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Testing the integrated package of tools supporting decision making on identification, prediction and optimization of complex technical systems operation, reliability and safety
Part 2
IS&RDSS Application – Exemplary system operation and reliability unknown parameters identification

Keywords

reliability function, operation processes, estimating

Abstract

There is presented the IS&RDSS application to the operation and reliability models of an exemplary complex technical system unknown parameters identification. There are performed in the paper, the exemplary system operation and reliability analysis and modelling. The identification of the probabilities of transitions this system operation process between the operation states and the conditional mean values of this process sojourn times in the particular operation states because of the lack of statistical data is performed throw the arbitrary fixing their values. assumption. The evaluation of the system components unknown intensities of departures the reliability state subsets and the identification of the exponential forms of their multistate reliability functions on the arbitrarily fixed statistical data coming from the system components states changing processes are performed as well.

3. The exemplary system operation process unknown parameters identification

3.1. The exemplary system analysis

We analyze the reliability of an exemplary system S that consists of two subsystems S_1, S_2. The subsystem S_1 is composed of two series subsystems, each of them composed of 3 components, denoted respectively by

$$E_{ij}^{(1)}, \quad i = 1,2, \quad j = 1,2,3,$$

with the reliability structure presented in *Figure 1.*

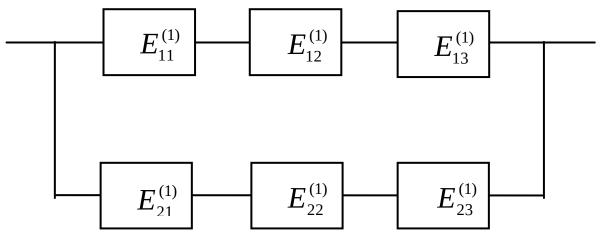

Figure 1. The scheme of the system S_1 reliability structure

The subsystem S_2 is composed of four series subsystems, each of them composed of 2 components, denoted respectively by

$$E_{ij}^{(2)}, \quad i = 1,2,3,4 \quad j = 1,2,$$

with the reliability structure presented in *Figure 2.*

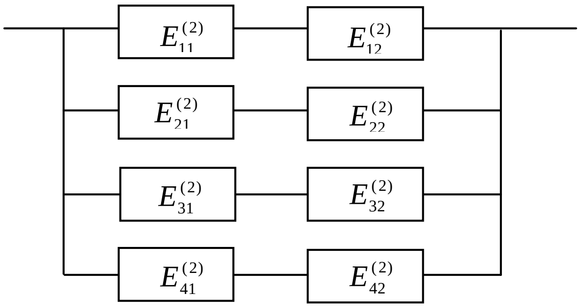

Figure 2. The scheme of the system S_2 reliability structure

The subsystems S_1, S_2, illustrated in *Figures 1–2* are forming a series reliability structure presented in *Figure 3.*

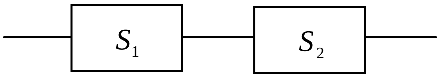

Figure 3. The general scheme of the system S reliability structure

3.2. The exemplary system operation process modelling

Under the assumption that the exemplary system structure and the subsystem components reliability depend on its changing in time operation states, we arbitrarily fix the number of the system operation process states $\nu = 4$ and we distinguish the following as its four operation states:

- an operation state z_1 – the system is composed of the subsystem S_1, with the scheme showed in *Figure 1*, that is a series-parallel system,
- an operation state z_2 – the system is composed of the subsystem S_2, with the scheme showed in *Figure 2* that is a series-parallel system,
- an operation state z_3 – the system is composed of the subsystems S_1 and S_2, with the scheme showed in *Figure 3* that are series-parallel system with the schemes given in *Figures 1-2,*
- an operation state z_4 – the system is composed of the subsystem S_1 and S_2, with the scheme showed in *Figure 3,* while the subsystem S_1 is a series-parallel system with the scheme given in *Figure 1* and the subsystem S_2 is a series-“2 out of 4” system.

Moreover, we assume that there are possible the transitions between all system operation states. Thus, according to Section 2 of [1], the parameters of the system operation process semi-Markov model are:

- the initial probabilities $p_b(0)$, $b = 1,2,3,4,$ of the system operation process $Z(t)$ staying in the particular states z_b at the moment $t = 0$,
- the matrix $[p_{bl}]_{4x4}$ of probabilities of the exemplary system operation process $Z(t)$ transitions between the operation states,
- the matrix $[H_{bl}(t)]_{4x4}$ of conditional distribution functions of the exemplary system operation process $Z(t)$ conditional sojourn times θ_{bl} in the operation states,
- the mean values of the conditional sojourn times θ_{bl}

To identify all these parameters of the exemplary system operation process the statistical data about this process is needed. As the considered system is an exemplary one and its operation process parameters are arbitrarily assumed then we do not have the statistical data collected that are needed for estimating these parameters.

3.3. The exemplary system operation process identification

We do not have statistical data on the exemplary system operation process and to illustrate the procedure comprehensively, we fix the process parameters defined by (2.1) and (2.3) in [1] arbitrarily.

The arbitrarily fixed probabilities of the system operation process transitions from the operation state z_b into the operation state z_l, defined by p_{bl} (2.1) [1], are given in the matrix below

$$[p_{bl}] = \begin{bmatrix} 0 & 0.22 & 0.32 & 0.46 \\ 0.20 & 0 & 0.30 & 0.50 \\ 0.12 & 0.16 & 0 & 0.72 \\ 0.48 & 0.22 & 0.30 & 0 \end{bmatrix} \quad (1)$$

As we do not have the realizations of the conditional sojourn times θ_{bl}, $b,l = 1,2,3,4,$ of the exemplary system operation process at the particular operation states, then it is not possible to identify their distributions defined by (2.2) in [1].

The arbitrarily fixed conditional mean values $M_{bl} = E[\theta_{bl}]$, $b,l = 1,2,.3.4,$ defined by (2.3) in [1], of the system sojourn times in the particular operation states are as follows:

$$M_{12} = 192,\ M_{13} = 480,\ M_{14} = 200,$$

$$M_{21} = 96,\ M_{23} = 81,\ M_{24} = 55,$$

$$M_{31} = 870,\ M_{32} = 480,\ M_{34} = 300,$$

$$M_{41} = 325,\ M_{42} = 510,\ M_{43} = 438. \qquad (2)$$

4. The exemplary system reliability model unknown parameters identification

4. 1. The exemplary system components reliability modelling

We assume that the exemplary system and its components have four reliability states 0, 1, 2 3, i.e. $z = 3$. And consequently, at all operation states z_b, $b = 1,2,3,4$, we arbitrarily distinguish the following reliability states of the system and its components:

- a reliability state 3 – the system operation is fully effective,
- a reliability state 2 – the system operation is less effective because of ageing,
- a reliability state 1 – the system operation is less effective because of ageing and more dangerous,
- a reliability state 0 – the system is destroyed.

We assume that there are possible the transitions between the components reliability states only from better to worse ones and we fix that the system and components critical reliability state is $r = 2$. Moreover, we assume that the changes of the operation states of the system *S* operation process *Z*(*t*) have an influence on the system reliability structure and the system multi-state components reliability as well.

The system operation process influence on the system reliability structure is expressed as follows.

At the system operation state z_1, the system is composed of the series-parallel subsystem S_1 containing two series subsystems ($k = 2$), each composed of three components ($l_1 = 3,\ l_2 = 3$) with the reliability structure showed in *Figure 1*.

At the system operation state z_2, the system is composed of the series-parallel subsystem S_2 containing four series subsystems ($k = 4$), each composed of two components ($l_1 = 2,\ l_2 = 2,\ l_3 = 2,\ l_4 = 2$) with the reliability structure showed in *Figure 2*.

At the system operational state z_3, the system is a series system with the reliability structure showed in *Figure 3*, composed of two series-parallel subsystems S_1, S_2 illustrated in *Figures 1-2*.

The subsystem S_1 consists of two series subsystems ($k = 2$), each composed of three components ($l_1 = 3,\ l_2 = 3$) with the reliability structure showed in *Figure 1*. The subsystem S_2 consists of four series subsystems ($k = 4$), each composed of two components ($l_1 = 2,\ l_2 = 2,\ l_3 = 2,\ l_4 = 2$) with the reliability structure showed in *Figure 2*.

At the system operation state z_4, the system is a series system with the scheme showed in *Figure 3*, composed of the subsystem S_1 and S_2 illustrated in *Figures 1-2*, whereas the subsystem S_1 is a series-parallel system and the subsystem S_2 is a series-"2 out of 4" system.

The subsystem S_1 consists of two series subsystems ($k = 2$), each composed of three components ($l_1 = 3,\ l_2 = 3$) with the reliability structure showed in *Figure 1*. The subsystem consists of four series subsystems ($k = 4$), each composed of two components ($l_1 = 2,\ l_2 = 2,\ l_3 = 2,\ l_4 = 2$) and is a series-"2 out of 4" system ($m = 2$).

The system operation process influence on the system components reliability is expressed by the assumption that the subsystems S_υ, $\upsilon = 1,2$, are composed of four-state, i.e. z = 3, components $E_{ij}^{(\upsilon)}$, $\upsilon = 1,2$, having the conditional four-state reliability functions

$$[R_{ij}^{(\upsilon)}(t,\cdot)]^{(b)}$$

$$= [1,\ [R_{ij}^{(\upsilon)}(t,1)]^{(b)}, [R_{ij}^{(\upsilon)}(t,2)]^{(b)}, [R_{ij}^{(\upsilon)}(t,3)]^{(b)}],$$

$$t \ge 0,\ b = 1,2,3,4,\ \upsilon = 1,2, \qquad (3)$$

with the exponential co-ordinates

$$[R_{ij}^{(\upsilon)}(t,1)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(1)]^{(b)} t],$$

$$[R_{ij}^{(\upsilon)}(t,2)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(2)]^{(b)} t],$$

$$[R_{ij}^{(\upsilon)}(t,3)]^{(b)} = \exp[-[\lambda_{ij}^{(\upsilon)}(3)]^{(b)} t], \qquad (4)$$
$$t \ge 0,\ b = 1,2,3,4,\ \upsilon = 1,2,$$

different in various operation states z_b, $b = 1,2,3,4$, where $[\lambda_{ij}^{(\upsilon)}(1)]^{(b)}$, $[\lambda_{ij}^{(\upsilon)}(2)]^{(b)}$, $[\lambda_{ij}^{(\upsilon)}(3)]^{(b)}$, $b = 1,2,3,4$, $\upsilon = 1,2$, are the subsystems components unknown intensities of departures respectively from the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$.

4.2. The exemplary system components reliability identification

4.2.1. Data collections coming from system components reliability state changing processes

To estimate existing in the formulae (3)-(4) the subsystems components unknown intensities $[\lambda_{ij}^{(\upsilon)}(1)]^{(b)}$, $[\lambda_{ij}^{(\upsilon)}(2)]^{(b)}$, $[\lambda_{ij}^{(\upsilon)}(3)]^{(b)}$, $b = 1,2,3,4,$ $\upsilon = 1,2,$ of departure respectively from the reliability state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$, we suppose that we have in disposal data collected from the system components reliability states changing processes due to the experiment *Case 2* described in Section 6.1.1 of [1]. Namely, we have in disposal the following data for particular components $E_{ij}^{(\upsilon)}$, $\upsilon = 1,2,$ of the system:

- the numbers of identical experiment posts $n^{(b)} = n_{ij}^{(b)}$,
- the observation times $\tau^{(b)} = \tau_{ij}^{(b)}$,
- the numbers $m^{(b)}(u) = m_{ij}^{(b)}(u)$ of components that have left the reliability states subset $\{u, u+1,...,3\}$, $u = 1,2,3,$
- the sets $A^{(b)}(u) = A_{ij}^{(b)}(u) = \{t_i^{(b)}(u) :$ $i = 1,2,..., m^{(b)}(u)\}$ of realizations $t_i^{(b)}(u) = t_{ij}^{(b)}(u)$ of the component lifetimes $T_{ij}^{(b)}(u)$ in the reliability states subset $\{u, u+1,...,3\}$, $u = 1,2,3,$ at the operation state z_b, $b = 1,2,3,4$.

The exemplary data for the component $E_{11}^{(1)}$ of the subsystem S_1 at the operation state z_1 are as follows [2]:

$n^{(1)} = 40,\ \tau^{(1)} = 2600,\ m^{(1)}(1) = 32,$

$A^{(1)}(1) = \{$30, 37, 37, 60, 63, 65, 89, 89, 80, 85, 88, 452, 490, 490, 490, 441, 350, 302, 307, 381,400, 430, 737, 824, 836, 769, 976, 991, 1153, 1697, 1700, 2454$\}$, (5)

$n^{(1)} = 40,\ \tau^{(1)} = 2600,\ m^{(1)}(2) = 32,$

$A^{(1)}(2) = \{$30, 37, 37, 60, 63, 65, 69, 69, 80, 85, 88, 352, 462, 470, 490, 441, 350, 302, 307, 381, 400, 430, 637, 652, 656, 669, 776, 891, 1053, 1597, 1600, 2254$\}$, (6)

$n^{(1)} = 40,\ \tau^{(1)} = 2600,\ m^{(1)}(3) = 32,$

$A^{(1)}(3) = \{$20, 27, 27, 50, 53, 65, 69, 69, 70, 75, 78, 252, 462, 470, 490, 241, 250, 302, 307, 381, 400, 430, 437, 552, 556, 569, 776, 861, 953, 1497, 1400, 2054$\}$. (7)

4.2.2. Estimating system components intensities of departures from reliability state subsets

As there are data collected from the exemplary system components reliability states changing processes, then their reliability functions unknown parameters identification using the methods described in Section 6.2.1.1 of [1] is possible. To find the approximate values $[\hat{\lambda}_{ij}^{(\upsilon)}(1)]^{(b)}$, $[\hat{\lambda}_{ij}^{(\upsilon)}(2)]^{(b)}$ and $[\hat{\lambda}_{ij}^{(\upsilon)}(3)]^{(b)}$ of the subsystems S_υ, $\upsilon = 1,2,$ components unknown intensities $[\lambda_{ij}^{(\upsilon)}(1)]^{(b)}$, $[\lambda_{ij}^{(\upsilon)}(2)]^{(b)}$ and $[\lambda_{ij}^{(\upsilon)}(3)]^{(b)}$ of departure respectively from the reliability states subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$, while the system is operating in the operation state z_b, $b = 1,2,3,4,$ existing in (3)-(4), we can use statistical data presented in Section 4.2.1 and the formula (6.9) from [1]. We can also use the formula (6.10) from [1] to get their pessimistic evaluations.

To illustrate this procedure, we will find the evaluations $[\hat{\lambda}_{11}^{(1)}(1)]^{(1)}$, $[\hat{\lambda}_{11}^{(1)}(2)]^{(1)}$ and $[\hat{\lambda}_{11}^{(1)}(3)]^{(1)}$ of the intensities $[\lambda_{11}^{(1)}(1)]^{(1)}$, $[\lambda_{11}^{(1)}(2)]^{(1)}$ and $[\lambda_{11}^{(1)}(3)]^{(1)}$ of departures respectively from the reliability state subsets $\{1,2,3\}$, $\{2,3\}$ and $\{3\}$ of the component $E_{11}^{(1)}$ of the subsystem S_1, while the system is operating at the operation state z_1.

We proceed as follows:

- from data specified in (5), we have

$$n^{(1)} = 40,\ \tau^{(1)} = 2600,\ m^{(1)}(1) = 32,$$

$$\sum_{i=1}^{m^{(1)}(1)} t_1^{(1)}(1) = 20 + 37 + ...$$

$$+ 1700 + 2454 = 17393,$$

then, according to (6.9) from [1], the evaluations of the intensity $[\lambda_{11}^{(1)}(1)]^{(1)}$ of departure from the reliability state subset $\{1,2,3\}$ is

$$[\lambda_{11}^{(1)}(1)]^{(1)} \cong [\hat{\lambda}_{11}(1)]^{(1)}$$

$$= \frac{m^{(1)}(1)}{\sum_{i=1}^{m^{(1)}(1)} t_i^{(1)}(1) + \tau^{(1)}[n^{(1)} - m^{(1)}(1)]}$$

$$= \frac{32}{17393 + 2600[40-32]} \cong 0.0008 \quad (8)$$

and according to (6.10) from [1], its pessimistic evaluation is

$$[\lambda_{11}^{(1)}(1)]^{(1)} \cong [\hat{\lambda}_{11}(1)]^{(1)}$$

$$= \frac{n^{(1)}}{\sum_{i=1}^{m^{(1)}(1)} t_i^{(1)}(1) + \tau^{(1)}[n^{(1)} - m^{(1)}(1)]}$$

$$= \frac{40}{17393 + 2600[40-32]} \cong 0.0010.$$

- from data specified in (6), we have

$$n^{(1)} = 40,\ \tau^{(1)} = 2600,\ m^{(1)}(2) = 32,$$

$$\sum_{i=1}^{m^{(1)}(2)} t_1^{(1)}(2) = 20 + 37 + ...$$

$$+ 1600 + 2254 = 15853,$$

then, according to (6.9) from [1], the evaluations of the intensity $[\lambda_{11}^{(1)}(2)]^{(1)}$ of departure from the reliability state subset $\{2,3\}$ is

$$[\lambda_{11}^{(1)}(2)]^{(1)} \cong [\hat{\lambda}_{11}(2)]^{(1)}$$

$$= \frac{m^{(1)}(2)}{\sum_{i=1}^{m^{(1)}(2)} t_i^{(1)}(2) + \tau^{(1)}[n^{(1)} - m^{(1)}(2)]}$$

$$= \frac{32}{15853 + 2600[40-32]} \cong 0.0009 \quad (9)$$

and according to (6.10)) from [1], its pessimistic evaluation is

$$[\lambda_{11}^{(1)}(2)]^{(1)} \cong [\hat{\lambda}_{11}(2)]^{(1)}$$

$$= \frac{n^{(1)}}{\sum_{i=1}^{m^{(1)}(2)} t_i^{(1)}(2) + \tau^{(1)}[n^{(1)} - m^{(1)}(2)]}$$

$$= \frac{40}{15853 + 2600[40-32]} \cong 0.0011.$$

- from data specified in (7), we have

$$n^{(1)} = 40,\ \tau^{(1)} = 2600,\ m^{(1)}(3) = 32,$$

$$\sum_{i=1}^{m^{(1)}(3)} t_1^{(1)}(3) = 20 + 27 + ...$$

$$+ 1400 + 2054 = 14243,$$

then, according to (6.9) from [1], the evaluations of the intensity $[\lambda_{11}^{(1)}(3)]^{(1)}$ of departure from the reliability state subset $\{3\}$ is

$$[\lambda_{11}^{(1)}(3)]^{(1)} \cong [\hat{\lambda}_{11}(3)]^{(1)}$$

$$= \frac{m^{(1)}(3)}{\sum_{i=1}^{m^{(1)}(3)} t_i^{(1)}(3) + \tau^{(1)}[n^{(1)} - m^{(1)}(3)]}$$

$$= \frac{32}{14243 + 2600[40-32]} \cong 0.0009 \quad (10)$$

and according to (6.10)) from [1], its pessimistic evaluation is

$$[\hat{\lambda}_{11}(3)]^{(1)} = \frac{m^{(1)}(3)}{\sum_{i=1}^{m^{(1)}(3)} t_i^{(1)}(3) + \tau^{(1)}[n^{(1)} - m^{(1)}(3)]}$$

$$= \frac{40}{14243 + 2600[40-32]} \cong 0.0011.$$

This way obtained evaluations of the unknown intensities of departures from the reliability state subsets for all components of the subsystems at various operation states are presented in [27].

4.2.3. Identifying system components exponential reliability functions

As there are data collected from the system components reliability states changing processes, then it is possible to verify the hypotheses on the exponential forms of the system components conditional reliability functions. To this end, we use the procedure given in Section 6.2.2.1 of [1] and data collected in Section 4.2.1. We may verify the hypotheses on the conditional exponential four-state exemplary components reliability functions

$[R_{ij}^{(\upsilon)}(t,\cdot)]^{(b)}$, $\upsilon = 1,2$, $b = 1,2,3$, at the particular operation states z_b, $b = 1,2,3$. To do this, we need a sufficient number of realizations of the system components lifetime in the reliability state subsets, namely the sets of their realizations should contain at least 30-40 realizations coming from the experiment. This condition is satisfied for the statistical data we have in disposal. To make the procedure familiar to the reader, we perform it for the conditional reliability function of the subsystem S_1 component $E_{11}^{(1)}$ at the system operation state z_1. Considering the evaluated values of the unknown intensities of the component $E_{11}^{(1)}$ departure from the reliability state subsets given by (8)-(10), we formulate the null hypothesis H_0, concerned with the form of its multistate reliability $[R(t,\cdot)]^{(1)}$ in the following form:

H_0: The conditional multistate reliability function of the system component $E_{11}^{(1)}$ at the operation state z_1

$$[R_{11}^{(1)}(t,\cdot)]^{(1)} = [1, [R_{11}^{(1)}(t,1)]^{(1)}, [R_{11}^{(1)}(t,2)]^{(1)}, [R_{11}^{(1)}(t,3)]^{(1)}],$$

has the exponential reliability functions coordinates of the forms

$$[R_{11}^{(1)}(t,1)]^{(1)} = \exp[-0.0008t],$$

$$[R_{11}^{(1)}(t,2)]^{(1)} = \exp[-0.0009t],$$

$$[R_{11}^{(1)}(t,3)]^{(1)} = \exp[-0.0009t] \text{ for } t \in <0,\infty). \quad (11)$$

To verify the hypothesis concerning the exponential form of the coordinate $[R_{11}^{(1)}(t,1)]^{(1)}$ defined by (11), it is due to act according to the scheme below:

- we fix the number of observed components and the number of realizations of the system component conditional lifetimes $T_{11}^{(1)}(1)$ in the reliability states subsets $\{1,2,3\}$, that according to (6.1)[1], are

$$\bar{n}^{(1)} = n^{(1)} = 40, \ \bar{m}^{(1)} = m^{(1)}(1) = 32,$$

- we fix the realizations $t_i^{(1)}(1)$, $i = 1,2,...,32$, of realizations of the exemplary system component conditional lifetimes $T_{11}^{(1)}(1)$ in the reliability states subsets $\{1,2,3\}$ that are given by (6.1)[1],
- we determine the number $\bar{r}^{(1)}(1)$ of the disjoint intervals $I_j^{(1)}(1) =< x_j^{(1)}(1), y_j^{(1)}(1))$, $j = 1,2,...,\bar{r}^{(1)}(1)$, that include the realizations $t_i^{(1)}(1)$, $i = 1,2,...,32$, of the system component conditional lifetimes $T_{11}^{(1)}(1)$ in the reliability states subsets $\{1,2,3\}$

$$\bar{r}^{(1)}(1) \cong \sqrt{\bar{m}^{(1)}(1)} = \sqrt{32} \cong 6,$$

- we determine the length $d^{(1)}(1)$ of the intervals

$$I_j^{(1)}(1) =< x_j^{(1)}(1), y_j^{(1)}(1)), \ j = 1,2,...,6,$$

$$\bar{R}^{(1)}(1) = \max_{1\le i\le 32}\{t_i^{(1)}(1)\} - \min_{1\le i\le 32}\{t_i^{(1)}(1)\}$$

$$= 2454 - 30 = 2424,$$

$$d^{(1)}(1) = \frac{2424}{6-1} \cong 485,$$

- we determine the ends $x_j^{(1)}(1)$, $y_j^{(1)}(1)$, of the intervals $I_j^{(1)}(1) =< x_j^{(1)}(1), y_j^{(1)}(1))$, $j = 1,2,...,6$,

$$\min_{1\le i\le 32}\{t_i^{(1)}(1)\} - \frac{d^{(1)}}{2} = 30 - 242.5 = -212.5,$$

$$x_1^{(1)}(1) = \max\{-212.5, 0\} = 0,$$

$$y_1^{(1)}(1) = x_1^{(1)}(1) + 1d^{(1)}(1) = 0 + 1 \cdot 485 = 485,$$

$$x_2^{(1)}(1) = y_1^{(1)}(1) = 485,$$

$$y_2^{(1)}(1) = x_1^{(1)}(1) + 2d^{(1)}(1) = 0 + 2 \cdot 485 = 970,$$

$$x_3^{(1)}(1) = y_2^{(1)}(1) = 970,$$

$$y_3^{(1)} = x_1^{(1)} + 3d^{(1)} = 0 + 3 \cdot 485 = 1455,$$

$$x_4^{(1)} = y_3^{(1)} = 1455,$$

$$y_4^{(1)} = x_1^{(1)} + 4d^{(1)} = 0 + 4 \cdot 485 = 1940,$$

$x_5^{(1)}(1) = y_4^{(1)}(1) = 1940,$

$y_5^{(1)}(1) = x_1^{(1)}(1) + 5d^{(1)}(1) = 0 + 5 \cdot 485 = 2425,$

$x_6^{(1)}(1) = y_5^{(1)}(1) = 2425,$

$y_6^{(1)}(1) = x_1^{(1)}(1) + 6d^{(1)}(1) = 0 + 6 \cdot 485 = 2910,$

and the ends of the interval $I_7^{(1)}(1) = < x_7^{(1)}(1), y_7^{(1)}(1))$, including the remaining unknown realizations

$x_7^{(1)}(1) = y_6^{(1)}(1) = 2910, \quad y_7^{(1)}(1) = +\infty,$

- we determine the numbers of realizations $n_j^{(1)}(1)$ in particular intervals $I_j^{(1)}(1)$, $j = 1,2,...,7,$

$n_1^{(1)}(1) = 19,\ n_2^{(1)}(1) = 7,\ n_3^{(1)}(1) = 3,\ n_4^{(1)}(1) = 2,$

$n_5^{(1)}(1) = 0,\ n_6^{(1)}(1) = 1,\ n_7^{(1)}(1) = 40 - 32 = 8,$

we find the realization of the histogram of the exemplary system component conditional lifetimes $T_{11}^{(1)}(1)$ that is presented in *Table 1*,

Table 1. The realization of the histogram of the conditional lifetime $T_{11}^{(1)}(1)$

Histogram of the conditional lifetime $T_{11}^{(1)}(1)$							
$I_j^{(1)}(1) = < x_j^{(1)}(1), y_j^{(1)}(1))$	0 – 485	485– 970	970– 1455	1455– 1940	1940– 2425	2425– 2910	2910– +∞
$n_j^{(1)}(1)$	19	7	3	2	0	1	8
$\bar{f}_{40}(t,1) = n_j^{(1)}(1)/\bar{n}^{(1)}$	19/40	7/40	3/40	2/40	0/40	1/40	8/40

we join the intervals specified in *Table 1* in the realization of the histogram $\bar{f}_{40}(t,1)$ that have the numbers $n_j^{(1)}(1),\ \ j = 1,2,...,6,$ of realizations less than 4 with the neighbouring intervals into new intervals to have this condition satisfied and after this operation we join the interval $I_7^{(1)}(1)$ with its new neighbouring interval,

- we fix the new number of intervals $\bar{r}^{(1)}(1) = 3$,
- we determine the new intervals

$\bar{I}_1^{(1)}(1) = < 0, 485),\ \ \bar{I}_2^{(1)}(1) = < 485, 970),$

$\bar{I}_3^{(1)}(1) = < 970, +\infty),$

- we fix the numbers of realizations in the new intervals

$\bar{n}_1^{(1)}(1) = 21,\ \bar{n}_2^{(1)}(1) = 5,\ \ \bar{n}_3^{(1)}(1) = 14,$

- we calculate, using (6.25) from [1], the hypothetical probabilities that the conditional lifetime $T_{11}^{(1)}(1)$ takes values from the new intervals

$p_1^{(1)}(1) = P(T_{11}^{(1)}(1) \in \bar{I}_1^{(1)}(1))$

$= P(0 \le T_{11}^{(1)}(1) < 485)$

$= [R_{11}^{(1)}(0,1)]^{(1)} - [R_{11}^{(1)}(485,1)]^{(1)}$

$= \exp[-0.0008 \cdot 0] - \exp[-0.0008 \cdot 485]$

$= 1 - \exp[-0.388] = 1 - 0.6784 = 0.3216,$

$p_2^{(1)}(1) = P(T_{11}^{(1)}(1) \in \bar{I}_2^{(1)}(1))$

$= P(485 \le T_{11}^{(1)}(1) < 970)$

$= [R_{11}^{(1)}(485,1)]^{(1)} - [R_{11}^{(1)}(970,1)]^{(1)}$

$= \exp[-0.0008 \cdot 485] - \exp[-0.0008 \cdot 970]$

$= \exp[-0.388] - \exp[-0.776]$

$= 0.6784 - 0.4602 = 0.2182,$

$p_3^{(1)}(1) = P(T_{11}^{(1)}(1) \in \bar{I}_3^{(1)}(1))$

$= P(970 \le T_{11}^{(1)}(1) < +\infty)$

$= 1 - [p_1^{(1)}(1) + p_2^{(1)}(1)]$

$= 1 - [0.3216 + 0.2182] = 0.4602,$

- we calculate, using (6.26) from [1], the realization of the χ^2 (chi-square)-Pearson's statistics

$$u_{40} = \sum_{j=1}^{3} \frac{(\overline{n}_j^{(1)}(1) - n^{(1)} p_j^{(1)}(1))^2}{n^{(1)} p_j^{(1)}(1)}$$

$$= \frac{(19 - 40 \cdot 0.3216)^2}{40 \cdot 0.3216} + \frac{(7 - 40 \cdot 0.2182)^2}{40 \cdot 0.2182}$$

$$+ \frac{(14 - 40 \cdot 0.4602)^2}{40 \cdot 0.4602}$$

$$\cong 2.93 + 0.34 + 1.06 = 4.33,$$

- we assume the significance level $\alpha = 0.01$,
- we fix the number of degrees of freedom $\overline{\overline{r}}^{(1)}(1) - l - 1 = 3 - 1 - 1 = 1,$

- we read from the Tables of the χ^2 – Pearson's distribution the value u_α for the fixed values of the significance level $\alpha = 0.05$ and the number of degrees of freedom $\overline{\overline{r}}^{(1)}(1) - l = 1 = 1$, such that, according to (6.27) from [1], the following equality holds

$$P(U_{40} > u_\alpha) = 1 - \alpha = 1 - 0.01 = 0.99$$

that amounts $u_\alpha = 6.63$ and we determine the critical domain in the form of the interval $(6.63, +\infty)$ and the acceptance domain in the form of the interval $< 0, 6.63 >$,

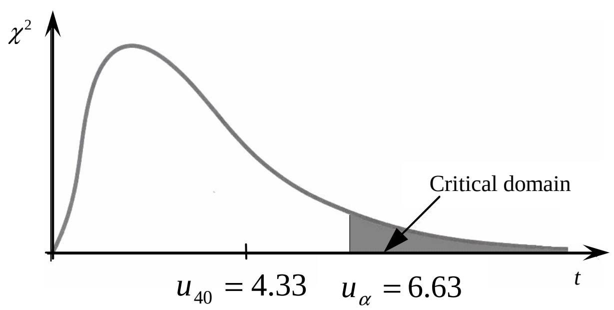

Figure 4. The graphical interpretation of the critical interval and the acceptance interval for the chi-square goodness-of-fit test

- we compare the obtained value $u_{40} = 4.33$ of the realization of the statistics U_{40} with the read from the Tables of the chi-square distribution critical value $u_\alpha = 6.63$ and since the value $u_{40} = 4.33$ does not belong to the critical domain, i.e.

$$u_{40} = 4.33 \le u_\alpha = 6.63,$$

then we do not reject the hypothesis H_0 in its part concerned with the coordinate $[R_{11}^{(1)}(t,1)]^{(1)}$.

To verify the hypothesis concerning the exponential form of the coordinate $[R_{11}^{(1)}(t,2)]^{(1)}$ defined by (11), it is due to act according to the scheme below:

- we fix the number of observed components and the number of realizations of the system component conditional lifetimes $T_{11}^{(1)}(2)$ in the reliability states subsets {2,3}, according to (6.2)[1], is

$$\overline{n}^{(1)} = n^{(1)} = 40, \ \overline{m}^{(1)}(2) = m^{(1)}(2) = 32,$$

- we fix the realizations $t_i^{(1)}(2)$, $i = 1,2,...,32$, of realizations of the exemplary system component conditional lifetimes $T_{11}^{(1)}(2)$ in the reliability states subsets {2,3} that are given by (6),
- we determine the number $\overline{r}^{(1)}(2)$ of the disjoint intervals $I_j^{(1)}(2) =< x_j^{(1)}(2), y_j^{(1)}(2))$, $j = 1,2,...,\overline{r}^{(1)}(2)$, that include the realizations $t_i^{(1)}(2)$, $i = 1,2,...,32$, of realizations of the system component conditional lifetimes $T_{11}^{(1)}(2)$ in the reliability states subsets {2,3}

$$\overline{r}^{(1)}(2) \cong \sqrt{\overline{m}^{(1)}(2)} = \sqrt{40} \cong 6,,$$

- we determine the length $d^{(1)}(2)$ of the intervals $I_j^{(1)}(2) =< x_j^{(1)}(2), y_j^{(1)}(2))$, $j = 1,2,...,6$,

$$\overline{R}^{(1)}(2) = \max_{1 \le i \le 32}\{t_i^{(1)}(2)\} - \min_{1 \le i \le 32}\{t_i^{(1)}(2)\}$$

$$= 2254 - 30 = 2224,$$

$$d^{(1)}(2) = \frac{2224}{6-1} \cong 445,$$

- we determine the ends $x_j^{(1)}(2)$, $y_j^{(1)}(2)$, of the intervals $I_j^{(1)}(2) =< x_j^{(1)}(2), y_j^{(1)}(2))$, $j = 1,2,...,6$, according to the formulae

$$\min_{1\le i\le 32}\{t_i^{(1)}(2)\} - \frac{d^{(1)}(2)}{2} = 30 - 222.5 = -192.5,$$

$$x_1^{(1)}(2) = \max\{-192.5,\ 0\} = 0,$$

$$y_1^{(1)}(2) = x_1^{(1)}(2) + 1d^{(1)}(2) = 0 + 1\cdot 445 = 445,$$

$$x_2^{(1)}(2) = y_1^{(1)}(2) = 445,$$

$$y_2^{(1)}(2) = x_1^{(1)}(2) + 2d^{(1)}(2) = 0 + 2\cdot 445 = 890,$$

$$x_3^{(1)}(2) = y_2^{(1)}(2) = 890,$$

$$y_3^{(1)}(2) = x_1^{(1)}(2) + 3d^{(1)}(2) = 0 + 3\cdot 445 = 1335,$$

$$x_4^{(1)}(2) = y_3^{(1)}(2) = 1335,$$

$$y_4^{(1)}(2) = x_1^{(1)}(2) + 4d^{(1)}(2) = 0 + 4\cdot 445 = 1780,$$

$$x_5^{(1)}(2) = y_4^{(1)}(2) = 1780,$$

$$y_5^{(1)}(2) = x_1^{(1)}(2) + 5d^{(1)}(2) = 0 + 5\cdot 445 = 2225,$$

$$x_6^{(1)}(2) = y_5^{(1)}(2) = 2225,$$

$$y_6^{(1)}(2) = x_1^{(1)}(2) + 6d^{(1)}(2) = 0 + 6\cdot 445 = 2670,$$

and the ends of the interval $I_7^{(1)}(2) =< x_7^{(1)}(2), y_7^{(1)}(2))$, including the remaining unknown realizations

$$x_7^{(1)}(2) = y_6^{(1)}(2) = 2670,\quad y_7^{(1)}(2) = +\infty,$$

- we determine the numbers of realizations $m_j^{(1)}(2)$ in particular intervals $I_j^{(1)}(2)$, $j = 1,2,...,7$,

$$n_1^{(1)}(2) = 18,\ n_2^{(1)}(2) = 9,\ n_3^{(1)}(2) = 2,\ n_4^{(1)}(2) = 2,$$

$$n_5^{(1)}(2) = 0,\ n_6^{(1)}(2) = 1,\ n_7^{(1)}(2) = 40 - 32 = 8,$$

we find the realization of the histogram of the exemplary system component conditional lifetimes $T_{11}^{(1)}(2)$ is presented in *Table 2*,

Table 2. The realization of the histogram of the conditional lifetime $T_{11}^{(1)}(2)$

Histogram of the conditional lifetime $T_{11}^{(1)}(2)$							
$I_j^{(1)}(2) =< x_j^{(1)}(2), y_j^{(1)}(2))$	0 – 445	445–890	890–1335	1335–1780	1780–2225	2225–2670	2670–$+\infty$
$n_j^{(1)}(2)$	18	9	2	2	0	1	8
$\bar{f}_{40}(t,2) = n_j^{(1)}(2)/\bar{n}^{(1)}$	18/40	9/40	2/40	2/40	0/40	1/40	8/40

- we join the intervals specified in *Table 2* in the realization of the histogram $\bar{f}_{40}(t,2)$ that have the numbers $n_j^{(1)}(2)$, $j = 1,2,...,6$, of realizations less than 4 with the neighbouring intervals into new intervals to have this condition satisfied and after this operation we join the interval $I_7^{(1)}(2)$ with its new neighbouring interval,
- we fix the new number of intervals $\bar{\bar{r}}^{(1)}(2) = 3$,
- we determine the new intervals

$$\bar{I}_1^{(1)}(2) =< 0,\ 445),\quad \bar{I}_2^{(1)}(2) =< 445,\ 890),$$

$$\bar{I}_3^{(1)}(2) =< 890,\ +\infty),$$

- we fix the numbers of realizations in the new intervals

$$\bar{n}_1^{(1)}(2) = 22,\ \bar{n}_2^{(1)}(2) = 5,\ \bar{n}_3^{(1)}(2) = 13,$$

- we calculate, using (6.25) from [1], the hypothetical probabilities that the conditional lifetime $T_{11}^{(1)}(2)$ takes values from the new intervals

$$p_1^{(1)}(2) = P(T_{11}^{(1)}(2) \in \bar{I}_1^{(1)}(2))$$

$$= P(0 \le T_{11}^{(1)}(2) < 445)$$

$$= [R_{11}^{(1)}(0,2)]^{(1)} - [R_{11}^{(1)}(445,2)]^{(1)}$$

$$= \exp[-0.0009 \cdot 0] - \exp[-0.0009 \cdot 445]$$

$$= 1 - \exp[-0.4005]$$

= 1 - 0.6700 = 0.3300,

$$p_2^{(1)}(2) = P(T_{11}^{(1)}(2) \in \bar{I}_2^{(1)}(2))$$

$$= P(445 \le T_{11}^{(1)}(2) < 890)$$

$$= [R_{11}^{(1)}(445,1)]^{(1)} - [R_{11}^{(1)}(890,1)]^{(1)}$$

$$= \exp[-0.0009 \cdot 445]$$

$$- \exp[-0.0009 \cdot 890]$$

$$= \exp[-0.4005] - \exp[-0.8010]$$

= 0.6700 – 0.4489 = 0.2211,

$$p_3^{(1)}(2) = P(T_{11}^{(1)}(2) \in \bar{I}_3^{(1)}(2))$$

$$= P(890 \le T_{11}^{(1)}(2) < +\infty)$$

$$= 1 - [p_1^{(1)}(2) + p_2^{(1)}(2)]$$

= 1 – [0.3300 + 0.2211] = 0.4489,

- we calculate, using (6.26) from [1] the realization of the χ^2 (chi-square)-Pearson's statistics

$$u_{40} = \sum_{j=1}^{3} \frac{(\bar{n}_j^{(1)}(2) - \bar{n}^{(1)} p_j^{(1)}(2))^2}{\bar{n}^{(1)} p_j^{(1)}(2)}$$

$$= \frac{(18 - 40 \cdot 0.3300)^2}{40 \cdot 0.3300} + \frac{(9 - 40 \cdot 0.2211)^2}{40 \cdot 0.2211}$$

$$+ \frac{(13 - 40 \cdot 0.4489)^2}{40 \cdot 0.4489}$$

$$\cong 1.75 + 0.003 + 1.37 = 3.12,$$

- we assume the significance level $\alpha = 0.01$,
- we fix the number of degrees of freedom

$$\bar{\bar{r}}^{(1)}(2) - l - 1 = 3 - 1 - 1 = 1,$$

- we read from the Tables of the χ^2 – Pearson's distribution the value u_α for the fixed values of the significance level $\alpha = 0.05$ and the number of degrees of freedom $\bar{\bar{r}}^{(1)}(2) - l = 1 = 1$, such that, according to (5.27)[1], the following equality holds

$$P(U_{40} > u_\alpha) = 1 - \alpha = 1 - 0.01 = 0.99$$

that amounts $u_\alpha = 6.63$ and we determine the critical domain in the form of the interval $(6.63, +\infty)$ and the acceptance domain in the form of the interval $< 0, 6.63 >$

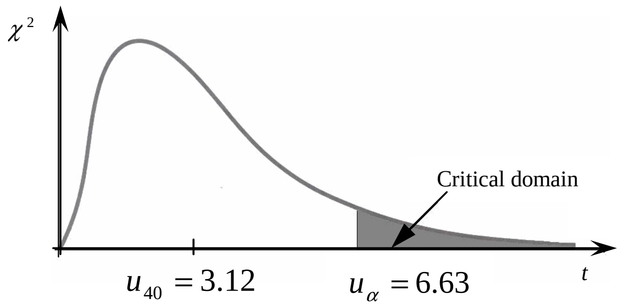

Figure 5. The graphical interpretation of the critical interval and the acceptance interval for the chi-square goodness-of-fit test

- we compare the obtained value $u_{40} = 3.12$ of the realization of the statistics U_{40} with the read from the Tables of the chi-square distribution critical value $u_\alpha = 6.63$ and since the value $u_{40} = 3.12$ does not belongs to the critical domain, i.e.

$$u_{40} = 3.12 \le u_\alpha = 6.63,$$

then we o not reject the hypothesis H_0 in its part concerned with the coordinate $[R_{11}^{(1)}(t, 2)]^{(1)}$.

To verify the hypothesis concerning the exponential form of the coordinate $[R_{11}^{(1)}(t, 3)]^{(1)}$ defined by (11), it is due to act according to the scheme below:
- we fix the number of observed components and the number of realizations of the system

component conditional lifetimes $T_{11}^{(1)}(3)$ in the reliability states subsets $\{3\}$, according to (6.3)[1], is

$$\bar{n}^{(1)} = n^{(1)} = 40,\ \bar{m}^{(1)}(3) = m^{(1)}(3) = 32,$$

- we fix the realizations $t_i^{(1)}(3),\ \ i = 1,2,...,32,$ of realizations of the exemplary system component conditional lifetimes $T_{11}^{(1)}(3)$ in the reliability states subsets $\{3\}$ that are given by (7),
- we determine the number $\bar{r}^{(1)}(3)$ of the disjoint intervals $I_j^{(1)}(3) =< x_j^{(1)}(3), y_j^{(1)}(3))$, $j = 1,2,...,\bar{r}^{(1)}(3)$, that include the realizations $t_i^{(1)}(3),\ \ i = 1,2,...,32,$ of realizations of the system component conditional lifetimes $T_{11}^{(1)}(3)$ in the reliability states subsets $\{3\}$

$$\bar{r}^{(1)}(3) \cong \sqrt{\bar{m}^{(1)}(3)} = \sqrt{32} \cong 6,,$$

- we determine the length $d^{(1)}(3)$ of the intervals $I_j^{(1)} =< x_j^{(1)}, y_j^{(1)})$, $j = 1,2,...,6$,

$$\bar{R}^{(1)} = \max_{1 \le i \le 32}\{t_i^{(1)}(3)\} - \min_{1 \le i \le 32}\{t_i^{(1)}(3)\}$$

$$= 2054 - 20 = 2034,$$

$$d^{(1)}(3) = \frac{2034}{6-1} \cong 407,$$

- we determine the ends $x_j^{(1)}(3),\ \ y_j^{(1)}(3)$, of the intervals $I_j^{(1)}(3) =< x_j^{(1)}(3), y_j^{(1)}(3))$, $j = 1,2,...,6$, according to the formulae

$$\min_{1 \le i \le 32}\{t_i^{(1)}(3)\} - \frac{d^{(1)}(3)}{2} = 20 - 203.5 = -183.5,$$

$$x_1^{(1)}(3) = \max\{-182.5, 0\} = 0,$$

$$y_1^{(1)}(3) = x_1^{(1)}(3) + 1d^{(1)}(3) = 0 + 1 \cdot 407 = 407\ ,$$

$$x_2^{(1)}(3) = y_1^{(1)}(3) = 405,$$

$$y_2^{(1)}(3) = x_1^{(1)}(3) + 2d^{(1)}(3) = 0 + 2 \cdot 407 = 814,$$

$$x_3^{(1)}(3) = y_2^{(1)}(3) = 810,$$

$$y_3^{(1)}(3) = x_1^{(1)}(3) + 3d^{(1)}(3) = 0 + 3 \cdot 407 = 1221,$$

$$x_4^{(1)}(3) = y_3^{(1)}(3) = 1215,$$

$$y_4^{(1)}(3) = x_1^{(1)}(3) + 4d^{(1)}(3) = 0 + 4 \cdot 407 = 1628,$$

$$x_5^{(1)}(3) = y_4^{(1)}(3) = 1680,$$

$$y_5^{(1)}(3) = x_1^{(1)}(3) + 5d^{(1)}(3) = 0 + 5 \cdot 407 = 2035,$$

$$x_6^{(1)}(3) = y_5^{(1)}(3) = 225,$$

$$y_6^{(1)}(3) = x_1^{(1)}(3) + 6d^{(1)}(3) = 0 + 6 \cdot 407 = 2442,$$

and the ends of the interval $I_7^{(1)}(3) =< x_7^{(1)}(3), y_7^{(1)}(3))$, including the remaining unknown realizations

$$x_7^{(1)}(3) = y_6^{(1)}(3) = 2442,\ \ y_7^{(1)}(3) = +\infty,$$

- we determine the numbers of realizations $n_j^{(1)}(3)$ in particular intervals $I_j^{(1)}(3)$, $j = 1,2,...,6,$ according

$$n_1^{(1)}(3) = 18,\ n_2^{(1)}(3) = 9,\ n_3^{(1)}(3) = 2,\ n_4^{(1)}(3) = 2,$$

$$n_5^{(1)}(3) = 0,\ n_6^{(1)}(3) = 1, n_7^{(1)}(3) = 8,$$

- we find the realization of the histogram of the exemplary system component conditional lifetimes $T_{11}^{(1)}(3)$ is presented in *Table 3*,

*Table 3***.** The realization of the histogram of the conditional lifetime $T_{11}^{(1)}(3)$

Histogram of the conditional lifetime $T_{11}^{(1)}(3)$							
$I_j^{(1)}(3) =< x_j^{(1)}(3), y_j^{(1)}(3))$	0 – 407	407– 814	814– 1221	1221– 1628	1628– 2035	2035– 2442	2442– +∞
$n_j^{(1)}(3)$	18	9	2	2	0	1	8
$\bar{f}_{40}(t,3) = n_j^{(1)}(3)/\bar{n}^{(1)}$	18/40	9/40	2/40	2/40	0/40	1/40	8/40

- we join the intervals specified in *Table 3* in the realization of the histogram $\bar{f}_{40}(t,3)$ that have the numbers $n_j^{(1)}(3)$, $j = 1,2,...,6$, of realizations less than 4 with the neighbouring intervals into new intervals to have this condition satisfied and after this operation we join the interval $I_7^{(1)}(3)$ with its new neighbouring interval,
- we fix the new number of intervals $\bar{r}^{(1)}(3) = 3$,
- we determine the new intervals

$$\bar{I}_1^{(1)}(3) =< 0, 407), \quad \bar{I}_2^{(1)}(3) =< 407, 814),$$

$$\bar{I}_3^{(1)}(3) =< 814, +\infty),$$

- we fix the numbers of realizations in the new intervals

$$\bar{n}_1^{(1)}(3) = 21, \ \bar{n}_2^{(1)}(3) = 6, \ \bar{n}_3^{(1)}(3) = 13,$$

- we calculate, using (6.25) from [1], the hypothetical probabilities that the conditional lifetime $T_{11}^{(1)}(1)$ takes values from the new intervals

$$p_1^{(1)}(3) = P(T_{11}^{(1)}(3) \in \bar{I}_1^{(1)}(3))$$

$$= P(0 \le T_{11}^{(1)}(3) < 405)$$

$$= [R_{11}^{(1)}(0,3)]^{(1)} - [R_{11}^{(1)}(407,1)]^{(1)}$$

$$= \exp[-0.0009 \cdot 0] - \exp[-0.0009 \cdot 407]$$

$$= 1 - \exp[-0.3663]$$

$$= 1 - 0.6933 = 0.3067,$$

$$p_2^{(1)}(3) = P(T_{11}^{(1)}(3) \in \bar{I}_2^{(1)}(3))$$

$$= P(407 \le T_{11}^{(1)}(3) < 814)$$

$$= [R_{11}^{(1)}(407,1)]^{(1)} - [R_{11}^{(1)}(814,1)]^{(1)}$$

$$= \exp[-0.0009 \cdot 407]$$

$$- \exp[-0.0009 \cdot 814])$$

$$= \exp[-0.3663] - \exp[-0.7326]$$

$$= 0.6933 – 0.4807 = 0.2126,$$

$$p_3^{(1)}(3) = P(T_{11}^{(1)}(3) \in \bar{I}_3^{(1)}(3))$$

$$= P(814 \le T_{11}^{(1)}(3) < +\infty)$$

$$= 1 - [p_1^{(1)}(3) + p_2^{(1)}(3)]$$

$$= 1 – [0.3067 + 0.2126] = 0.4807,$$

- we calculate, using (6.26) from [1], the realization of the χ^2 (chi-square)-Pearson's statistics

$$u_{40} = \sum_{j=1}^{3} \frac{(\bar{n}_j^{(1)}(3) - \bar{n}^{(1)} p_j^{(1)}(3))^2}{\bar{n}^{(1)} p_j^{(1)}(3)}$$

$$= \frac{(18 - 40 \cdot 0.3067)^2}{40 \cdot 0.3067} + \frac{(9 - 40 \cdot 0.2126)^2}{40 \cdot 0.2126}$$

$$+ \frac{(13 - 40 \cdot 0.4807^2}{40 \cdot 0.4807}$$

$$\cong 2.68 + 0.03 + 2.02 = 4.73,$$

- we assume the significance level $\alpha = 0.01$,

- we fix the number of degrees of freedom

$$\bar{\bar{r}}^{(1)}(3) - l - 1 = 3 - 1 - 1 = 1,$$

- we read from the Tables of the χ^2 – Pearson's distribution the value u_α for the fixed values of the significance level $\alpha = 0.05$ and the number of degrees of freedom $\bar{\bar{r}}^{(1)}(3) - l = 1 = 1,$ such that, according to (6.27) from [1], the following equality holds

$$P(U_{40} > u_\alpha) = 1 - \alpha = 1 - 0.01 = 0.99$$

that amounts $u_\alpha = 6.63$ and we determine the critical domain in the form of the interval $(6.63, +\infty)$ and the acceptance domain in the form of the interval $< 0, 6.63 >$,

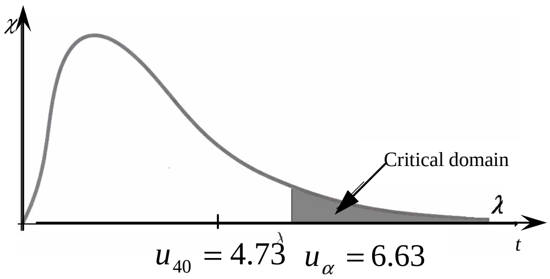

Figure 6. The graphical interpretation of the critical interval and the acceptance interval for the chi-square goodness-of-fit test

- we compare the obtained value $u_{40} = 4.73$ of the realization of the statistics U_{40} with the read from the Tables of the chi-square distribution critical value $u_\alpha = 6.63$ and since the value $u_{40} = 4.73$ does not belongs to the critical domain, i.e.

$$u_{40} = 4.73 \le u_\alpha = 6.63,$$

then we do not reject the hypothesis H_0 in its part concerned with the coordinate $[R_{11}^{(1)}(t, 3)]^{(1)}$.

In the conclusion, we accept in full the hypothesis H_0 that the reliability function of the component $E_{11}^{(1)}$ at the operation state z_1 is exponential with the coordinates given by (11).

The procedure of the hypotheses formulation and verification for the remaining components of the subsystems at various operation sates is analogous as for the component $E_{11}^{(1)}$ at the operation state z_1 [2].

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