

# Setting the range of reactive power regulation for a salient-poles motor based on the measurement of the synchronous work state

*The article features a method of determining synchronous reactance in the axes  $d$  and  $q$  for a synchronous motor with salient poles based on measuring basic electrical quantities in the state of synchronous work. The admissible operating range of the machine was discussed. The motor with salient poles was described by means of a phasor diagram. The author presented dependencies which allow to determine the solution searched for. The method was presented in a graphical manner. Finally, the author discussed the possibility to reduce the search range of available solutions.*

Key words: *synchronous motor, electrical drive, reactive power compensation*

## 1. INTRODUCTION

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Slow-speed high power synchronous motors are usually manufactured as salient-pole machines. In motors with salient poles the excitation winding is wound onto the poles of the rotor (field system). The core of the rotor is symmetrical to the salient pole axis which is marked as axis  $d$  and to the longitudinal axis marked as axis  $q$  which is perpendicular to  $d$ . Synchronous reactance has different values in both axes. This allows to achieve a starting torque, called reluctance torque, at the direct start-up from the supply network [1, 5, 8]. However, the lack of reactance symmetry in the axes  $d$  and  $q$  complicates significantly the mathematical description and analysis of the motor operations, particularly in dynamic states [6, 7].

In the state of synchronous work, when the motor is loaded with a braking torque (active power), the rotor axis does not coincide with the axis of the rotating field generated by the windings of the stator. The axis is shifted by the  $\vartheta$  angle, also called power angle. Synchronous work is possible in a limited range of the power angle.

Under-loaded synchronous motors are often used as regulated reactive power compensators. Reactive power regulation is administered by changing the current in the excitation winding of the motor [3, 4]. The assessment of the reactive power regulation range requires that one

should be familiar with synchronous reactance in the axes  $d$  and  $q$ . Improperly determined excitation current may cause the power angle to exceed the admissible value and the motor to fall out of synchronism.

There are methods to determine electromagnetic parameters of a synchronous machine by measuring current quenching at a motionless rotor [6]. This requires, however, that the axis of the field system should be placed very accurately with respect to the flow of the stator supplied from a DC current source.

The rotors of high power machines weigh several tonnes and are usually permanently integrated with the operated devices whose weights are similar or bigger. Setting the motor shaft in a proper position, in order to determine the reactance with the use of above mentioned method, is practically impossible in such cases.

Another method to determine synchronous reactance is the small slip method based on measuring the pulsations of the stator current envelope at an open excitation circuit. Voltages induced in the open excitation circuit during sub-synchronous work require that the supply voltage should be reduced to about 10% of nominal voltage. Obtaining such voltage values is impossible in industrial power supply grids and requires a special testing stand.

Determining the reactance by means of a synchronous short circuit trial requires that the motor should be driven by a different machine. This is difficult or even impossible to fulfil.

It is possible to determine the  $X_d$  and  $X_q$  reactance based on the measurements of active power, reactive power and excitation current during synchronous work of the machine.

The author of the article assumed motor-based work of the synchronous machine and a plus sign of reactive power when the motor absorbs inductive reactive power and a minus sign when the power is returned to the network (absorbing capacitive reactive power from the network).

## 2. SYNCHRONOUS STATE OF A MACHINE WITH SALIENT POLES

Using a synchronous machine as a reactive power compensator requires that an admissible work range should be determined. Figure 1 features a diagram of boundary loads of the synchronous machine [2].

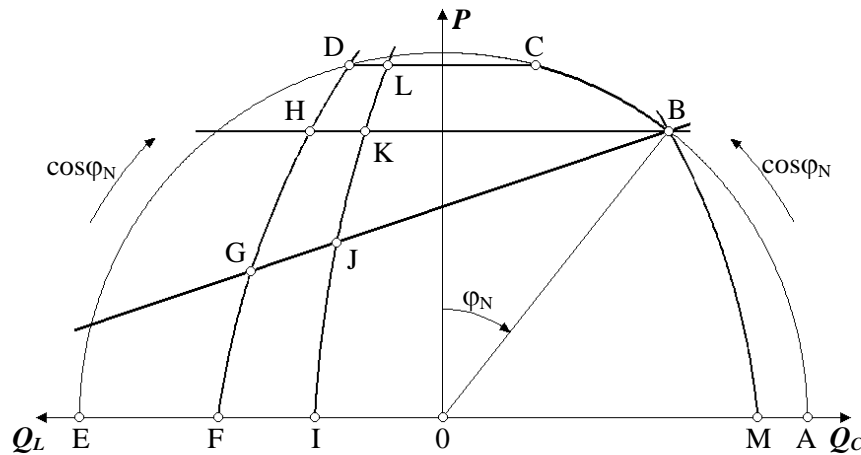


Fig. 1. Boundary loads of a synchronous machine at motor-based work

The  $B$  point is the nominal operating point of the motor. The  $ABCD$  curve is a restriction related to the nominal current of the stator (apparent power). The  $BM$  curve is a restriction related to the nominal excitation current. The  $BH$  section stands for a restriction related to the nominal active power, while the  $BCDH$  curve determines the boundary of temporary overload.

The  $FGHD$  curve is a restriction related to the maximal static power angle which does not result in falling out of synchronism. In reality, it is necessary to take into account the boundary of dynamic stability determined by the  $IJKL$  curve. In practice, the boundary of dynamic stability is the boundary of static stability with a suitable security margin. The manufacturers of motors recommend that the motors should not work with an angle bigger than the nominal one, determined by the  $BJG$  straight. Numerous industrial experiences show that working with a power angle bigger than the nominal one (in a proper range) does not bring any negative consequences. The area restricted by the  $BJ$  section can be thus enlarged to the boundary of the  $BKJ$  curve.

Figure 2 presents a diagram of the electromagnetic torque of a salient-pole motor depending on the power angle. The torque achieved by the motor during synchronous work is the sum of the reluctance torque

related to the lacking symmetry of the rotor magnetic circle and the synchronous torque coming from the excitation current. Stable synchronous work of the motor is possible for an operating point between the points  $A$  and  $B$  from Fig. 2.

The maximal power angle for the given active power load (point  $A$  in Fig. 2), being the boundary of static stability, can be determined with the use of dependencies based on the phasor diagram of the synchronous machine.

In order to determine the coordinates of the admissible work range of the synchronous motor as a reactive power compensator, it is necessary to know the data written on the nameplate of the machine and synchronous reactance values.

The input equations to describe the synchronous machine are vector equations describing a 3-phase AC machine saved in the common rotating coordinate system [1, 4, 5, 6, 7, 8]. However, it is most convenient to describe it in the system of  $d, q, 0$  coordinates rotating concurrently with the rotor. In such a system the windings of the stator and the rotor, placed in the axes  $d$  and  $q$  respectively, are motionless with respect to each other, thus they can be treated as the windings of a transformer with stable inductance values.

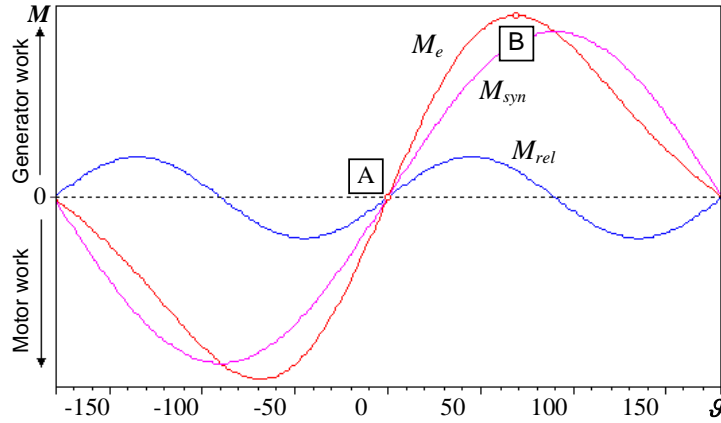


Fig. 2. Electromagnetic torque of a motor with salient poles:  $M_{rel}$  – reluctance torque,  $M_{syn}$  – synchronous torque,  $M_e$  – resultant torque

Figure 3 features a diagram of windings of the salient-pole machine in the axes  $d$  and  $q$ . The symbols  $d$  and  $q$  stand for quantities related to the windings of the stator,  $f$  represents the quantities related to excitation winding, while  $D$  and  $Q$  – quantities related to damping windings.

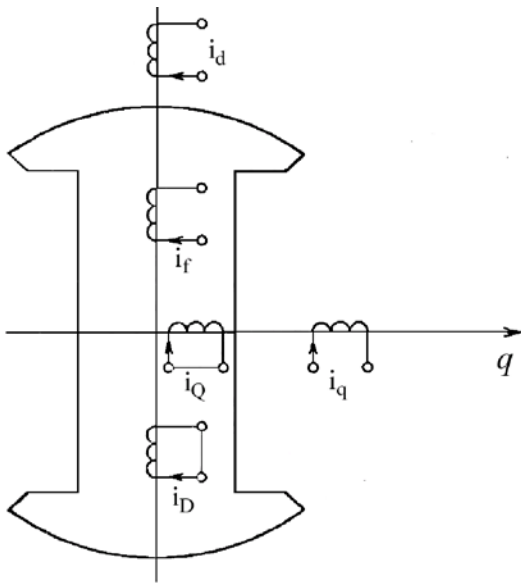


Fig. 3. Winding of a salient-pole synchronous machine in the  $dq$  coordinate system [author's own elaboration]

The equations describing the state of the synchronous machine operations [1, 4, 5, 6, 7] are significantly simplified when it is assumed that the work proceeds in the synchronous state. If we assume that the waveforms of voltages, currents and linked fluxes are symmetrical sinusoids, then in the steady state the time derivatives of fluxes and currents in damping circuits can be equated to zero, which allows to disregard the impact of damping windings.

A convenient way to illustrate the work of the motor is to present mutual relations by means of a phasor diagram. When determining a phasor diagram for high-power motors, the resistance values of stator windings are disregarded, as their impact on the motor operations is insignificant in comparison with the impact of synchronous reactance values in the longitudinal and lateral axis.

Figure 4 features a phasor diagram of the synchronous motor with salient poles in the  $dq$  coordinate system, disregarding losses in the stator winding.

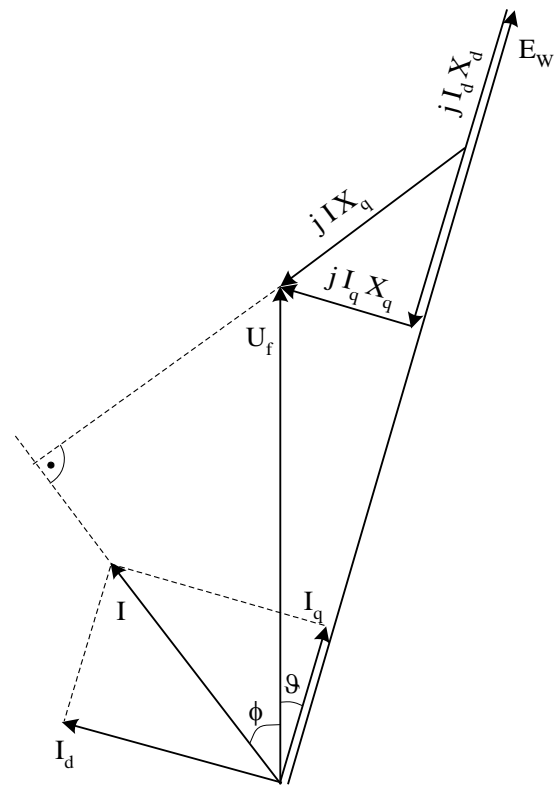


Fig. 4. Phasor diagram of a synchronous motor with salient poles

Based on the phasor diagram from Fig. 4 it is possible to write three independent equations:

$$\begin{cases} P = mU_f \frac{E_w}{X_d} \sin \vartheta + \frac{mU_f^2}{2} \sin 2\vartheta \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \\ Q = -\frac{mU_f^2}{X_q} + m \frac{U_f E_w}{X_d} \cos \vartheta + mU_f^2 \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \cos^2 \vartheta \\ E_w = \frac{I_w}{k_w} \end{cases} \quad (1)$$

where:

- $m$  – number of phases,
- $P$  – active power
- $Q$  – reactive power
- $U_f$  – phase-to-phase voltage,
- $E_w$  – electromotive force induced in the winding of the stator,
- $I_w$  – excitation current,
- $X_d$  – synchronous reactance in axis  $d$ ,
- $X_q$  – synchronous reactance in axis  $q$ .
- $\vartheta$  – power angle,
- $k_w$  – motor constant.

Taking into account the dependency on  $E_w$ , it is possible to write the following:

$$\begin{cases} P = mU_f \frac{I_w}{k_w X_d} \sin \vartheta + \frac{mU_f^2}{2} \sin 2\vartheta \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \\ Q = -\frac{mU_f^2}{X_q} + m \frac{U_f I_w}{k_w X_d} \cos \vartheta + mU_f^2 \cos^2 \vartheta \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \end{cases} \quad (2)$$

The power angle  $\vartheta$  can be expressed as follows:

$$\frac{\cos \vartheta}{\sin \vartheta} = \frac{mU_f^2}{X_q P} - \frac{Q}{P} \quad (3)$$

Assuming that the values  $P$ ,  $Q$ ,  $U_f$ ,  $I_w$  (e.g. nominal values) are known, it means that two independent equations (2) are a function of three unknown variables:

$$\begin{cases} P = f(k_w, X_d, X_q) \\ Q = f(k_w, X_d, X_q) \end{cases} \quad (4)$$

Thus it is impossible to determine the variables searched for with the use of analytical calculations.

Determining the reactance values  $X_d$ ,  $X_q$  and the  $k_w$  coefficient requires that we know at least 3 points of the motor work described by  $P$ ,  $Q$ ,  $I_w$ . The  $P$ ,  $Q$ ,  $I_w$  values can be read from measurement devices during synchronous work of the motor. Assuming that one of necessary operating points is determined by nominal parameters given on the nameplate of the motor, the measurements have to be done for two different operating points, other than the nominal operating point.

Knowing the values  $P$ ,  $Q$ ,  $I_w$  for three different operating points of the motor, it is possible to enter equations for the values of  $X_d$ ,  $X_q$  and  $k_w$ . However, high degree of the equations complexity makes it impossible to solve them in an analytical manner..

The solution can be achieved by means of an iterative algorithm which investigates mutual relations of the  $X_d$ ,  $X_q$   $k_w$  parameters for the known operating points of the machine.

For the set changes of  $X_q$ , based on (3) it is possible to determine the value of the power angle  $\vartheta$  and then, for the set changes of  $X_d$ , determine the value of the  $k_w$  coefficient in compliance with the following dependency:

$$k_w = \frac{I_w}{\frac{PX_d}{mU_f \sin \vartheta} - U_f \left( \frac{X_d}{X_q} - 1 \right) \cos \vartheta} \quad (5)$$

What we are looking for are such values of reactance  $X_d$  and  $X_q$  for which the value of the  $k_w$  coefficient is the same in each of the selected operating points.

In order to illustrate the method, the GAe-1716t/01 synchronous motor was used with rated data presented in Tab.1.

**Table 1.**  
**Rated data of GAe-1716t/01 motor**

| Rated power (mechanical) | $P_{mN}$ [kW]          | 3150         |
|--------------------------|------------------------|--------------|
| Active power             | $P_N$ [kW]             | 3274         |
| Stator voltage           | $U_{N(Y)}$ [V]         | 6000         |
| Stator current           | $I_N$ [A]              | 350          |
| Excitation voltage       | $U_{wN}$ [V]           | 90           |
| Excitation current       | $I_{wN}$ [A]           | 313          |
| Power factor             | $\cos \varphi_N$       | 0.9 capacity |
| Rotating speed           | $n_n$ [rotations/ min] | 375          |
| Frequency                | $f_n$ [HZ]             | 50           |

### 3. DETERMINING SYNCHRONOUS REACTANCE VALUES

The impact of reactance  $X_d$  and  $X_q$  on the value of the  $k_w$  coefficient was illustrated for the nominal operating point of the motor which is described by the values  $P_N$ ,  $Q_N$ ,  $U_{fN}$ ,  $I_{wN}$ . Figure 5 features a diagram of the  $k_w=f(X_d, X_q)$  function for the nominal operating point of the GAe-1716t/01 motor.

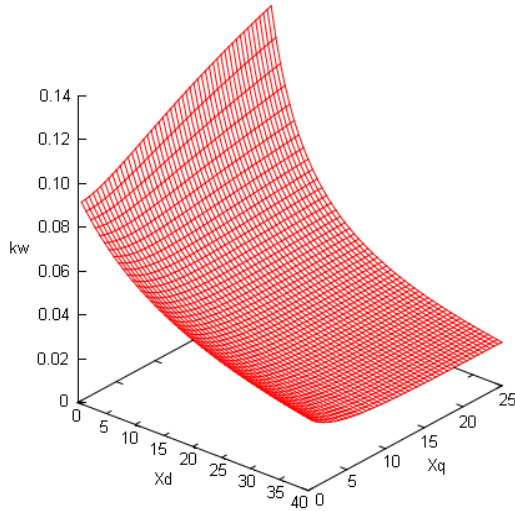


Fig. 5. Impact of  $X_d$ ,  $X_q$  parameters on the value of the  $k_w$  coefficient at rated parameter values for the GAe-1716t/01 motor

It is important to note that the  $k_w$  parameter is the motor constant, which means that it remains unchanged irrespective of the operating state of the driving system for constant values of  $X_d$  and  $X_q$ . The changing value of the  $k_w$  coefficient in Fig. 5 results from the changes in the  $X_d$  and  $X_q$  reactance values in the process of the diagram preparation.

The shape presented in Fig. 5 is described in a 3D space, which means that in order to determine the value of the  $k_w$  coefficient, it is necessary to investigate its variability values for at least three different operating points of the motor. Table 2 features measurement values for two remaining operating states at the rated supply voltage  $U_{fN}$ .

**Table 2.**  
Measurement values for the GAe-1716t/01 motor

| Active power $P$ [kW] | Reactive power $Q$ [kVAr] | Excitation current $I_w$ [A] |
|-----------------------|---------------------------|------------------------------|
| 1636.8                | -1127.5                   | 250                          |
| 327.4                 | -2556.0                   | 313                          |

For the presented measurement values the variability of the  $k_w$  coefficient was drawn, depending on the  $X_d$  and  $X_q$  parameters and was placed on a common diagram with the waveform determined for rated parameters. The results are shown in Fig. 6.

The drawn shapes cross each other in places where the  $k_w$  coefficient values are equal, while the points where all three shapes cross are points of potential values of reactance  $X_q$  and  $X_d$ .

Figure 7 features the projections of the drawn shapes against the  $X_d X_q$  plane. Potential values of the  $X_d$  and  $X_q$  reactance are placed in the points where three colours meet.

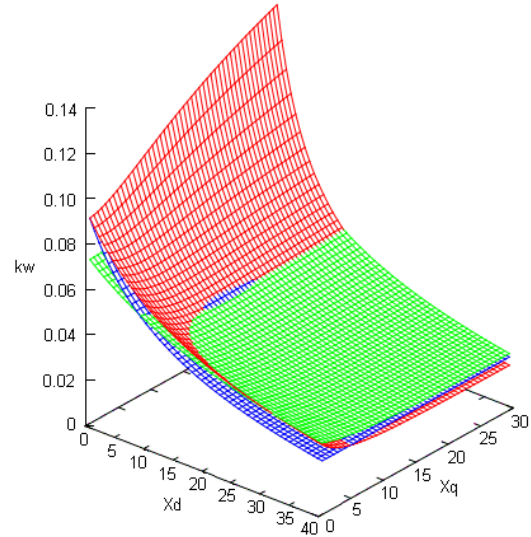


Fig. 6. Impact of the  $X_d$ ,  $X_q$  parameters on the value of the  $k_w$  coefficient for selected operating points of the GAe-1716t/01 motor

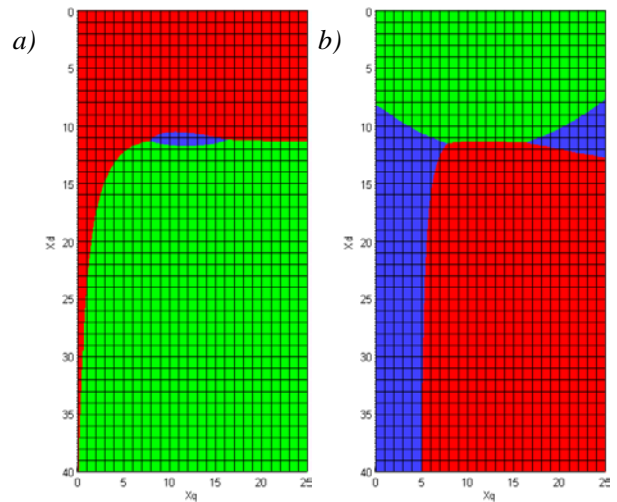


Fig. 7. Projection of the diagram of  $X_d$ ,  $X_q$  parameters impact on the value of the  $k_w$  coefficient for selected operating points of the GAe-1716t/01 motor against the  $X_d X_q$  plane: a) top view, b) bottom view

Figure 8 features a projection of common edges of particular shapes from Fig. 6 against the  $X_q X_d$  plane.

Determining common points of three curves in a 2D space, it is possible to determine potential values of the  $X_q$  and  $X_d$  reactance.

As one can see in Fig. 8, there are two points,  $A(X_{q1}, X_d)$  and  $B(X_{q2}, X_d)$  which fulfil the condition of the  $k_w$  coefficient invariability. Due to the lack of symmetry of the rotor of the salient-poles motor, the reactance in the axis  $d$  is always bigger than the reactance in the axis  $q$ . For this reason it is necessary to reject point  $B$  and the solution sought for are the coordinates of point  $A$  from Fig. 8. For the presented example, the determined reactance values are the following:  $X_q=7,82 \Omega$  and  $X_d=11,38 \Omega$ .

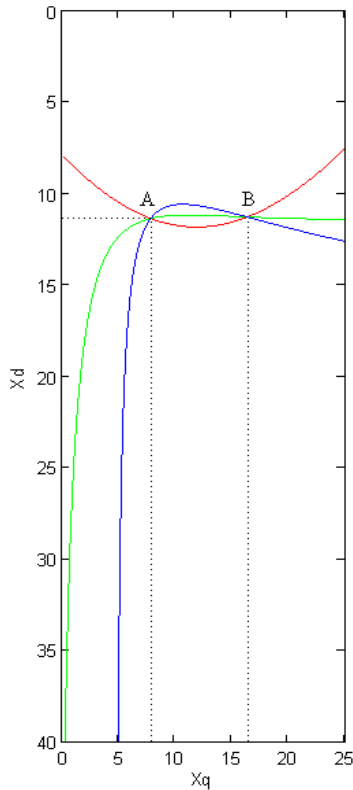


Fig. 8. Projection of common edges of the  $k_w$  coefficient dependency shapes against the  $X_q X_d$  plane for different operating points of the GAe-1716t/01 motor

Using an analogical method, it is possible to determine the values of the  $X_q$  and  $X_d$  reactance, at the assumption that the  $E_w$  electromotive force is stable. However, in this case the measurements of operating states should be conducted at the same excitation current and the same change of the active power load.

The method of looking for a common value of the  $k_w$  coefficient in order to determine synchronic reactance values assumes that the magnetic circuit is linear. To minimize the impact of saturation, it is necessary to select measurement points at the possibly small values of the excitation current.

#### 4. RESTRICTING THE SEARCH AREA

The shapes of the  $k_w=f(X_d, X_q)$  function diagram are not always as regular as those in Fig. 6.

Figure 9 features a diagram of the  $k_w=f(X_q, X_d)$  function for the operating point  $P=0,6P_N$ ,  $Q=2159001$  kVA,  $I_w=110$  A, which is a stable point of synchronous operations for the considered motor at an assumed load.

The “ragged” edges of the shape result from the adopted calculation step with respect to the axes  $X_q$  and  $X_d$  when the diagram is drawn. In reality, in these

places there is a full band with no ragged edges that are seen in Fig.9. A too big calculation step may lead to bigger errors or even make it impossible to determine common values of the  $k_w$  coefficient, which means that it would be impossible to determine, even roughly, the  $X_d$  and  $X_q$  reactance values.

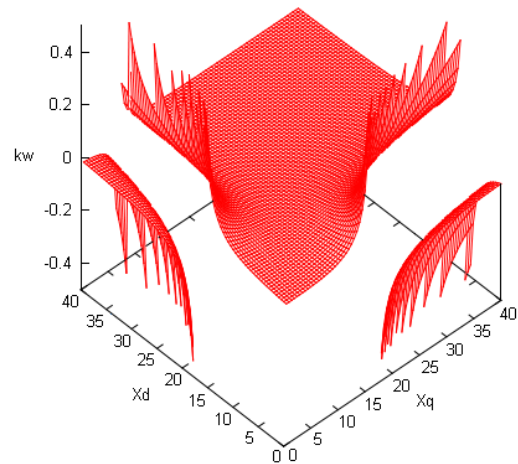


Fig. 9. Diagram of function  $k_w=f(X_q, X_d)$  for the GAe-1716t/01 motor at  $P=0,6P_N$ ,  $I_w=110$  A,  $Q=2159001$  kVA

As one can see in Fig. 9, the obtained shape consists of several segments. The segmentation is caused by the discontinuity of the  $\text{ctg}$  function. It is impossible to determine the  $\vartheta$  angle from the dependency (3) for some combinations of the  $X_d$  and  $X_q$  values. Such a shape of the function produces extra intersections of the edges of all considered areas for different operating points of the motor, thus giving extra solutions which are inaccurate from the point of view of the motor operations.

For motor-based work the minimal power angle  $\vartheta$  is 0. With consideration of the condition  $\vartheta \geq 0$  in (3), we obtain the following:

$$X_q \leq \frac{mU_f^2}{Q} \tag{6}$$

For the considered example the search area of a solution can be limited to the value  $X_{q\max}=16,6744 \Omega$  determined on the basis of the dependency (6).

The  $k_w$  coefficient, which is the motor constant, must be higher than 0. In the light of two above assumptions it is possible to restrict the range of the sought solution.

Figure 10 features a graphic representation of the impact of the  $X_d, X_q$  parameters on the value of the  $k_w$  coefficient for selected operating points of the motor, with respect to the restrictions in the area where the solution is sought.

The figures are graphic illustration of the iterative method for determining parameters. The method enables to read reactance parameters in both axes.

The use of the iterative calculation method to determine the values of the  $X_d$  and  $X_q$  reactance enables to achieve a result of the assumed accuracy. The iterative method should narrow the search to the values close to the  $k_w$  coefficient until the achieved error is smaller than the required accuracy. It is important to have a suitably small initial step in order to avoid situations presented in Fig. 9 and 10 in the form of ragged bands for the drawn functions  $k_w=f(X_q, X_d)$ .

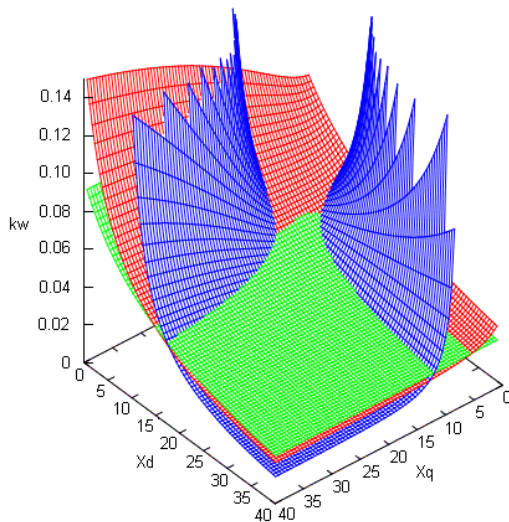


Fig. 10. Impact of  $X_d$ ,  $X_q$  parameters on the value of the  $k_w$  coefficient for the selected operating points of the GAe-1716t/01 motor with restricted search area for  $P=0,6P_N$ ,  $I_w=110$  A,  $Q=2159001$  kVA

## 5. CONCLUSIONS

The knowledge of synchronous reactance values in the longitudinal and lateral axis for a salient-pole machine is indispensable for determining the range of reactive power regulation which ensures safe operations of the motor in the synchronous state [3]. The

$X_d$  and  $X_q$  parameters can be determined on the basis of measurements of basic electrical quantities (active power, reactive power, excitation current) in the state of synchronous work. However, it is necessary to take into account a security margin related to the admissible power angle in dynamic states.

As the dependencies between different operating points of the synchronous-state motor are very complex, it is not possible to achieve the result in an analytical manner. The solution to this problem is the iterative method which looks for a solution according to the criterion of the same value of the  $k_w$  coefficient. Restricting the search area to values related to possible states of the machines with working motors eliminates potentially inaccurate values of the determined synchronous reactance and allows to achieve a solution of any assumed accuracy.

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