ISSN 0208-7774 TRIBOLOGIA 1/2017 p. 31–34

Myron CZERNIEC[*], Jerzy KIEŁBIŃSKI[*], Jurij CZERNIEC[**]

THE EFFECT OF TEETH CORRECTION IN AN ARCHIMEDES WORM GEAR ON THE CONTACT STRENGTH, WEAR, AND LIFE OF THE WORM GEAR TEETH

WPŁYW KOREKCJI ZAZĘBIENIA PRZEKŁADNI ŚLIMAKOWEJ ZE ŚLIMAKIEM ARCHIMEDESA NA JEJ WYTRZYMAŁOŚĆ STYKOWĄ, ZUŻYCIE ORAZ TRWAŁOŚĆ

Key words: worm gear, teeth of Archimedes, correction, contact strength, wear, life.

Abstract: The paper presents a method for determining the effect of teeth correction in an Archimedes worm gear on the contact strength, wear, and life of teeth of the worm wheel. The regularities regarding the effect of correction on contact and tribocontact parameters are established.

Słowa kluczowe: przekładnia ślimakowa, zazębienie Archimedesa, korekcja, wytrzymałość stykowa (kontaktowa), zużycie, trwałość.

Streszczenie: Przedstawiono metodę oceny obliczeniowej wpływu korekcji uzębienia przekładni ślimakowej ze ślimakiem Archimedesa na wytrzymałość stykową, zużycie oraz trwałość zębów koła ślimakowego. Ustalono prawidłowości wpływu korekcji na parametry kontaktu oraz kontaktu tribologicznego.

INTRODUCTION

Archimedes worm gears are widely used in various devices and equipment. On meshing a sliding friction is generated, which leads to the wear of the teeth of the worm wheel. For this reason, it is required that the life of such a gear and the wear of its wheel be determined at the stage of design. The literature reports methods for determining the abrasive wear of worm gear teeth **[L. 1–4]**; however, the proposed methods cannot be used for uncorrected and corrected gears. The studies **[L. 1, 2]** report the results regarding the determination of teeth wear in an uncorrected gear according to a modified Archard law which takes into account the changes in contact pressures and oil film thickness in the contact zone based on elastohydrodynamic lubrication theory. A method **[L. 5, 6]** was developed for the determination of contact parameters in uncorrected worm gears. Later, it was generalized and applied to corrected worm gears.

UNIT LINEAR WEAR FUNCTION

According to **[L. 5],** the function of liner wear of the teeth of a worm gear per one revolution is calculated from the following formula:

$$h'_{2j} = \frac{\nu_j t'_j \left(f p^{(w)}_{j\max} \right)^{m_2}}{C_2 \left(\tau_{s2} \right)^{m_2}}, \tag{1}$$

where

$t'_j = 2b_j / v_j$ is the time of contact between the meshing components at j-th point on the friction path with a length of $2b_j$;

v_j is the velocity of slide at j-th point of meshing at the height of worm coils;

f is the sliding friction coefficient;

C_2, m_2 are the indicators of wear resistance of the material of the worm wheel in a selected pair and the conditions of wear as determined in the experimental tests;

[*] Lublin University of Technology, Mechanical Engineering Faculty, Institute of Technological Information Systems, Nadbystrzycka 36, 20-618 Lublin, tel.: (81) 538-42-76, e-mail:m.czerniec@pollub.pl.

[**] Ivan Franko State Pedagogical University in Drohobych (Ukraine), Scientific Research Department, Ivan Franko 24, 82100 Drohobych, Ukraine.

$\tau_{s2} \approx 0.35 R_{m2}$ is the temporary shear strength of material the worm wheel;
R_{m2} is the temporary tensile strength of material of the worm wheel;
$2b_j^{(w)} = 2.256\sqrt{\Theta N' \rho_j / bw}$ is the width of contact area;
$p_{j\max}^{(w)} = 0.564\sqrt{N' / w\theta\rho_{2j} b}$ are the maximum contact pressured determined according to the Hertz formula depending on the number of meshing pairs w of teeth of the worm wheel,
$\theta = \left(1-\mu_1^2\right)/E_1 + \left(1-\mu_1^2\right)/E_2$ is the Kirchhoff modulus;
μ, E are the Poisson ratio and Young modulus of materials of the worm wheel, respectively;
ρ_{2j} is the radius of curvature of teeth of the worm wheel at j-th point of meshing;
b is the worm wheel width.

Meshing force

The meshing force is calculated according to the following formula **[L. 5]:**

$$N' = \frac{2T}{d_1 \cos\alpha_{pxj} \sin\left(\gamma + \rho'\right)}, \tag{2}$$

where
$T = 9550 \cdot 10^3 (N/n_1)$ is the torque transmitted by the worm,
$\rho' = arctg\left(f / \cos\alpha\right)$ is the friction angle,
N is the transmitted power,
n_1 is the number of revolutions of the worm,
$\alpha = 20^\circ$ is the angle of meshing,
γ is the angle of elevation of the screw line of worm coils,
d_1 is the reference diameter of the worm,
α_{pxj} see below.

Curvature radius of the worm gear teeth and gear geometry

The curvature radius of the worm wheel teeth is calculated with the following formula:

$$\rho_{2j} = \left(\frac{d_2}{2}\sin\alpha_{xj} + e_{pAj}\right). \tag{3}$$

The coordinate x is be in a range of $x_A < x < x_B$, respectively, $x_A = r_{f_1} + 0,2m$, $x_B = r_{a_1}$. The section of meshing $[x_A, x_B]$ must be divided proportionately into smaller sections: $x_A = j_A = j_1$, $x_2 = j_2$, $x_3 = j_3$, ..., $x_B = j_n = j_B$.

The parameters and geometrical relationships of the Archimedes worm gear are as follows **[L. 5]:**

$$r_{f_1} = 0.5\left(d_1 - 2h_{f_1}\right),\ h_{f_1} = 1.2m \text{ (when } \gamma \le 15^\circ\text{)},$$
$$h_{f_1} = 1,2m_n \text{ (when } \gamma \rangle 15^\circ\text{)};$$
$$tg\gamma = mz_1 / d_1,\ d_1 = qm\ ;\ r_{a_1} = 0.5\left(d_1 + 2h_{a_1}\right),$$
$$h_{a_1} = m \text{ (when } \gamma \le 15^\circ\text{)},\ h_{a_1} = m_n \text{ (when } \gamma > 15^\circ\text{)};$$
$$r_2 = 0.5 z_2 m,\ r_2 = 0.5 d_2,\ z_2 = uz_1,\ q = 2\left(1+\sqrt{z_2}\right);$$
$$\alpha_{pxj} = arctg(-tg\alpha_{xj}),\ tg\alpha_{xj} = \frac{180}{\pi}\frac{mz_1}{2x},$$
$$e_{pAj} = \frac{r_1 - x}{\sin\alpha_{pxj}},\ r_1 = 0.5d_1,\ b = 2m\sqrt{q+1},$$

where
r_{f1} is the radius of a circle of worm cavity,
m is the axial modulus of meshing,
$m_n = m\cos\gamma$ is the normal modulus of meshing,
z_1 is the number of worm coils,
q is the diametral quotient of the worm gear,
r_{a1} is the radius of a circle of worm coil prongs,
h_{a1} is the height of head of the worm coil,
d_2 is the reference diameter of the worm wheel,
z_2 is the number of teeth in the worm wheel,
u is the gear ratio,
e_{pA} is the distance of j point from the contact point.

The slipping velocity v_j is determined according to the formula:

$$v_j = \frac{\omega_1 x}{\cos\gamma_A} \tag{4}$$

where
$tg\gamma_A = mz_1 / 2x$,
$\omega_1 = \pi n_1 / 30$ is the angular velocity of the worm.

Wear and life of the worm gear

The wear of the worm wheel teeth within one hour of gear operation is as follows **[L. 5, 6]:**

$$\overline{h}_{2j} = 60 n_2 h'_{2j},\quad n_2 = n_1 / u, \tag{5}$$

where
n_2 is the number of revolutions of the worm wheel per minute;
h'_{2j}, $n_2 h'_{2j}$ are the linear wear of the worm wheel teeth within one revolution and one minute of operation, respectively.

The life of the worm gear for the acceptable wear h_{2*} of the worm wheel teeth is calculated according to the following formula:

$$t_* = \left(h_{2*} / \overline{h}_{2j}\right). \tag{6}$$

Worm wheel teeth correction

As a result of the teeth correction, the profile of the worm wheel teeth undergoes displacement (positive or negative) with respect to the initial profile (uncorrected) and a formation of a tooth profile by some other section of the involute. As a result, the contact pressures and teeth wear decrease while the gear life increases. In worm gears, we can only apply correction to the worm wheel teeth. Hence, the applied displacement of the gear-cutting hob is as follows:

$$\xi = x_2 m,$$

where $x_2 \le \pm 1$ is the correction correlation.

The interaxial distance is

$$a_{wk} = a_w + x_2 m, \qquad (7)$$

where $a_w = r_1 + r_2$ is the interaxial distance in the uncorrected gear.

The reference diameter of the worm in the uncorrected gear is

$$d_{w1} = d_1 + x_2 m.$$

Therefore, the distance e_{pAj} between the j-th point of contact and the point of contact in **[L. 6]** will be as follows:

$$e_{pAj} = \frac{r_{w1} - x}{\sin \alpha_{pxj}}, \quad r_{w1} = d_{w1}/2.$$

The teeth curvature radius for the corrected worm wheel is as follows:

$$\rho_{2j} = \left(\frac{d_2}{2}\sin \alpha_{xj} + \frac{r_{w1} - x}{\sin \alpha_{pxj}}\right).$$

For the uncorrected worm gear, the force in meshing is as follows:

$$N' = \frac{2T}{d_{w1}\cos \alpha_{pxj} \sin(\gamma + \rho')}.$$

NUMERICAL SOLUTION

Other geometrical parameters are determined according to the formulae for the uncorrected worm gear. The computations were performed using the following set of data: $N = 3.5$ kW, $n_1 = 1410$ rev/min, $m = 6$ mm, $z_1 = 2$, $u = 25.5$, $f = 0.05$, $q = 8$; worm- hardened steel grade 45 (HRC 50) described by $E_1 = 2.1 \cdot 10^5$ MPa, $\mu_1 = 0.3$; worm wheel ring – bronze CuSn6Zn6Pb6 described by $E_2 = 1.1 \cdot 10^5$ MPa, $\mu_2 = 0.34$; $C_2 = 7.6 \cdot 10^6$, $m_2 = 0.88$; $\tau_{s2} = 75$ MPa; for $j = 1$; 2; 3; 4 and 5, respectively $x = 18$; 20; 22; 24 and 26 mm; $h_{2*} = 0.5$ mm; with double-pair meshing.

The results of the numerical solution are given in **Figs. 1–6. Fig. 1** shows the relationships between the maximum contact pressures p_{max} and the displacement coefficient x_2 on entering the mesh ($j = 1$) and on leaving the mesh ($j = 5$). It was found that the positive correction of the worm wheel teeth leads to reduced pressures, while the negative correction results in their increase compared to the observations made regarding the uncorrected gear. This results in an increase in the teeth curvature radius ρ_2 at their acceptable wear $h_{2*} = 0.5$ mm when x_2 is positive and it decrease at negative values (**Fig. 2**).

The effect of teeth correction of the linear wear $\bar{h}_2$ of the worm wheel teeth during one hour of gear operation is illustrated in **Fig. 3**. **Fig. 4** shows the minimal life t_{min} of the corrected gear ($j = 1$; $x = 18$ mm) at the acceptable wear $h_{2*} = 0.3$ та 0.5 mm.

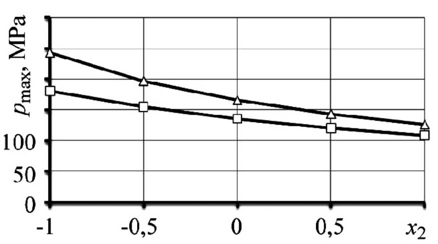

Fig. 1. The effect of teeth correction on p_{max}: □ – value p_1 by $j = 1$; △ – value p_5 by $j = 5$

Rys. 1. Wpływ korekcji na wartość p_{max}: □ – wartości p_1, gdy $j = 1$; △ – wartości p_5, gdy $j = 5$

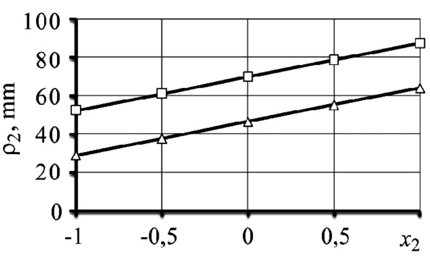

Fig. 2. The effect of teeth correction on variations in curvature radius: □ – value ρ_{2max} by $j = 1$; △ – value ρ_{2min} by $j = 5$

Rys. 2. Wpływ korekcji na zmianę promieni krzywizny: □ – wartości ρ_{2max}, gdy $j = 1$; △ – wartości ρ_{2min}, gdy $j = 5$

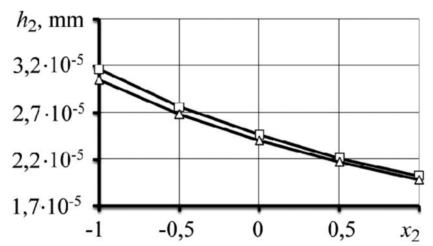

Fig. 3. Gear teeth wear versus x_2: □ – value $\bar{h}_{2max}$ by $j = 1$; △ – value $\bar{h}_{2min}$ by $j = 5$

Rys. 3. Zależność zużycia zębów koła od zmiany x_2: □ – wartości $\bar{h}_{2max}$, gdy by $j = 1$; △ – wartości $\bar{h}_{2min}$, gdy $j = 5$

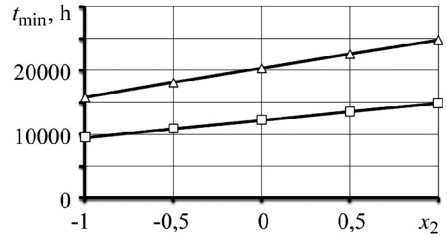

Fig. 4. The effect of teeth correction on gear life t_{min}: △ – value t_{min} by h_{2*} = 0.3 mm; □ – value t_{min} by h_{2*} = 0.5 mm

Rys. 4. Wpływ korekcji na trwałość przekładni t_{min}: △ – wartości t_{min}, gdy h_{2*} = 0,3 mm; □ – wartości t_{min}, gdy h_{2*} = 0,5 mm

As a result of the applied teeth correction, when $x_2 > 0$, ρ_2 increases and so the wear $\overline{h}_2$ decreases and the gear life t_{min} increases; when $x_2 < 0$, ρ_2 decreases, which causes consequences which are undesired from a practical point of view. It must be stressed that the linear wear of teeth on entering the mesh (j = 1) and on leaving the mesh (j = 5) is similar (**Fig. 3**), although the pressures p_{max} (see **Fig. 1**) differ to a significant degree. The applied teeth correction does not affect the sliding velocity v_j in meshing, and its value only depends on the position of contact point at the height of the tooth (**Fig. 5**).

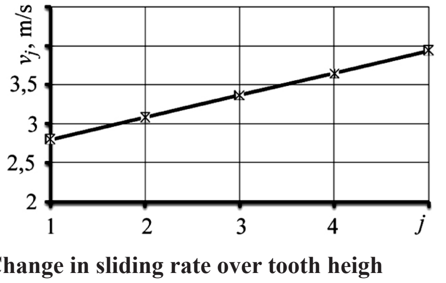

Fig. 5. Change in sliding rate over tooth heigh

Rys. 5. Zmiana szybkości poślizgu wzdłuż zęba

We also investigated the contact area width $2b_j$ at meshing points j = 1, 2, …, 5 (**Fig. 6*a***) and the time t'_j of unit contact. It was found that the unit sliding friction path $2b$ decreases from j = 1 to j = 5, so the life of the gear decreases too. When the gear teeth enter the mesh, the parameter $2b_1$ remains practically unchanged, but

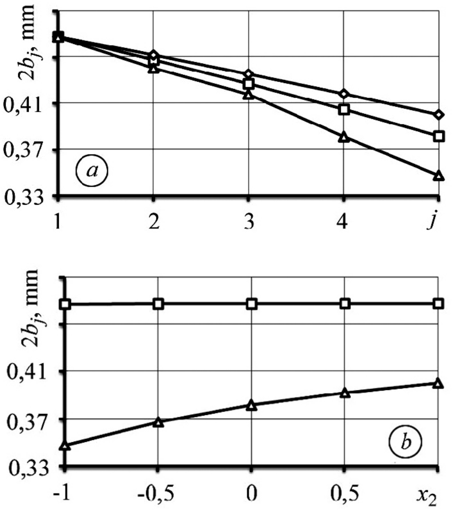

Fig. 6. Effect of teeth correction on contact area per one interaction of teeth and worm wheel revolution: □ – x_2 = 0; ○ – 1; △ – x_2 = –1

Rys. 6. Wpływ korekcji na strefę styku w ciągu jednej interakcji zęba ze zwojem ślimaka: □ – x_2 = 0; ○ – 1; △ – x_2 = –1

when the teeth leave the mesh, the value of $2b_5$ (bottom curve) slightly changes (**Fig. 6*b***).

The discovered qualitative and quantitative relationships with respect to the effect of teeth correction on the load capacity, wear, life, and sliding velocity of the gear are reported in the conclusions section below.

CONCLUSIONS

1. It has been found that, in contrast to the uncorrected gear, when the correction coefficient x_2 is positive, the maximum contact pressures decrease. When the correction coefficient has a negative value, the pressures increase (**Fig. 1**).
2. The radii of teeth curvature increase with an increase in x_2, and they decrease with a decrease in x_2 (**Fig. 2**).
3. The wear of the worm wheel teeth decreases at $x_2 > 0$ and increases at $x_2 < 0$ compared to the case when $x_2 = 0$ (**Fig. 3**).
4. The life of the gear increases at $x_2 > 0$, while at $x_2 < 0$, it decreases (**Fig. 4**).

REFERENCES

1. Sharif K.J., Evans H.P., Snidle R.W., Barnett D., Egorov I.M.: Effect of elastohydrodynamic film thickness on a wear model for worm gears. IMechE, Vol. 220, 2006, Part J: J. Engineering Tribology, p. 295–306.
2. Sharif K.J., Evans H.P., Snidle R.W.: Prediction of the wear pattern in worm gears. Wear, Vol. 261, 2006, p. 666–673.
3. Sabiniak H.G.: Obciążalność i trwałość przekładni ślimakowych, Wydawnictwa Politechniki Łódzkiej, Łódź 2007.
4. Sabiniak H., Woźniak K.: Analityczna ocena trwałości ściernej przekładni ślimakowej. Zagadnienia Eksploatacji Maszyn, Zeszyt 2–3, 1990, s. 82–83.
5. Czerniec M., Kiełbiński J.: Metoda badania przekładni ślimakowej ze ślimakiem Archimedesa. Tribologia №3, 2009, s. 31–40.
6. Czerniec M., Jarema R. Prognozowanie trwałości przekładni ślimakowych ze ślimakiem Archimedesa oraz ewolwentowym. Problemy Tribologii, № 2, 2011, s. 21–25.