 3.0 \cdot 0.13v = 0.39v.$

3. The estimation of average speed of a vehicle should be calculated according to the weighted average formula. Looking forward, modern speedometers should send their readings to control points or save information on single trips in the internal memory for further analysis, for example, by controlling officers. Such data will be also extremely important in the processes of analysing accidents, directing the cases to court, *etc*.

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