TEMPERATURE EFFECTS ON THE PARAMETRIC GENERATION IN A WATER-FILLED ACOUSTICAL RESONATOR

ISABEL PÉREZ- ARJONA, ALEJANDRO CEBRECOS,
VÍCTOR ESPINOSA, VÍCTOR J. SÁNCHEZ-MORCILLO

Institut de Investigació per a la Gestió Integrada de Zones Costaneres
Universitat Politècnica de València (IGIC-UPV)
C/Paranimf, 1, 46730 Grau de Gandia, Spain
iparjona@upv.es

We consider the problem of parametric sound generation in an acoustic interferometer containing a fluid. We have taken explicitly into account both the influence of the nonlinearly generated second harmonic and the effect of the unavoidable rising of the temperature due to the acoustic wave. Two main new features under these considerations can be observed. On one hand, the fundamental field does not achieve a saturating value when subharmonics fields appear, but still grows with the driving. On the other hand, the subharmonics fields can exist only for a limited range of driving value, due to the dependence of the cavity resonances with the temperature of the medium. The experimental setup was designed and we show here the experimental measurements, which are in good agreement with the predicted values for the mode amplitudes and the parametric threshold.

INTRODUCTION

The observation of nonlinear effects and in particular, parametric phenomena in acoustics requires high-intensity fields. High quality resonators are the most adequate systems for achieving intense fields. One simple configuration consists in two plane and parallel walls, one of them vibrating with a driving frequency f. In this configuration, nearly one-dimensional standing waves are formed along the cavity. This system was the basis of the first experimental observation of acoustic parametric oscillation in a fluid [1,2]. This phenomenon [3] consists in the emergence of oscillation modes at frequencies smaller than that of the driving when a parameter is varied periodically in time. This is a universal phenomenon, which has been demonstrated in a large variety of physical systems. [4]. In the case of the acoustical resonator, the length of the cavity (and thus the eigenfrequencies of the normal modes) is the time-dependent parameter, and the parametric excitation is achieved when the input energy is high enough to overcome the dissipative losses. The parametric fields usually appear as doublets, whose frequencies f_1 and f_2 add to match the driving frequency f_0, i.e., $f_0= f_1+ f_2$, although in some circumstances the half-frequency mode (degenerated) is observed.

A theoretical description, based on the Mathieu equation, has been successfully applied to the description of these processes in an acoustical interferometer, allowing predicting the subharmonic spectrum and its excitation threshold [4,5]. This theory was also applied to the interferometer case,[6-9] but the agreement with the experiment [6] was mainly qualitative. The discrepancies can be interpreted in terms of the influence of the first higher harmonics, introducing additional features that cannot be captured by the fully dispersive model, where higher harmonics of the pump are absent.

In a previous work [10] we extended the previous theoretical description of parametric sound generation in a fluid-filled interferometer, to include the coupling with the second harmonic in the case of degenerated parametric acoustic generation, giving rise to new phenomena as bistability or hysteresis. In this work we derived the model to include higher harmonic influence in the most usual case of nondegenerated parametric oscillation ($f_1\neq f_2$) and we considered in the model the unavoidable effect of the rising of the temperature induced by the acoustic wave in the medium. When the heating of the medium taken into account, the system exhibits new features due to the change of the system resonances: the threshold value of the parametric oscillation is shifted, the fundamental mode does not saturate after the subharmonic generation and the existence of parametrically generated subharmonic field can be limited to a finite range of driving amplitudes. All these phenomena have been experimentally observed, in very good agreement with the theoretical prediction of the model.

1. THEORY

The acoustical interferometer considered is composed by two parallel and solid walls, with thicknesses D and H, respectively, located at a distance L from each other, containing a fluid medium inside. Each medium involved in the model is acoustically characterized by its density ρ, bulk modulus κ, and sound velocity c, related as $c^2= \kappa / \rho$. The resonance modes (eigenfrequencies) of the resonator depend on these parameters. We consider now the driven system, assuming that one of the walls vibrates with frequency f0. Then, above the parametric generation threshold the spectrum inside the resonator can be decomposed in two sets: the subharmonics resulting from the parametric instability and the higher harmonics of the driving $n\ f_0$, with n an integer number. The amplitude of any of these harmonics decreases with the detuning, defined as the difference between the corresponding field frequency ω_n and the frequency of the closest mode, $\omega^c{}_n$, i.e., $\delta_n=\omega_n-\omega^c{}_n$. Usually only harmonics with small n reach significant amplitude, but the effects of the second harmonic must be taken into account in most of the cases [10].

The existence of the acoustic wave provokes an intrinsic change in the temperature of the medium. When temperature increases, the sound velocity changes and then the cavity modes are shifted, so changing unavoidably the detuning values. The previous models of parametric acoustic generation in acoustic resonators had ignored the temperature evolution with driving amplitude. We extend the model for including both the effects of the second harmonic and temperature rising in the general case of nondegenerated parametric generation in an plane-wall acoustic interferometer.

Under these assumptions, and following the technique described in detail in [8] for dispersive resonators, the evolution of the slowly-varying amplitude for each mode is given by the given system equations (Eqs. 1):

$$dP_1/dt = -(\gamma_1 + i(\delta_1 + \alpha\Delta T))P_1 - i\sigma P_0 P_2^* \omega_1,$$
$$dP_2/dt = -(\gamma_2 + i(\delta_2 + \alpha\Delta T))P_2 - i\sigma P_0 P_1^* \omega_2,$$
$$dP_0/dt = -(\gamma_0 + i(\delta_0 + \alpha\Delta T))P_0 - i\sigma\omega_0(P_1 P_2 + P_4 P_0^*) + c P_{in}/L,$$
$$dP_h/dt = -(\gamma_h + i(\delta_h + \alpha\Delta T))P_h - i\sigma P_0^2 \omega_0.$$

corresponding to the first and second subharmonic fields, second harmonic and fundamental (driving), respectively. Other parameters are as follows: P_{in} is the driving amplitude, γ_n (n=0,1,2,h) are the decay rates of each mode in the cavity, $\sigma=(1+B/(2A))/(4\rho c^2)$, is related to the nonlinearity parameter B/A of the fluid, $\delta_n=\omega_n-\omega^c_n$ are the detunings and α_n are the coefficients describing the resonance shift when temperature changes ΔT from the equilibrium value. In first approximation (an ideal situation), the eigenmodes of the cavity are proportional to the sound velocity in the fluid (f_n= n c/2L). This sound velocity linearly changes with the temperature in a enough small range of temperatures around the equilibrium, so it can be assumed that the effect of temperature variation on the cavity resonances is a modification on the detuning values, proportional to the temperature variation. The value of α can be obtained from the evaluation of the sound velocity dependence on temperature (Fig. 1).

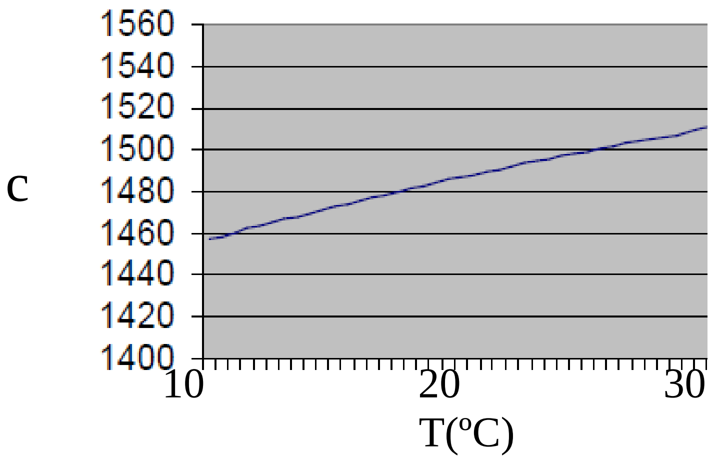

Fig.1. Sound velocity evolution with temperature from 10 to 30 ºC in water.

The dynamical system given by Eq. 1 can be reduced to a simpler, dimensionless form, defining the new variables A_n and ε by

$$P_0 = \frac{i A_0 \rho c^2 \sqrt{\gamma_1 \gamma_2}}{\sigma \sqrt{\omega_1 \omega_2}}, \quad P_h = \frac{i \rho c^2 A_h \gamma_0}{\sigma \omega_0}, \quad P_1 = \frac{i A_1 \rho c^2 \sqrt{\gamma_2 \gamma_0}}{\sigma \sqrt{\omega_2 \omega_0}},$$

$$P_2 = \frac{i \rho c^2 A_2 \sqrt{\gamma_1 \gamma_0}}{\sigma \sqrt{\omega_1 \omega_0}}, \quad P_{in} = \frac{i \in L \sqrt{\gamma_1 \gamma_2}}{\sigma c \sqrt{\omega_1 \omega_2}}$$

and the parameters $\Gamma=\gamma_1\gamma_2/\gamma_0\gamma_h$, $\Omega=\omega_1\omega_2/\omega_0^2$ $\Delta_n=\delta_n/\gamma_n$. The model obtained reads (Eqs.2)

$$\gamma_1^{-1}\frac{dA_1}{dt} = -\left(1 + i\left(\Delta_1 + \frac{\alpha}{\gamma_1}\Delta T\right)\right)A_1 - i\, A_0 A_2^{*},$$

$$\gamma_2^{-1}\frac{dA_2}{dt} = -\left(1 + i\left(\delta_2 + \frac{\alpha}{\gamma_2}\Delta T\right)\right)A_2 - i\, A_0 A_1^{*},$$

$$\gamma_0^{-1}\frac{dA_0}{dt} = -\left(1 + i\left(\Delta_0 + \frac{\alpha}{\gamma_0}\Delta T\right)\right)P_0 + (A_1 A_2 - A_h Ac_0) + \epsilon,$$

$$\gamma_h^{-1}\frac{dA_h}{dt} = -\left(1 + i\left(\Delta_h + \frac{\alpha}{\gamma_h}\Delta T\right)\right)A_h - \frac{\Gamma}{\Omega}A_0^2 .$$

The stationary solution of Eqs. 2 when the system is below the parametric generation threshold, $A_1=A_2=0$, are known and given in [10]. When the driving amplitude exceeds the parametric generation threshold, which has been numerically evaluated in the model including the evolution of the second harmonic, the subharmonics fields emerge. This threshold is modified due to the temperature change. In the case of the system described in the present work, the variation of the temperature depends on the amplitude of the driving field. In the simplest approach, for a range of driving amplitude values, it can be considered that temperature grows linearly with driving, $\Delta T=\beta\ \varepsilon$.This assumption is in good agreement with the experimental observations, and β can be evaluated from the registered measurements.

The numerical integration of (Eqs.2) is shown in Fig.2 for a set of parameters (see figure caption)

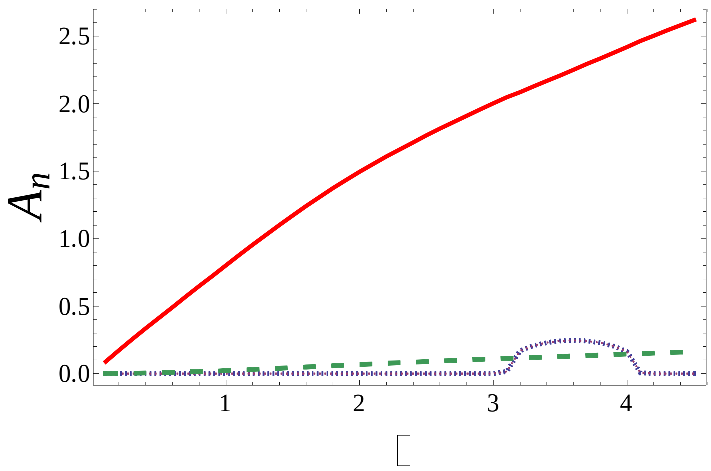

Fig.2. Numerical simulation of evolution of fundamental (continous line), subharmonics (tiny dashed line) and second harmonic (large dashed) in front of driving amplitude. Parameter values are Δ_1=0.8, Δ_2=0, Δ_0=0.5, Δ_h =3, γ_1=73, γ_2=100, γ_0=100, γ_h=120 Ω=4.02 and $\alpha\beta$= 0.14.

2. EXPERIMENT

The resonator consists in two piezoceramic disks (ρ=7700 kg/m^3 and c=4400 m/ s) with radius of 1.5 cm and thicknesses of 1 and 2 mm (corresponding to resonance frequencies around 2 and 1 MHz, respectively), mounted in a Plexiglas tank containing distilled and degassed water (ρ =1000 kg/m^3 and c=1480 m/s at T=20 °C). Both sides are located at a variable distance L(L=3.4cm), and its parallelism can be carefully adjusted to get a high-Q

interferometer. One of the piezoceramics—that with resonance frequency around 2 MHz—is driven by the signal provided by a function generator (Agilent 33220) and a broadband rf power amplifier ENI 240L. The experimental setup is completed by a needle
hydrophone (TNUA200 NTR Systems) to measure the intracavity pressure field. In this way, by changing the amplitude of the driving source, we are able to follow the bifurcation diagram of the resonator for a given set of parameters. Although the pump value and the decay rates γ_n can be unambiguously determined by measuring the line width of the cavity modes, the detuning parameters change when driving increases, due to the intrinsic temperature rising, and are in the experiment are measured at the equilibrium temperature, T=23ºC.

One of the experimental results is shown in Fig. 3

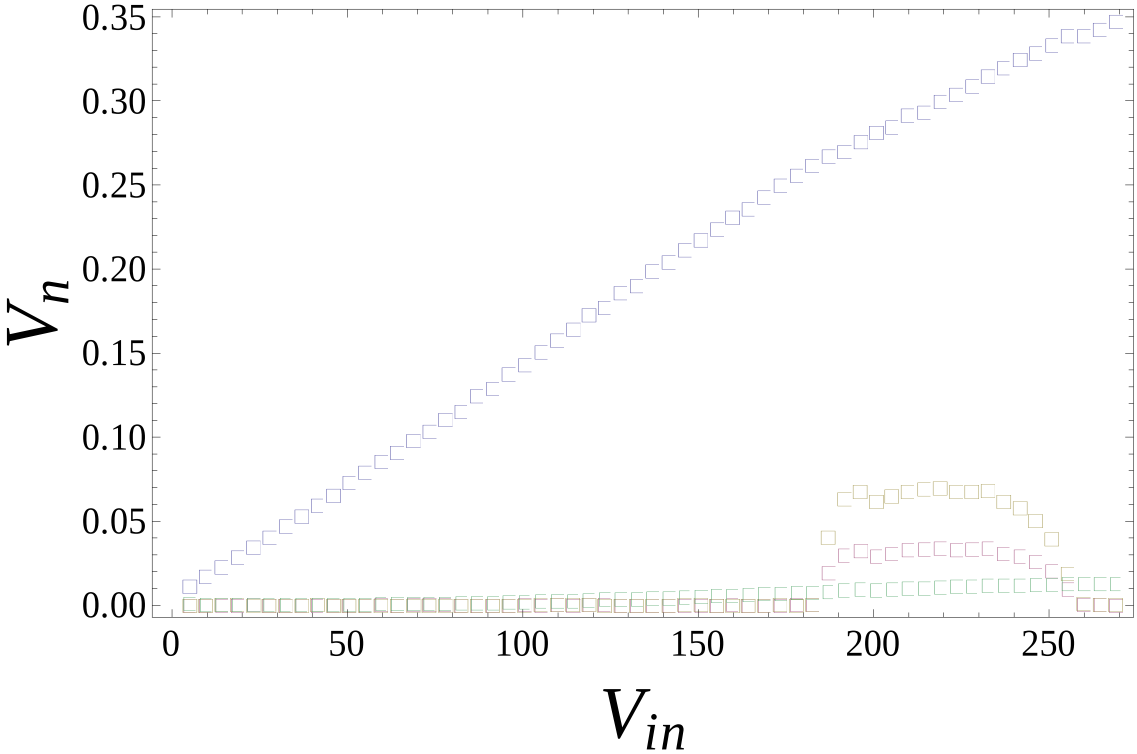

Fig.3. Evolution of fundamental (dots), subharmonics(diamonds and squares) and second harmonic (triangles) in front of driving amplitude. Parameter values are Δ_1=0.8, Δ_2=0, Δ_0=0.5, Δ_h =3, γ_1=73, γ_2=100, γ_0=100, γ_h=120.

We note the good qualitative agreement with the numerical results shown in Fig.2: shift of parametric threshold, finite range in driving amplitude of existence of parametric generation and growing of fundamental and second harmonic fields far from the threshold, with a different slopes when subharmonic fields are present.

First approach confirm that this agreement is not only quantitative but also quantitative when the sensitivity of hydrophones is considered, as is shown in Fig.4.

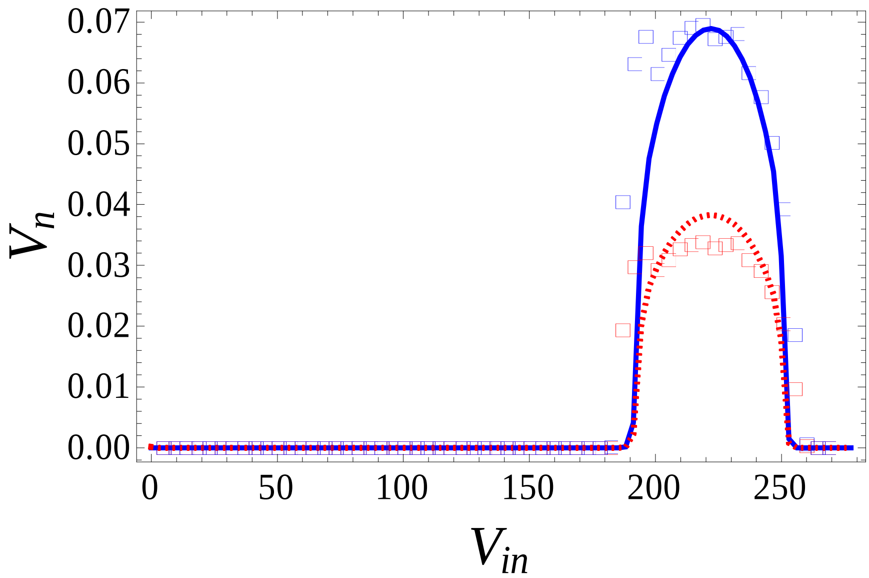

Fig.4. Numerical simulation (continous line) and experimental result (points) for the evolution of first (red) and second (blue) subharmonic field in the case correspondig to Fig.1 and Fig.2

REFERENCES

[1] A. Korpel and R. Adler, "Parametric phenomena observed on ultrasonic waves in water", Appl. Phys. Lett. 7, 106–106 (1965).

[2] L. Adler and M. A. Breazeale, "Excitation of subharmonics in a resonant ultrasonic wave system," Naturwiss. 55, 385–385 (1968).

[3] M. Faraday, "On a peculiar class of acoustical figures; and on certain forms assumed by particles upon vibrating elastic surfaces," Philos. Trans. R. Soc. London 121, 299–340 (1931).

[4] L. Adler and M. A. Breazeale, "Parametric phenomena in physics," Am. J. Phys. 39, 1522–1527 (1971)

[5] A. Teklu, M. S. McPherson, M. A. Breazeale, R. D. Hasse, and N. F. Declercq, "Measurement of the frequency dependence of the ultrasonic parametric threshold amplitude for a fluid-filled cavity," J. Acoust. Soc. Am. 120, 657–660 (2006).

[6] N. Yen, "Experimental investigation of subharmonic generation in an acoustic interferometer," J. Acoust. Soc. Am. 57, 1357–1362 (1975).

[7] V. J. Sánchez-Morcillo, "Spontaneous pattern formation in an acoustical resonator," J. Acoust. Soc. Am. 115, 111–119 (2004).

[8] I. Pérez-Arjona and V. J. Sánchez-Morcillo, "Pattern formation in parametric sound generation," Phys. Rev. E 72, 066202 (2005).

[9] V. J. Sánchez-Morcillo, V. Espinosa, J. Redondo, and J. Alba, "Nonlinear dynamics and chaos in parametric sound generation," Acta. Acust. Acust. 92, 210–216 (2006).

[10] I. Pérez-Arjona, V. J. Sánchez-Morcillo and V. Espinosa, "Bistable and dynamic states of parametrically excited ultrasouund in a fluid-filled interferometer", J. Acoust. Soc. Am. 125, 3555-3560 (2009).