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A MATHEMATICAL MODEL FOR EVALUATION THE EFFICIENCY OF GAS-MAIN PIPELINES IN TRANSIENT OPERATIONAL MODES

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Abstract. A mathematical model for control by transient modes of gas flows in the long-distance gas pipeline is considered in the paper. The long-distance pipeline is considered in the model as the system of line segments serially connected via compressor stations. Gas motion in such system is described by the non-linear system of equations of gas dynamics. In the frame of this model the integral parameters which determine the expenditure of energy and durations of the transient mode are introduced. These parameters can be used for formulation the problems for optimal control steady-state and transient modes of operation of main-gas pipelines.

Key words: gas-main pipeline, transient modes of operation, models of gas dynamics, control by transient process, efficiency of gas transportation, duration of transient process.

INTRODUCTION

Long-distance gas pipelines are used in many countries for natural gas delivery from the producing areas to market areas. The gas transmission system (GTS) of Ukraine resolves two main functions: supplies the internal customers by natural gas and transits it to Central and Eastern European's countries [1]. The length of the Ukrainian gas-main pipelines is over 37 thousands km. They are provided by 71 compressor stations, the overall power of which exceeds 5400 MW. High power of the compressor stations, high pipeline's capacity, which substantially exceeds the average annual gas flow through it, and availability of underground gas storage facilities of capacity $31 \cdot 10^9$ m^3 make it possible to deliver big amounts of gas on long distances in short time periods [1].

The expenditures of energy for gas transportation are dependent on mode of GTS operation. Steady-state operate modes are the most effective ones if the maximal pressure in the pipeline is close to maximum allowable pressure and the compressor stations work with the highest efficiency. But in practice the necessity to use transient modes under which the inlet and outlet pressures and flow density in the pipelines vary in time. In the transient modes the expenditures of energy can be considerably higher in comparison with the stationary modes. In this connection the problem of minimization of energy costs of gas transportation in the transient modes of operation of gas-main pipelines becomes actual.

A mathematical model for control by transient modes of gas flows in the long-distance gas pipeline is considered in the paper. In the frame of this model the integral parameters which determine the expenditure of energy and durations of the transient mode are introduced. These parameters can be used for formulation the problems for optimal control by steady-state and transient modes of operation of main-gas pipelines.

THE PROBLEM STATEMENT

We consider a long-distance pipeline as the system of line pipelines (segments) serially connected via compressor stations. Each segment is long pipeline. Gas motion in such system is described by the non-linear system of equations of gas dynamics. The equations of this system bound the parameters of gas state (pressure P, mass density ρ, and temperature T) and the parameters of gas motion (velocity V and mass flux J) in each segments. The system should be complemented by the conditions, which take into account the effect of the compressor stations on the gas flow. These conditions couple the gas-dynamics parameters of adjoining segments.

In stationary modes the state and flow parameters in each segment are dependent just on the spatial coordinate along he tube's axis whereas the interface conditions are constant. In transient modes the interface conditions change with time whereas the state and flow parameters are functions of the spatial coordinate x and time t:

$$P = P(x,t),\ \rho = \rho(x,t),\ T = T(x,t)\ \ V = V(x,t),$$
$$J = J(x,t).$$

These functions can be determined by solving the boundary-initial value problem, formulated within the mathematical model describing the mass, momentum and energy transfer in each segment. With the use of the obtained solution one can calculate integral parameters of the transient flow – the mass of the gas containing in each segment, the duration of the transient process, the expenditure of energy spent on its realization etc.

We suppose that inlet and outlet pressures can be measured with sufficient precision as functions of time:

$$P_\lambda^{in} = P_\lambda^{in}(t),\ P_\lambda^{out} = P_\lambda^{out}(t).$$

We can use these functions as the boundary conditions for each segment. In such way we define two functionals, which for any pair of functions:

$$\{P_\lambda^{in}(t), P_\lambda^{out}(t)\}$$

determine two positive numbers: τ – the duration of the transient process, and W – the expenditure of energy spent on this process realization.

Similarly, if to suppose that inlet and outlet mass fluxes can be measured:

$$J_\lambda^{in} = J_\lambda^{in}(t),\ J_\lambda^{out} = J_\lambda^{out}(t),$$

we can consider another three pair of boundary conditions:

$$\{P_\lambda^{in}(t), J_\lambda^{out}(t)\},\ \{J_\lambda^{in}(t), P_\lambda^{out}(t)\} \text{ and } \{J_\lambda^{in}(t), J_\lambda^{out}(t)\}.$$

The boundary-value problems, formulated with the use of these conditions, define the functionals for determination τ and W for corresponding transient process, controlled by given pair of the parameters.

THE MODEL FOR GAS-DYNAMIC IN A LONG PIPELINE

We will conduct the study within the one-dimensional model of gas-dynamics, describing non-stationary motion of natural gas in a cylindrical pipe of constant diameter. Such models were considered by many authors [2-12]. Here we restrict ourselves by the isothermal case, supposing the temperature T as the given parameter, independent on coordinate x and time t. In this case the flows in the segment are described by two partial differential equations – the equation for mass balance (equation of continuity):

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho V}{\partial x} = 0 \qquad (1)$$

and equation for balance of momentum:

$$\frac{\partial \rho V}{\partial t} + \frac{\partial}{\partial x}\left(\rho V^2 + P\right) + \rho g \frac{\partial h}{\partial x} + f_R = 0 \qquad (2)$$

where: g stands for acceleration of gravity; $h = h(x)$ is the function, determining the elevation of the pipeline's axis; f_R stands for the density of average frictional force [4].

Density ρ of the gas is depended on its pressure P and temperature T. The dependence is defined by the equation of state:

$$P = R/\mu_g\ z\rho T = c_0{}^2 z\rho, \quad c_0^2 \equiv RT/\mu_g, \qquad (3)$$

where: R - stands for the universal gas constant, μ_g - is molar mass of the gas, z - compressibility factor of the gas, c_0 - stands for sound velocity in the gas with molar mass μ_g at temperature T.

The compressibility factor z takes into account departure of thermodynamic properties of real gas from the properties of ideal gas. The are different empirical formulas representing this thermodynamic parameter as function of pressure P and temperature T. Among them the formula of American Gas Association (AGA), which is valid for pressures up to 7 MPa:

$$z(P,T) = 1 + 0,257\cdot(P/P_c) - 0,533\cdot(P/P_c)\cdot(T_c/T), \quad (4)$$

where: T_c and P_c stand for critical temperature and pressure

The force f_R in formula (2) takes into account the viscous friction in gas volume, in the boundary layer and on the inside surface of the pipe's wall. For sufficiently high flow's velocity (turbulent flow mode) f_R can be determined by formula:

$$f_R = \frac{\lambda |V|}{2D}\rho V, \qquad (5)$$

where: λ - stands for the coefficient of hydraulic resistance, D - is pipe's inner diameter.

The Haaland explicit equation [6] enables to express parameter λ through the height ε of roughness on the inner surface of the pipe's wall and Reynolds number Re of the flow:

$$\frac{1}{\sqrt{\lambda}} = -1.8\lg\left(\frac{6.9}{Re} + \left(\frac{\varepsilon D}{3.7}\right)^{1.11}\right).$$

Using formulas (3)-(5) one can reduce mathematical model for motion of the gas in the pipeline to non-linear system of two partial differential equations. Any pair of the parameters (ρ, V), (ρ, J), (P, V) or (P, Q) can be used as key functions for this system. Here:

$$J = \rho V \quad \text{and} \quad Q = V\pi D^2/4$$

are the mass and volumetric flow rates. Though the systems of equations formulated for different pairs of parameters are mathematically equivalent, the chosen key functions can be significant for algorithms for numerical solving of the corresponding problems. With this in view we chose the parameters $\rho = \rho(x,t)$ and $J = J(x,t)$ as the key functions for mathematical model. So, we obtain

$$\frac{\partial \rho}{\partial t}+\frac{\partial J}{\partial x}=0 \quad (6)$$

$$\frac{\partial J}{\partial t}+\frac{\partial P}{\partial \rho}\frac{\partial \rho}{\partial x}+\frac{\partial}{\partial x}\left(\frac{J^2}{\rho}\right)+\frac{\lambda}{2D}\frac{|J|J}{\rho}+g\frac{dH}{dx}\rho=0\,. \quad (7)$$

As we can see the equation (6) of the system (6), (7) is a linear one in this case.

Without restriction of generality we can treat the compressibility factor z as a function of the state parameters ρ and T. Hence of the base of (3) we have

$$\frac{\partial P}{\partial \rho}=c_0^2 Z(\rho,T),\quad Z(\rho,T)\equiv z(\rho,T)+\rho\frac{\partial z(\rho,T)}{\partial \rho}\,. \quad (8)$$

In particular, for compressibility factor in the form (4) relations (8) look like

$$z(\rho,T)=\frac{1}{1-\frac{\rho}{\rho_c}\left(0,257-0,533\frac{T_c}{T}\right)},$$
$$Z(\rho,T)=\frac{1}{\left[1-\frac{\rho}{\rho_c}\left(0,257-0,533\frac{T_c}{T}\right)\right]^2}, \quad (9)$$

where: $\rho_c \equiv P_c/c_0^2$.

If functions $\rho(x,t)$ and $J(x,t)$ are known one can easily calculate the pressure $P(x,t)$ and velocity $V(x,t)$ as functions of coordinate x and time t:

$$P=c_0^2 z(\rho)\rho\,,\qquad V=J/\rho\,. \quad (10)$$

Let P_0 and J_0 be some characteristic values of pressure and mass flux. We introduce the normalized dimensionless coordinate, time and dependent variables:

$$\xi=2\frac{x}{L}-1,\ \tau=\frac{t}{t_0},\ \tilde{\rho}=\frac{\rho}{\rho_0},$$
$$j=\frac{J}{J_0},\ p=\frac{P}{P_0},\ v=\frac{V}{V_0}, \quad (11)$$

where: L - is the section's length, $t_0=L/c_0$ - is the characteristic time, ρ_0 - stands for characteristic mass density, which corresponds to characteristic pressure:

$$P_0=c_0^2 z(\rho_0)\rho_0\,,$$

and characteristic velocity:

$$V_0=J_0/\rho_0\,.$$

In the dimensionless variables the system (6), (7) takes the form:

$$\frac{\partial \tilde{\rho}}{\partial \tau}+Ma\frac{\partial j}{\partial \xi}=0\,, \quad (12)$$

$$\frac{\partial j}{\partial t}+\frac{1}{Ma}Z(\tilde{\rho},T)\frac{\partial \tilde{\rho}}{\partial \xi}+Ma\frac{\partial}{\partial \xi}\left(\frac{j^2}{\tilde{\rho}}\right)+$$
$$+Ma\beta\left(\frac{j^2}{\tilde{\rho}}\right)+\frac{d\gamma}{d\xi}\tilde{\rho}=0. \quad (13)$$

Here the denotations are used:

$$Ma=\frac{J_0}{\rho_0 c_0}=\frac{V_0}{c_0},\ \beta=\frac{\lambda L}{2D},\ \ \gamma=\frac{H}{H_0},\ H_0=\frac{c_0 V_0}{g}\,.$$

In the stationary case:

$$\partial\tilde{\rho}/\partial\tau=\partial j/\partial\tau=0\,.$$

So, it follows from equation (12), that $j(\xi)=const$. With this, the equation (13) can be reduced to the form:

$$\frac{d\tilde{\rho}}{d\xi}=-\frac{Ma^2\beta j^2\tilde{\rho}+Ma\frac{d\gamma}{d\xi}\tilde{\rho}^3}{Z(\tilde{\rho})\tilde{\rho}^2-Ma^2 j^2}\,. \quad (14)$$

Obtaining ordinary differential equation describes the steady-state distribution of the mass density in a pipeline in stationary mode. This non-linear equation can be solved numerically with the use a Runge-Kutta method. When the density ρ of the gas is known one can calculate its pressure with the use of the state equation (3).

THE MODEL FOR CONTROLLING OF THE TRANSIENT PIPELINE'S OPERATIONAL MODES

The equations (12), (13) describe a wide class of non-stationary (dynamic) motion of the gas in pipelines. We chose from this class so-called transient motions. So, each time we will consider a dynamic motion, which has been started from some known steady-state motion and will be finished as other steady-state motion. I.e. we will consider dynamic processes being transitions between two steady-state processes.

Let J^0 and J^1 be the mass fluxes in steady-state modes 0 (beginning mode) and 1 (finishing mode), P_-^0 and P_+^0 be static inlet and outlet pressures acting in the mode 0, P_-^1 and P_+^1 be static inlet and outlet pressures acting in the mode 1.

We can find pressure distributions in modes 0 and 1 using the steady-state model (14). To do that we put in the equation (14) $j = j^0 \equiv J^0/J_0$, solve this equation subordinating its solution $\tilde{\rho}^0 \equiv \tilde{\rho}^0(\xi)$ to one of boundary conditions (15) at $\mu = 0$:

$$\tilde{\rho}^\mu\big|_{\xi=-1} = \tilde{\rho}^\mu_-, \ \tilde{\rho}^\mu\big|_{\xi=1} = \tilde{\rho}^\mu_+, \quad \lambda = 0,1, \tag{15}$$

where:

$$\tilde{\rho}^\mu_- = \frac{P^\mu_-}{c_0{}^2 z\left(P^\mu_-, T\right)\rho_0}, \quad \tilde{\rho}^\mu_+ = \frac{P^\mu_+}{c_0{}^2 z\left(P^\mu_+, T\right)\rho_0}. \tag{16}$$

Then we put in the equation (14) $j = j^1 \equiv J^1/J_0$, solve this equation, subordinating its solution $\tilde{\rho}^1 \equiv \tilde{\rho}^1(\xi)$ to one of boundary conditions (15) at $\mu = 1$.

We use the first of two conditions (15), when in the stationary mode μ inlet pressure is given, and we use the second condition (15), when in the stationary mode μ outlet pressure is given.

Using the obtained solution we can find the pressure distributions in stationary modes 0 and 1 $P^1(x)$ and $P^2(x)$ in the section:

$$P^\mu(x) = P_0 c_0{}^2 \tilde{\rho}^\lambda(x/L_0)\, z\left(\tilde{\rho}^\lambda(x/L_0), T\right). \tag{17}$$

We can find also the value of pressure on the opposite end of the pipe for both modes. For instance, if in the mode μ the inlet pressure P^μ_- is given, then the outlet pressure P^μ_+ for this mode can be can be found as:

$$P^\lambda_+ = P_0 c_0{}^2 \tilde{\rho}^\lambda(1)\, z\left(\tilde{\rho}^\lambda(1), T\right).$$

Otherwise, if the outlet pressure P^μ_+ is given, the outlet pressure will be found as:

$$P^\lambda_- = P_0 c_0{}^2 \tilde{\rho}^\lambda(-1)\, z\left(\tilde{\rho}^\lambda(-1), T\right).$$

To attain the required flow parameters in the pipeline one can change the mechanical power, which the compressor stations transfer into the moving gas. That will be attended by varying of values of the gas-dynamics parameters on the both ends of the each section.

We suppose, that inlet and outlet pressure, and, in some cases, inlet and/or outlet mass fluxes can be measured with necessary precision as. It means, that for each segment we dispose functions $P_\pm(t)$, representing the time variation of inlet and outlet pressures. In some case we can dispose functions $J_\pm(t)$, representing the time variation of the inlet and outlet fluxes.

Such assumptions enables to consider independent problems for each segment and use for it four types of boundary conditions: i) on both ends pressure as functions of time are given (the conditions of the first kind); ii) on both ends flux as functions of time are given (the conditions of the second kind); iii) inlet pressure and outlet flux as functions of time are given; iv) inlet flux and outlet pressure as functions of time are given (the mixed boundary conditions). Due to this we consider for system (12), (13) the next boundary conditions:

$$\tilde{\rho}\big|_{\xi=-1} = \tilde{\rho}_-(\tau), \qquad \tilde{\rho}\big|_{\xi=1} = \tilde{\rho}_+(\tau), \tag{18}$$

$$j\big|_{\xi=-1} = j_-(\tau), \qquad j\big|_{\xi=1} = j_+(\tau), \tag{19}$$

$$\tilde{\rho}\big|_{\xi=-1} = \tilde{\rho}_-(\tau), \qquad j\big|_{\xi=1} = j_+(\tau), \tag{20}$$

$$j\big|_{\xi=-1} = j_-(\tau), \qquad \tilde{\rho}\big|_{\xi=1} = \tilde{\rho}_+(\tau). \tag{21}$$

Here:

$$\tilde{\rho}_\pm(\tau) = \frac{P_\pm(\tau t_0)}{c_0{}^2 z\left(P_\pm(\tau t_0), T\right)\rho_0}, \quad j_\pm(\tau) = \frac{J_\pm(\tau t_0)}{J_0} \tag{22}$$

As the transient mode begins from the mode 0, we have the next initial conditions for system (12), (13):

$$\tilde{\rho}\big|_{\tau=0} = \tilde{\rho}^0(\xi), \qquad j\big|_{\tau=0} = j^0. \tag{23}$$

Four initial-boundary-value problems (12), (13), (18) (23); (12), (13), (18), (23); (12), (13), (19), (23); та (12), (13), (20), (23) define corresponding four models to control the transient flow in each section.

We will restrict our consideration by control functions $P_\pm(t)$ and $J_\pm(t)$, which are monotonous in the intervals $\left[P^0_\pm, P^1_\pm\right]$ and $\left[J^0, J^1\right]$ correspondingly.

To do this let consider the function:

$$\varphi(t,\Delta) = \begin{cases} 0, & t \le 0, \\ t/\Delta, & 0 < t \le \Delta, \\ 1, & \Delta < t, \end{cases} \quad \Delta > 0, \tag{24}$$

with the use of which we represent the control functions in the forms:

$$\begin{aligned} P_-(t) &= P^0_- + \left(P^1_- - P^0_-\right)\varphi\left(t, \Delta^P_-\right), \\ P_+(t) &= P^0_+ + \left(P^1_+ - P^0_+\right)\varphi\left(t - t^P, \Delta^P_+\right), \\ J_-(t) &= J^0 + \left(J^1 - J^0\right)\varphi\left(t, \Delta^J_-\right), \\ J_+(t) &= J^0 + \left(J^1 - J^0\right)\varphi\left(t - t^J, \Delta^J_+\right). \end{aligned} \tag{25}$$

where: t^P and t^J are real constants which define delay/lead the outlet control functions relative to inlet ones.

The boundary conditions (18)-(21) take the form:

$$\tilde{\rho}\big|_{\xi=-1} = \tilde{\rho}_-^0 + \left(\tilde{\rho}_-^1 - \tilde{\rho}_-^0\right)\varphi\left(\tau, \Delta_-^\rho\right),$$
$$\tilde{\rho}\big|_{\xi=+1} = \tilde{\rho}_+^0 + \left(\tilde{\rho}_+^1 - \tilde{\rho}_+^0\right)\varphi\left(\tau - \tau^\rho, \Delta_+^\rho\right) \quad (26)$$

$$j\big|_{\xi=-1} = j^0 + \left(j^1 - j^0\right)\varphi\left(\tau, \Delta_-^j\right),$$
$$j\big|_{\xi=+1} = j^0 + \left(j^1 - j^0\right)\varphi\left(\tau - \tau^j, \Delta_+^j\right) \quad (27)$$

$$\tilde{\rho}\big|_{\xi=-1} = \tilde{\rho}_-^0 + \left(\tilde{\rho}_-^1 - \tilde{\rho}_-^0\right)\varphi\left(\tau, \Delta_-^\rho\right),$$
$$j\big|_{\xi=+1} = j^0 + \left(j^1 - j^0\right)\varphi\left(\tau - \tau^j, \Delta_+^j\right), \quad (28)$$

$$j\big|_{\xi=-1} = j^0 + \left(j^1 - j^0\right)\varphi\left(\tau, \Delta_-^j\right),$$
$$\tilde{\rho}\big|_{\xi=+1} = \tilde{\rho}_+^0 + \left(\tilde{\rho}_+^1 - \tilde{\rho}_+^0\right)\varphi\left(\tau - \tau^\rho, \Delta_+^\rho\right). \quad (29)$$

where:

$$\tilde{\rho}_\pm^\lambda = \frac{P_\pm^\lambda}{c_0{}^2\rho_0 z\left(P_\pm^\lambda, T\right)},\ j^\lambda = \frac{J^\lambda}{J_0},\ \Delta_\pm^{\rho,j} = \frac{\Delta_\pm^{P,J}}{t_o},\ \tau^{\rho,j} = \frac{t^{P,J}}{t_0}.$$

Thereby due the suggested approach any of the control model (18) – (21) is defined by 7 scalar parameters. For instance for model defined by boundary conditions (28) they are P_-^0, P_-^1, Δ_-^P, J_-^0, J_-^1, Δ_-^J and t^J.

Solving the initial-boundary-value problem, corresponding to any of control model (18) – (21), we obtain two functions $\tilde{\rho}(\xi,\tau)$ and $j(\xi,\tau)$. With the use of these functions we can introduce integral measures which determine the duration of the transient process and the expenditure of energy for its realization.

EVALUATION THE DURATION OF TRANSIENT MODE

The gas containing in the pipeline is an inertial system. Transient processes in this system can still go on even after the compressor stations have gained the steady-state modes. To evaluate the transient mode's duration we introduce measures for deviation of the current state of the gas accumulated in the pipeline from its state in the stationary mode 1, to which the system tends.

We consider two kinds of such measures. The measures of the first kind determine the deviations of the current values of gas-dynamic parameters on the ends of the section from their corresponding values in the stationary mode 1. Depending on the chosen control model (18)-(21), we will consider four such measure:

$$\delta^j(\tau) = \max\left(\delta_-^j(\tau), \delta_+^j(\tau)\right),$$
$$\delta^\rho(\tau) = \max\left(\delta_-^\rho(\tau), \delta_+^\rho(\tau)\right),$$
$$\delta^{j\rho}(\tau) = \max\left(\delta_-^j(\tau), \delta_+^\rho(\tau)\right), \quad (30)$$
$$\delta^{\rho j}(\tau) = \max\left(\delta_-^\rho(\tau), \delta_+^j(\tau)\right),$$

where:

$$\delta_-^j(\tau) = \left|\frac{j(-1,\tau) - j^1}{j^1}\right|,\ \delta_+^j(\tau) = \left|\frac{j(1,\tau) - j^1}{j^1}\right|,$$
$$\delta_-^\rho(\tau) = \left|\frac{\tilde{\rho}(-1,\tau) - \tilde{\rho}_-^1}{\tilde{\rho}_-^1}\right|,\ \delta_+^\rho(\tau) = \left|\frac{\tilde{\rho}(1,\tau) - \tilde{\rho}_+^1}{\tilde{\rho}_1^1}\right|. \quad (31)$$

The measures of second kind determine deviations of the current values of integral characteristic in pipeline's volume. We will consider two kinds of such measures, which corresponds two equations of gas dynamics (6) and (7).

The first measure $\delta^M(\tau)$ defines deviation of the mass containing in the pipeline at the current moment τ from its value in the stationary state 1. The second one determines the current deviation the value of the momentum of gas containing in the pipelines:

$$\delta^M(\tau) = \frac{\int_{-1}^{1}\left|\tilde{\rho}(\xi,\tau) - \tilde{\rho}^1(\xi)\right| d\xi}{\int_{-1}^{1}\tilde{\rho}^1(\xi)\, d\xi},$$
$$\delta^J(\tau) = \frac{\int_{-1}^{1}\left|j(\xi,\tau) - j^1\right| d\xi}{j^1}. \quad (32)$$

Let ε a given positive real number:

$$\tau_\varepsilon^\omega : \delta^\omega\left(\tau_\varepsilon^\omega\right) = \varepsilon\ \ \forall\omega \in \left\{j, \rho, i\rho, \rho j, M, J\right\}.$$

Then we define the real number:

$$\tau_\varepsilon = \mathrm{Inf}\left\{\tau_\varepsilon^\omega, \omega = j, \rho, i\rho, \rho j, M, J\right\}$$

as dimensionless duration of the transient process from stationary state 0 to stationary state 1.

Introduced measures (30) and (31) of nonstationarity enable to evaluate quantitatively the duration of the transient process for any chosen control model (18) – (21).

EVALUATION THE POWER EFFICIENCY OF TRANSIENT MODES

Using the solution of the initial-boundary-value problems corresponding to any chosen control model (18) – (21), we can calculate the power of friction force f_R at any moment τ:

$$W_R(\tau) = \beta\frac{\pi D^2 J_0{}^3}{8\rho_0{}^2}\int_{-1}^{1}\frac{j^3(\xi,\tau)}{\tilde{\rho}^2(\xi,\tau)}d\xi. \quad (33)$$

Due to this the energy efficiency of the transient mode can be evaluated as:

$$\eta_\varepsilon = \beta \frac{{J_0}^2}{2{\rho_0}^2} \int_0^{\tau_\varepsilon} \int_{-1}^{1} \frac{j^3(\xi,\tau)}{\tilde{\rho}^2(\xi,\tau)} d\xi d\tau \Bigg/ \int_0^{\tau_\varepsilon^\omega} j(1,\tau)\, d\tau \qquad (34)$$

Then we can compare parameter η_ε to corresponding parameters η^0 and η^1, defining the expenditures of energy for the modes 0 and 1 correspondingly:

$$\eta^0 = \beta \frac{\left(J^0\right)^2}{2{\rho_0}^2} \int_{-1}^{1} \frac{d\xi}{\left(\tilde{\rho}^0(\xi)\right)^2},\ \eta^1 = \beta \frac{\left(J^1\right)^2}{2{\rho_0}^2} \int_{-1}^{1} \frac{d\xi}{\left(\tilde{\rho}^1(\xi)\right)^2}. \qquad (35)$$

Using introduced parameter η_ε we can compare quantitatively different transient modes and different control models.

NUMERICAL STUDY

Consider an example of application the developed mathematical tools to study the control model (20).

To numerical solving the initial-boundary-value problem (12), (13), (23), (28) we represent the sought-for functions $\tilde{\rho}(\xi,\tau)$ and $j(\xi,\tau)$ as Fourier-Legendre series expansions:

$$\tilde{\rho}(\xi,\tau) = \sum_{k=0}^{N} A_k(\tau) P_k(\xi),\ j(\xi,\tau) = \sum_{k=0}^{N} B_k(\tau) P_k(\xi). \qquad (36)$$

With the use of the spectral Galerkin method [13–16], we reduce equations (12), (13) to the system of ordinary differential equations:

$$\begin{aligned} &\frac{d\mathbf{A}}{dt} + \mathbf{C}_{12}\cdot\mathbf{B} + \mathbf{K}_1 = 0 \\ &\frac{d\mathbf{B}}{dt} + \mathbf{C}_{21}\cdot\mathbf{A} + \mathbf{C}_{22}\cdot\mathbf{B} + \mathbf{K}_2 = 0 \end{aligned}, \qquad (37)$$

where:

$$\begin{aligned} &\mathbf{A} = \left(A_1(\tau), A_2(\tau), \ldots, A_N(\tau)\right)^{\mathrm{T}}, \\ &\mathbf{B} = \left(B_0(\tau), B_1(\tau), \ldots, B_{N-1}(\tau)\right)^{\mathrm{T}} \end{aligned}.$$

The (**37**) is nonlinear one as elements of matrixes $\mathbf{C}_{21}$, $\mathbf{C}_{22}$ are functionals of sought-for solution (36). Elements of vectors $\mathbf{K}_1$ and $\mathbf{K}_2$ are functionals of the given control functions of boundary conditions (28).

System (37) was discretized on the uniform time grid with the use of the Crank–Nicolson's difference scheme [13]. To solve the obtained nonlinear algebraic system of equations an iterative algorithm was developed.

On fig.1,2 some results obtained by solving the problem (12), (13), (23), (28) are shown (arrows on the plots show time increasing). The numerical solution was obtained for the case, when mass flow rate in the pipeline is increased for 15 % under constant inlet pressure.

The calculations were made for $N = 15$, time discretization was made with time step $\delta\tau = 0,05$.

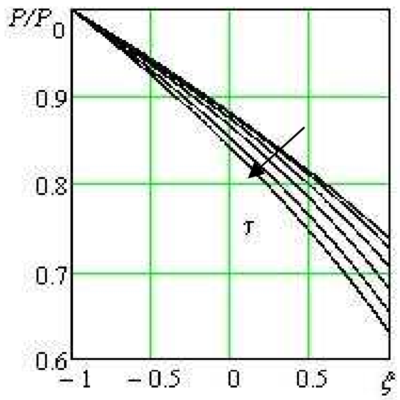

Fig.1. Pressure distributions along the pipeline at the moments τ = 0; 4; 8; 12; 20 and 40

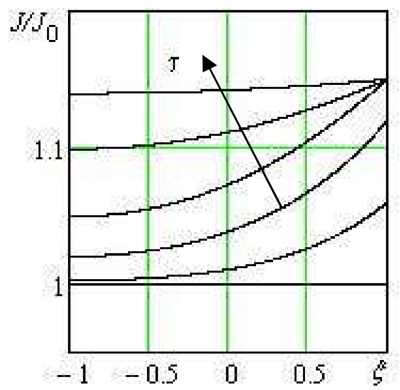

Fig.2. Flux distributions along the pipeline at moments τ = 0; 4; 8; 12; 20 and 40

The gas mixture in proportion: methane – 98.6 %, ethane – 0.15 %, carbon dioxide – 0.31 %, nitrogen – 1.24 %. was considered. The next values for the task's parameters were taken: the pipeline's length L = 120 km, the pipeline's inner diameter D = 1.338 m, the characteristic pressure P_0 = 6.65 MPa, the characteristic temperature T = 300 K, the characteristic mass flux J_0 = 468 kg / m^{2}s, the friction factor $\lambda = 9.22\cdot 10^{-3}$. Calculated characteristic time and Mach number are: $t_0 = 302.58\ s$, $Ma = 0.024$, dimensionless friction factor $\beta = 413.46$.

On the fig. 3 the time dependences for the measures of nonstationarity are shown: the solid, dashed, chain and dotted lines correspond:

$$\delta_-^J(\tau),\ \delta_+^P(\tau),\ \delta^M(\tau) \text{ and } \delta^J(\tau),$$

respectively.

We obtained the values:

$$\delta_-^J(\tau) = 4.79\cdot 10^{-3},\quad \delta_+^P(\tau) = 3.24\cdot 10^{-3},$$

$$\delta^M(\tau) = 1,28\cdot 10^{-3} \text{ and } \delta^J(\tau) = 6,42\cdot 10^{-3} \text{ at } \tau = 48.$$

On this basis we can conclude, that with precision $6,42\cdot 10^{-3}$ the transient process's duration, determined due to measure δ^J equals $48 t_0 \approx 1,452\cdot 10^4$ s, i.e. is more the four hours.

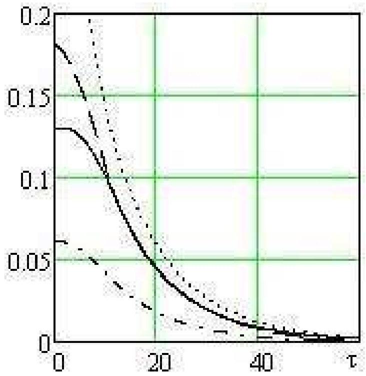

Fig. 3. Dependences of the measures of nonstationarity on time τ

The expenditure of energy referred to the constant $\left(\beta J_0^{\,2}\right)/\left(2\rho_0^{\,2}\right) = 8.143\ \mathrm{J/kg}$ equal: $\eta_\varepsilon^J = 3,689$ (for transient mode), $\eta^0 = 2.744$ (for stationary mode 0) and $\eta^1 = 3,207$ (for stationary mode 1).

CONCLUSIONS

1. The nonstationary one-dimensional initial-boundary-value problems for gas dynamics in long-distance pipeline have been considered. The problems describe transient processes arising in the pipeline under transition of the pipeline from one stationary operational mode to another. Several models to control the transient operational modes have been considered. They differ by the control functions, which determine time variations of the inlet and outlet control parameters (pressures and/or fluxes). The approach for parameterization of the control functions has been proposed. Due to it the control functions can be parameterized by several scalar parameters, among them – mean rates of inlet and outlet control parameters' variations, and time shift between the output and input control actions.

2. The approach to evaluation the duration and expenditure of energy of the transient operational mode has been proposed. The approach is based on the functionals which have been introduced to determine the measures for deviation of the current state of the gas accumulated in the pipeline from its state in the stationary mode, to which the system tends. The measures enable to compare quantitatively different transient modes which realized according to different control models with different control functions. The defined functionals can be used to formulate the problems for optimal control the steady-state and transient modes of operation of main-gas pipelines.

3. The iterative algorithm for numerical solving of the nonlinear initial-boundary-value problems, which correspond to the proposed models for controlling the transient modes, has been developed. The approach is based on representation the solutions as Fourie-Legendre series expansion and time discretization of the sought-for functions with the use of the Crank–Nicolson's difference scheme.

4. The numerical experiments, which have been conducted for different control models, corroborated the high efficiency of the developed approaches and possibility of their application to model the transient processes in long-distance pipelines. The developed approaches and mathematical tools can be used to formulate the problems for optimal control the pipeline's transient operational modes and to solve these problems by their reducing consequences of corresponding direct problems.

5. Considered in the paper model of isothermal flow do not includes the expenditure of energy caused be heat exchange with environment. To raise the practical value of the developed approach it should be improved by taking into consideration the process of energy transfer.

REFERENCES

1. **Tsarenko A. 2007.** Overview of Gas Market in Ukraine. www.case-ukraine.com.ua
2. **Kralik J., Stiegler P., Vostry Z. and Zavorka J. 1984.** Modelling the Dynamics of Flow in Gas Pipelines. IEEE Transactions on System, Man, and Cybernetics. Vol. 14, No 4, 586–596.
3. **Kralik J., Stiegler P., Vostry Z. and Zavorka J. 1984.** Universal Dynamic Simulation Model of Gas Pipeline Networks. IEEE Transactions on System, Man, and Cybernetics. Vol. 14, No 4, 597 – 606.
4. **Yedrouj M. Osiadacz, A.J. A 1989**. Comparison of a finite element method and a finite difference method for transient simulation of a gas pipeline. Appl. Math. Modelling, 13, 606-618.
5. **Gato L.M.C., Henriques J.C.C. 2005.** Dynamic behaviour of high-pressure natural-gas flow in pipelines. International Journal of Heat and Fluid Flow. Vol 26, 817–825.
6. **Prashanth R.H., Narasimhan S. Bhallamudi S.M. 2006.** Simulation and State Estimation of Transient Flow in Gas Pipeline Networks Using a Transfer Function Model. Ind. Eng. Chem. Res. Vol. 45, 3853-3863.
7. **Langelandsvik, L.I., Kunkel, G.J., Smits, A.J. 2008.** Flow in a commercial steel pipe. Journal of Fluid Mechanics. Vol. 595, 323-339.
8. **Behbahani-Nejad M., Shekari Y. 2009**. Reduced Order Modelling of Natural Gas Transient Flow in Pipelines. International Journal of Mathematical, Physical and Engineering Sciences. Vol. 3:3, 168-172
9. **Chaczykowski M. 2010.** Transient Flow in Natural Gas Pipeline – The Effect of Pipeline Thermal Model. Applied Mathematical Modelling. Vol. 34, 1051–1067
10. **Nouri-Borujerdi A. 2011.** Transient modelling of gas flow in pipelines following catastrophic failure// Mathematical and Computer Modelling. Vol. 54, 3037–3045.
11. **Helgaker J.F, Oosterkamp A., Langelandsvik L.I., Ytrehus T. 2014.** Validation of 1D flow model for high pressure offshore natural gas pipelines. Journal of Natural Gas Science and Engineering. Vol 16, 44-56.

12. **Oosterkampa A., Helgakera J.F., Ytrehus T. 2015.** Modelling of Natural Gas Pipe Flow with Rapid Transients.– Case Study of Effect of Ambient Model. Energy Procedia. Vol 64, 101 – 110.
13. **Fletcher C. A. J. 1984.** Computational Galerkin Methods. Springer-Verlag. 309.
14. **Dolinskii A., Draganov B., Kozirskii V. 2012.** Nonequilibrium State of Engineering systems – ECONTECHMOD. An international quarterly journal. Vol. 1, № 1, 33–34.
15. **Batluk V., Basov M. and Klymets'. 2013.** Mathematical model for motion of weighted parts in curled flow. - ECONTECHMOD. An international quarterly journal. Vol. 2, No 3, 17-24.
16. **Artyukh A., Sidorov M. 2014.** Mathematical modeling and numerical analysis of nonstationary plane-parallel flows of viscous incompressible fluid by R-functions and Galerkin method. - ECONTECHMOD. An international quarterly journal. Vol.3, No.3, 03-11.