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## A GENERALIZED METHOD FOR PREDICTING CONTACT STRENGTH, WEAR, AND THE LIFE OF INVOLUTE CONICAL SPUR AND HELICAL GEARS: PART 2. HELICAL GEARS

### UOGÓLNIONA METODA PROGNOZOWANIA WYTRZYMAŁOŚCI STYKOWEJ, ZUŻYCIA ORAZ TRWAŁOŚCI STOŻKOWEJ EWOLWENTOWEJ PRZEKŁADNI O ZĘBACH PROSTYCH ORAZ UKOŚNYCH: CZ. 2. PRZEKŁADNIA O ZĘBACH UKOŚNYCH

<b>Key words:</b>	involute conical helical gear, tooth correction, contact and tribo-contact pressures, tooth wear, gear durability.
<b>Abstract</b>	The paper presents the results of research undertaken to determine maximum contact pressures, wear, and the life of involute conical helical gear, taking account of gear height correction, tooth engagement, and wear-generated changes in the curvature of their involute profile. We have established the following: (a) initial maximal contact pressures will be almost the same at the engagement in external and internal segments; (b) their highest meanings occur in different points of engagement depending on the coefficients of displacement; (c) the maximal tooth wear of the rings in the internal section will be a little bit lower than in the external; (d) the coefficients of displacement have an optimum at which the highest gear life is possible; and, (e) the gear life in the frontal section will be 1.25 lower than in the internal section. The calculations were made for a reduced cylindrical gear using a method developed by the authors. The effect of applied conditions of tooth engagement in the frontal and internal sections of a cylindrical gear ring is shown graphically. In addition, optimal correction coefficients ensuring the longest possible gear life are determined.
<b>Słowa kluczowe:</b>	stożkowa ewolwentowa przekładnia o zębach ukośnych, korekcja technologiczna zazębienia, naciski kontaktowe oraz tribokontaktowe, zużycie zębów, trwałość przekładni.
<b>Streszczenie</b>	W artykule przedstawiono rezultaty oszacowania maksymalnych nacisków stykowych, zużycia oraz trwałości stożkowej przekładni o zębach ukośnych z uwzględnieniem korekcji technologicznej uzębienia, parzystości zazębienia zębów oraz zmiany krzywizny ich zarysów ewolwentowych wskutek zużycia. Ustalono, że: a) początkowe maksymalne naciski kontaktowe w zazębieniu będą prawie takie same jak w przekrojach czołowym i wewnętrznym; b) ich największe wartości występują w różnych punktach zazębienia w zależności od wartości współczynników korekcji; c) maksymalne zużycie zębów kół w przekroju wewnętrznym będzie nieco niższe niż w przekroju czołowym; d) współczynniki korekcji profilu mają optimum, przy którym trwałość przekładni będzie najwyższa; e) trwałość przekładni w przekroju czołowym będzie o 1,25 razy mniejsza niż w przekroju wewnętrznym. Obliczenia przekładni stożkowej przeprowadzono jako zredukowanej przekładni walcowej przy użyciu metody opracowanej przez autorów. Ustalono prawidłowości wpływu wymienionych warunków interakcji zębów w przekroju czołowym oraz wewnętrznym wieńca kół stożkowych przedstawiono w postaci graficznej. Ponadto określono optymalne współczynniki korekcji zapewniające maksymalnie możliwą trwałość przekładni.

## INTRODUCTION

Together with spur conical gears, conical gears with helical (Fig. 1) and spiral teeth also find wide application. Their advantages in comparison with spur

gears are a higher bearing capacity and service life and a lower wear. Positive effects arise due to the increase of the coefficient of tooth overlap in engagement, and it is also due to the change of teeth engagement conditions (parity of engagement). In the literature, the results of the

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research of helical conical gears, taking account of gear height correction, tooth engagement and wear-generated changes in curvature of their involute profile, are absent. The calculations for a conical helical gear were made in the same way as for a reduced cylindrical gear with frontal and internal modules of conical engagement made variable over a tooth length  $m_{min} \leq m \leq m_{max}$  [L. 1], as was done in Part 1 in the case of a spur gear, with the applied methods for assessing contact strength, and the wear and life of spur and helical gears [L. 2–6].



**Fig. 1. Involute conical helical gear**  
Rys. 1. Przekładnia stożkowa ewolwentowa o zębach ukosnych

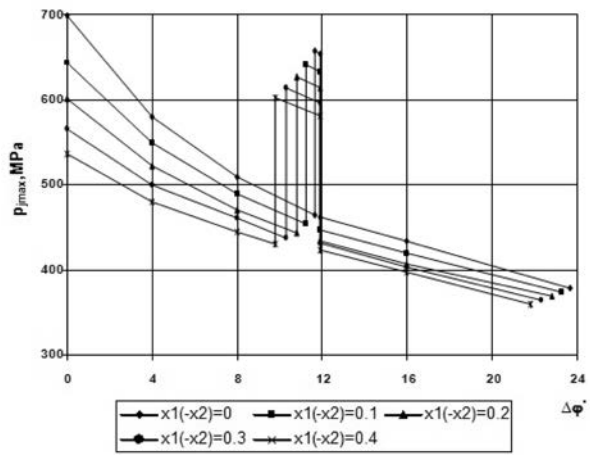
**NUMERICAL SOLUTION FOR THE CONICAL HELICAL GEAR PROBLEM**

The input data included the following:  $z_{1K} = 20$ ;  $u_K = 3$ ;  $n_1 = 750$  rpm;  $P = 20$  kW;  $b_w = 50$  mm;  $\beta = 10^\circ$ ;  $m_{max} = 4.617$  mm – a normal module of tooth engagement in the frontal section of a gear at  $\beta = 10^\circ$ ;  $m_{min} = 3.391$  mm – a normal module of tooth engagement in the internal section of a gear at  $\beta = 10^\circ$ ;  $\Delta\varphi = 4^\circ$ ;  $h_{k*} = 0,5$  mm – maximum acceptable wear of gear teeth;  $B = 900000 = 1200 n_1$  revolutions. We applied boundary lubrication with a sliding friction factor set to  $f = 0.05$ . The applied profile shift coefficients were  $x_1 = -x_2 = 0; 0,1; 0,2; 0,3; 0,4$ .

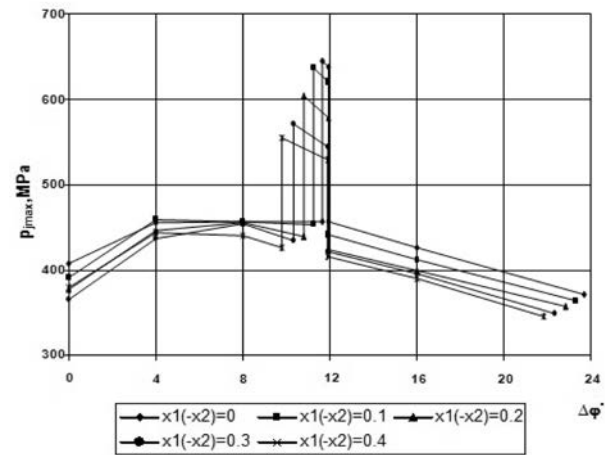
The gears were ascribed the following material properties: the pinion was made of 38HMJA steel after nitriding at a depth ranging from 0.4 mm to 0.5 mm described by 58 HRC,  $R_m = 1040$  MPa,  $C_1 = 3.5 \times 10^6$ ,  $m_1 = 2$ ; the gear was made of bulk hardened 40H steel with 53 HRC,  $R_m = 981$  MPa,  $C_2 = 0.17 \times 10^6$ ,  $m_2 = 2.5$ ;  $E = 2.1 \times 10^5$  MPa,  $\mu = 0.3$ .

The frontal section of the gear ring is characterized by a double-single-double tooth engagement at  $\beta = 10^\circ$ , while the internal section of the gear ring has a triple-double-triple tooth engagement at  $\beta = 10^\circ$ . The results are illustrated in the Figs. 2–11 below.

**I. Frontal section,  $m_n = m_{max} = 4.617$  mm, double – single – double tooth engagement**



**Fig. 2. Changes in  $p_{jhmax}$  during tooth interaction**  
Rys. 2. Zmiana  $p_{jhmax}$  w trakcie interakcji zębów



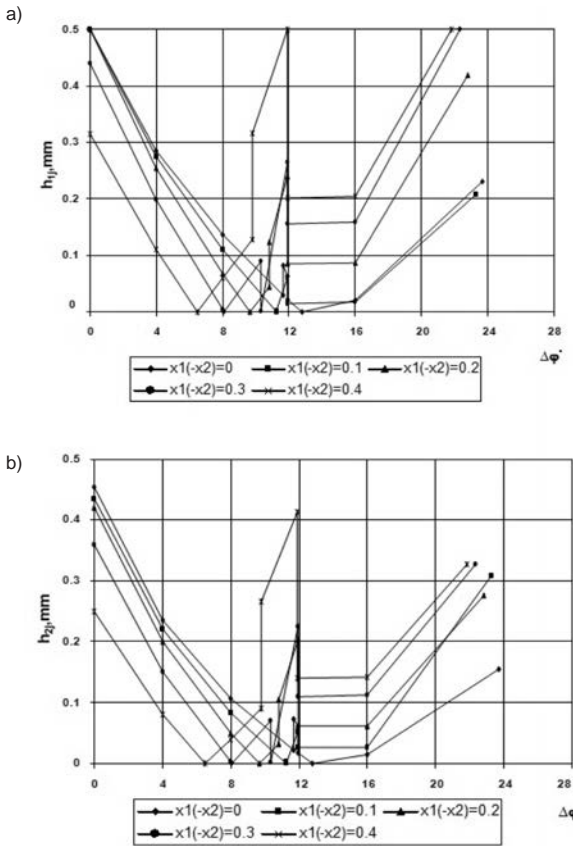
**Fig. 3. Tribo-contact pressures  $p_{jhmax}$**   
Rys. 3. Naciski tribokontaktowe  $p_{jhmax}$

Here, the single tooth engagement zone is much smaller (Fig. 2) than in the previous case with the spur gear. The changes in  $p_{jhmax}$  pressures are similar in the left-hand zone of the double tooth engagement, yet they somewhat differ in the single tooth engagement zone (Fig. 3).

Fig. 4 shows the diagrams of the linear wear of gear profiles in the engagement zone.

Here, the maximum acceptable tooth wear occurs at the entrance of the left-hand zone of double tooth engagement at  $x_1 = -x_2 = 0; 0.1; 0.2$ , at the exit of the double tooth engagement zone on the right ( $x_1 = -x_2 = 0.3; 0.4$ ), or at  $x_1 = -x_2 = 0.4$  at the exit of single- and double tooth engagement simultaneously.

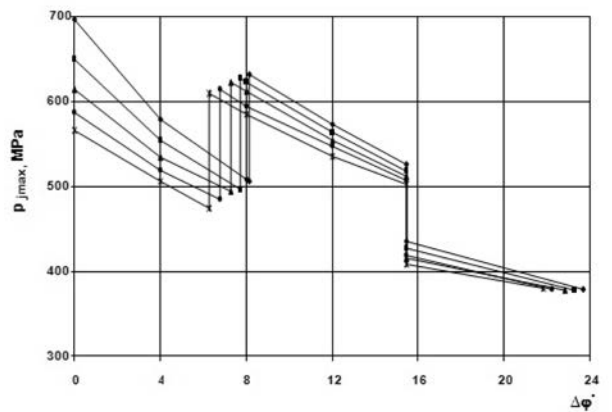
Fig. 5 depicts the influence of shift coefficients on minimal gear life for the profile point of the pinion tooth in which the permissible wear  $h_{k*} = 0.5$  mm is reached.



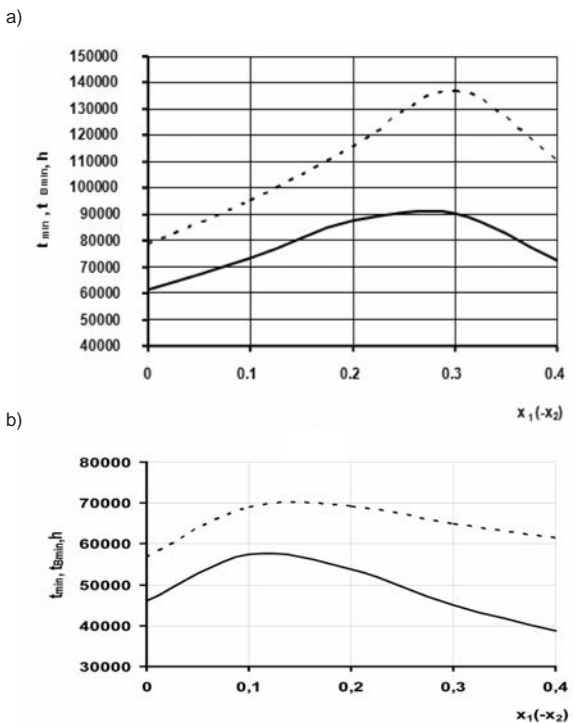
**Fig. 4. Linear wear of corrected teeth: a) pinion, b) gear**  
 Rys. 4. Zużycie liniowe zębów z korekcją technologiczną: a) zębniak, b) koło zębate

Here, the optimal profile shift coefficients with respect to gear life are  $x_1 = -x_2 = 0.3$ . The gear life  $t_{Bmin}$  (taking account of the effect of tooth wear on changes in the curvature radii of a tooth profile) is higher than the gear life  $t_{min}$  by 1.52 times. For comparison, in **Fig. 5b**, the life of spur gear is given along with the character of its change in this section together with the change of shift coefficients. The optimum helical gear life  $t_{Bmin}$  is 1.37 times higher than the optimum life of spur gear.

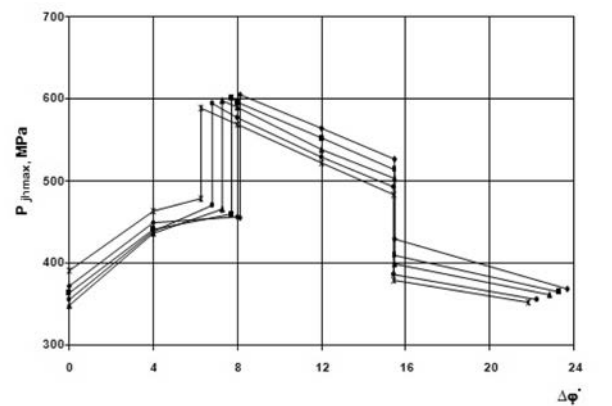
**II. Internal section,  $m_{min} = 3.391$  mm, triple – double – triple tooth engagement**



**Fig. 6. Changes in  $p_{jmax}$  during tooth interaction**  
 Rys. 6. Zmiana  $p_{jmax}$  w trakcie interakcji zębów



**Fig. 5. Gear life:  $t_{min}$  when  $p_{jmax} = const$ ,  $t_{Bmin}$  when  $p_{jmax} = var$  in the: a) helical gear, b) spur gear**  
 Rys. 5. Trwałość przekładni:  $t_{min}$  gdy  $p_{jmax} = const$ ,  $t_{Bmin}$  gdy  $p_{jmax} = var$ : a) zęby ukośne, b) zęby proste



**Fig. 7. Tribo-contact pressures  $p_{jmax}$**   
 Rys. 7. Naciski tribokontaktowe  $p_{jmax}$

The zones of triple tooth engagement (left and right) are narrower than in the case of double tooth engagement observed in the previous case, while the central zone is wider (**Fig. 2**). The change in  $p_{jmax}$  at the maximum acceptable wear of gear teeth in the left-hand zone of triple tooth engagement (**Fig. 6**) is similar to that shown in **Fig. 2**. For triple-double-triple tooth engagement,  $p_{jmax}$  is almost the same as at double-single-double tooth engagement in frontal section (**Figs. 2, 3**).

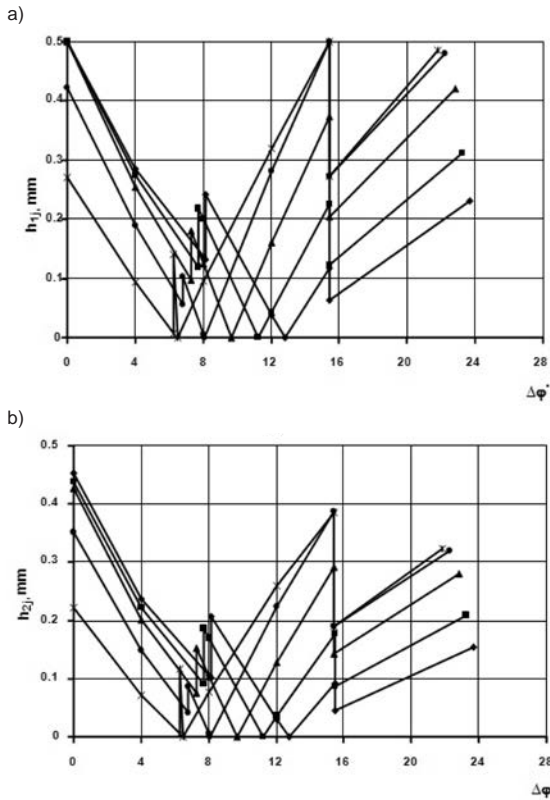


Fig. 8. Linear wear of corrected teeth: a) pinion, b) gear  
 Rys. 8. Zużycie liniowe zębów z korekcją technologiczną: a) zębniak, b) koło zębate

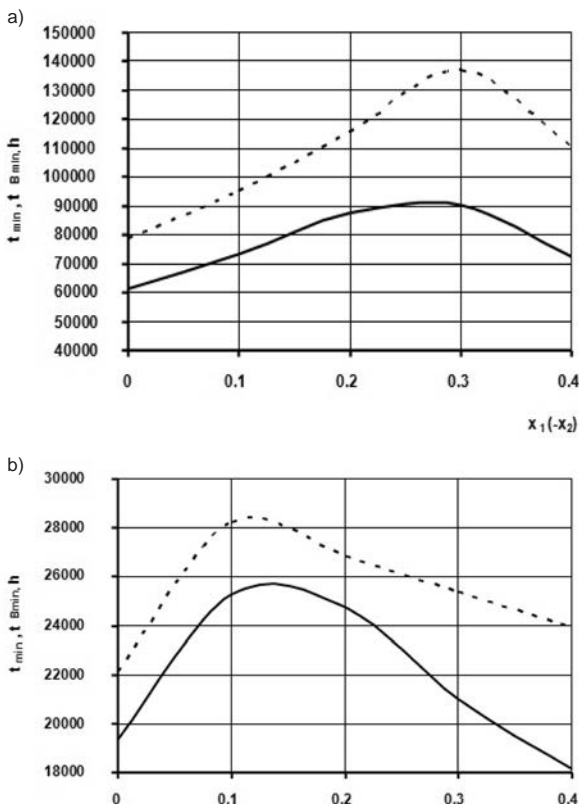


Fig. 9. Gear life  
 Rys. 9. Trwałość przekładni

The trends in tooth wear illustrated in Fig. 8 are somewhat different from those shown in Fig. 4, despite the fact that the maximum acceptable tooth wear is also attained here either at the entry of the left-hand zone of the triple tooth engagement ( $x_1 = -x_2 = 0; 0.1; 0.2$ ) or at the exit of the double tooth engagement ( $x_1 = -x_2 = 0.3; 0.4$ ). This difference lies in the fact that the zones of three-pair engagement (Fig. 8) are narrower than zones of two-pair engagement (Fig. 2), and the character of changes in wear values in the central zone is quite different (Figs. 2 and 8).

Both this section (Fig. 9) and the frontal (Fig. 6a) one are characterized by optimal values of profile shift coefficients  $x_1 = -x_2 = 0.3$ . The gear life will be higher here than that shown in Fig. 5a. Since the life in the frontal section is lower by 1.17 times, below are the results of  $p_{jmax}, h_{1j}, h_{2j}$  calculated for this gear life (Figs. 10 and 11).

III. Internal section, triple – double – triple tooth engagement,  $t_{min}$  set identical as in frontal section

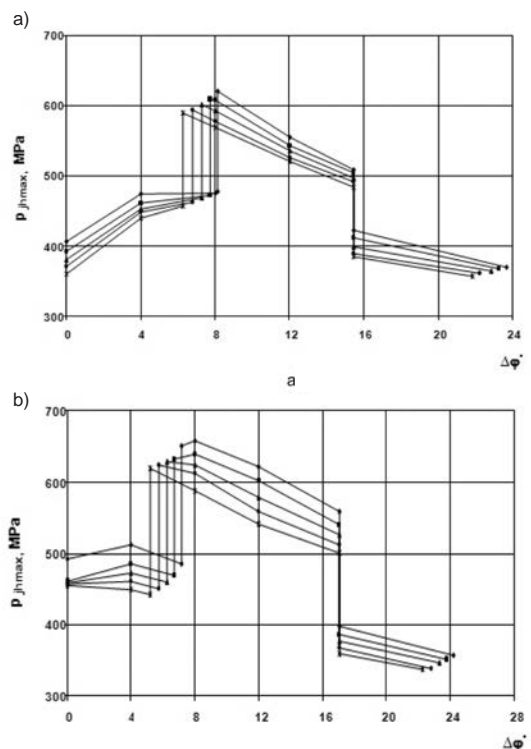


Fig. 10. Real transformation pressures  $p_{jmax}$  in the: a) internal section helical gear, b) frontal section spur gear

Rys. 10. Rzeczywista zmiana nacisków  $p_{jmax}$ : a) w przekroju wewnętrznym przekładni o zębach prostych, b) w przekroju czołowym przekładni o zębach ukośnych

The difference between the  $p_{jmax}$  pressures shown in Fig. 10 and those given in Fig. 7 is insignificant. Nonetheless, the real wear of gear teeth (Fig. 11) will be lower by 7.5 % and 12.7%, respectively, than that shown in spur gear.



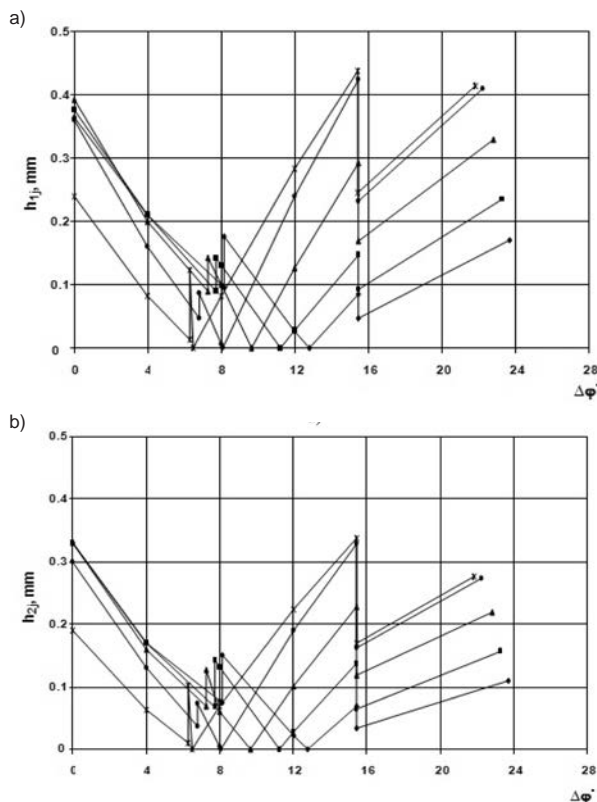


Fig. 11. Linear wear of gear teeth in internal section: a) pinion, b) gear

Rys. 11. Zużycie liniowe zębów kół w przekroju wewnętrznym: a) zębnik, b) koło zębate

## CONCLUSIONS

The results have demonstrated the following:

1. The  $p_{jmax}$  pressures will be almost identical in both sections due to the fact that the internal section is characterized by a triple-double-triple tooth engagement, whether the module is decreased or not.

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2. The highest  $p_{jmax}$  occurs either at the entry of the left-hand zone of engagement (uncorrected gear) or, depending on the coefficients of displacement, at the entry of the left-hand and central zone of engagement or at the exit of the central zone of engagement simultaneously.
3. The regularities concerning a decrease in  $p_{jmax}$  and  $p_{jhmax}$  in the left-hand zone of engagement can similarly be observed with increasing the profile shift coefficients  $x_1 = -x_2$ .
4. We determined optimal displacement coefficients  $x_1 = -x_2 = 0.3$ , which will produce the highest possible gear life.
5. The life of helical gears is the lowest in the frontal section of the gear ring in contrast to the spur gear where minimal durability would be in the internal section of the gear ring.
6. The real values of maximum wear would be 82.5% in the internal section of the pinion teeth and 75% for wheel teeth from the permissible  $h_{k*} = 0.5$  mm.

## DISCUSSION

The conducted researches of involute conical gears with straight and helical teeth confirm the efficiency of the developed calculation method. The obtained results of the numerical solution show the differences in the progress process of tribo-contact teeth interaction in these two types of conical gears and the values of calculation parameters dependently of teeth correction coefficients and their pitch angle. Having used the research results obtained due to the presented method, it is possible to provide detailed recommendations at the stage of gear design calculations at which its highest life at optimum teeth profile wear will be reached as well as proper teeth bearing capacity during operation.