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## **Determination the Accuracy of TELPOD SVP 45 Resistive Sensors as Tools for Measuring the Relative Displacement of Points\*\***

### **1. Description of the TELPOD Resistive Sensors and Their Applications**

These devices are slide potentiometers encased in a metal package with a carbon resistive element. The manufacturer has two types of potentiometers on offer. The first type has a linear resistance characteristics, and the second – a logarithmic one. The length of the slider path is 45 mm.

The primary use of potentiometers of the SVP 45 group is their application as a regulatory element in electronic audio-visual equipment and household appliances. Other applications include various measurement systems.

The main advantage of such devices is their low price – the cost of one potentiometer of the measurement devices group is an expense of circa 10 PLN. Individual sensors can be combined in the form of surveying rosettes, surveying micro-lines, parallel sensor units, etc. Reference points, depending on the needs, are stabilized in the form of concrete blocks with metal rods in the ground, plugs in the walls, etc.

The performance principle of a single element of the system is based on measuring changes in the distance between two points. A rod is permanently attached to one point, and resistive sensor to the second one. The resistive sensor is connected to the second end of the metal rod with a stiff leg. With the change in the distance between the test points, the rod causes a change in the slider position in the resistive sensor. The sensor is connected to the check chart with signal wires. A single check chart allows us to connect, depending on the model, from six to eight sensors. Further data transfer is carried via a USB cable to a PC. The record of changes in the section length is executed by a PC using an appropriate computer program. The data record is kept as a text file on the computer disc.

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## 2. Calibration of Resistive Sensors

Before field installation of the resistive sensors, their calibration is a necessity. In order to implement the scaling of the resistive sensors, a calibration position has been constructed (Fig. 1), which uses a micrometer with a reading precision of 0.001 mm. The leg of the resistive sensor was fixed to the micrometer measuring arm, so that its length changes were the same as the path of the slider in the resistive sensor. The Figure additionally presents the check chart and the direction of the micrometer measuring arm and resistive sensor leg movements.

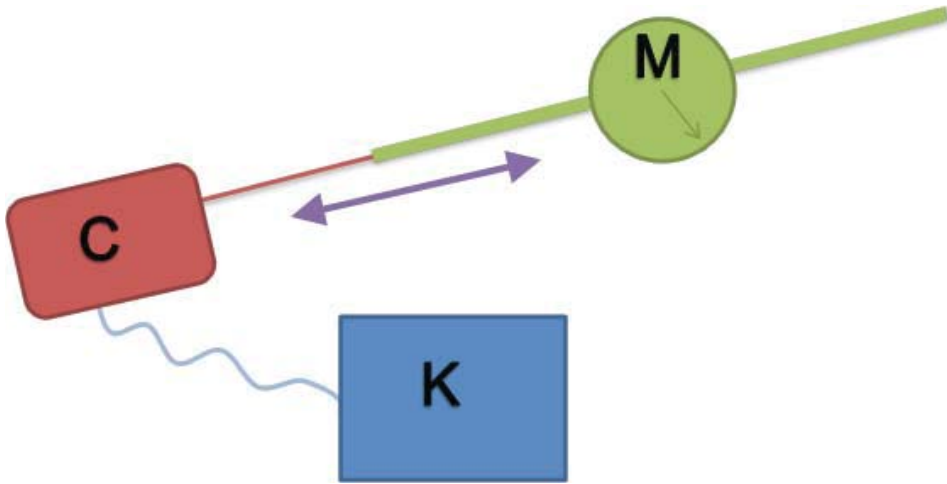


Fig. 1. Resistive sensors calibration position scheme: M – micrometer, C – resistive sensor, K – check chart

The basic values which are subject to designation in the calibration process is the constant multiplicand of each sensor and its measuring range. The constant multiplicand is used to convert the values given in volts by the software for linear displacements expressed in mm. The actual measurement range is shorter than the path of the slider, it amounts to about 30 mm and it is dependent on the characteristics of a particular device.

The calculation and introduction of these parameters to the software before fixing the sensors allows us to obtain and field-record deformation values in millimeters. The calibration procedure comprised the extreme values survey of the measurement range on the micrometer ( $O_{m-\min}$  and  $O_{m-\max}$ ) and on the resistive sensor ( $O_{V-\min}$  and  $O_{V-\max}$ ).

Constant multiplicand  $k$  was calculated from the formula below:

$$k = \frac{O_{m-\max} - O_{m-\min}}{O_{V-\max} - O_{V-\min}} \quad (1)$$

In table 1, measurement ranges and the constant multiplicand of the resistive sensors used in the research were used.

**Table 1.** Values of measurement ranges and constant multiplicand of resistive sensors

	C1	C2	C3	C4	C5	C6	C7	C8
$O_{m-\min}$ [mm]	7.000	7.000	7.000	6.000	6.000	7.000	5.000	6.000
$O_{m-\max}$ [mm]	38.000	36.000	37.000	37.000	35.000	38.000	37.000	37.000
$O_{V-\min}$ [V]	0.86884	0.86133	0.86800	0.74974	0.74046	0.86768	0.62239	0.74457
$O_{V-\max}$ [V]	4.71656	4.42971	4.58800	4.62339	4.31932	4.71027	4.60570	4.59152
$k$	8.05672	8.12694	8.06452	8.00278	8.10313	8.06747	8.03353	8.05833

### 3. Statistical Analysis of Survey Accuracy Performed with the SVP45 Sensors

After the calibration, studies were carried out which aimed at determining the accuracy of a single reading. For this purpose, for each sensor a survey was carried out in its entire measurement range. The survey was performed at each 0.500 mm. Depending on the sensor, from 59 to 65 paired observations were obtained. Readings from the micrometer were marked as  $O_M$  and from the resistive sensor as  $O_C$ .

Statistical evaluation of measurement results was based on the Bland–Altman test [1]. This test is applied to compare two measurement methods. A positive test result leads to a conclusion that the two investigated measurement methods do not significantly differ from each other. In the analysis of the results obtained during the calibration of resistive sensors, the null hypothesis was verified that the indications of the micrometer and the resistive sensors do not significantly differ from each other, against an alternative hypothesis saying about significant differences in indications of these sensors. The analyses were conducted separately for each pair of the micrometer-resistive sensor devices. Therefore, eight cases were examined.

The first stage of the Bland–Altman test is to draw up a chart presenting a dependence between the differences obtained from both methods ( $D_{ij}$ ) (vertical axis of the chart) and the mean ( $M_{ij}$ ) (horizontal axis). The values  $D_{ij}$  and  $M_{ij}$  were calculated from the following formulas:

$$D_{ij} = O_{Cij} - O_{Mij} \quad (2)$$

$$M_{ij} = \frac{O_{Cij} + O_{Mij}}{2} \quad (3)$$

In the equations (2) and (3):

$D$  – difference between the results obtained from both methods,

$M$  – mean of the results obtained from both methods,

$O_c$  – reading made with the resistive sensor,

$O_M$  – reading made with the micrometer,

$i$  – next observation,

$j$  – number of the tested sensor.

While creating a chart for the values  $D_{ij}$  and  $M_{ij}$  in the Bland–Altman test, limits are calculated as well:

$$U_j = \hat{D}_j + 2\sigma_j \quad (4)$$

$$L_j = \hat{D}_j - 2\sigma_j \quad (5)$$

where:

$U_j$  – upper limit for a given sensor,

$L_j$  – lower limit for a given sensor,

$\hat{D}_j$  – the mean of the indication differences for the micrometer and a given sensor,

$\sigma_j$  – standard deviation of the differences between the measurement methods.

If the condition is met, where  $D_{ij}$  implementations in the field of surveying are greater than  $L_j$  and less than  $U_j$ , therefore the dependence is met:

$$\forall_{i=1..n} D_{ij} \in \langle L_j, U_j \rangle \quad (6)$$

then the two measurement methods are considered to be consistent according to the Bland–Altman test.

Table 2 summarizes the values of mean indication differences between the readings from the micrometer and a given sensor ( $\hat{D}_j$ ), the standard deviations of differences between the measurement methods ( $\sigma_j$ ) and the upper  $U_j$  and lower  $L_j$  limits for all the sensors.

**Table 2.** Specification of the values  $\hat{D}_j$ ,  $\sigma_j$ ,  $U_j$  and  $L_j$  for observation differences between the resistive sensors and the micrometer

Sensor	$\hat{D}_j$ [mm]	$\sigma_j$ [mm]	$L_j$ [mm]	$U_j$ [mm]
C1	-0.002	0.132	-0.266	0.262
C2	0.001	0.125	-0.249	0.251
C3	0.000	0.125	-0.250	0.250
C4	0.000	0.128	-0.256	0.256
C5	0.009	0.109	-0.209	0.227
C6	0.000	0.118	-0.236	0.236
C7	0.000	0.135	-0.270	0.270
C8	0.002	0.128	-0.254	0.258

Based on the above data, charts for all the sensors were drawn up (an example of a chart has been presented in figure 2). In the case of the sensors C4, C5 and C7, all the points on the chart fall between the limits  $L_j$  and  $U_j$ , so that the first criterion of the Bland–Altman test was met. In the case of other sensors, each chart presents observations slightly exceeding the lower limit  $L_j$ . These observations are in this part of the chart, which corresponds to the resistive sensor readings for the end of the measurement range which, in practice, is not used. It was assumed then, that this test is also satisfied for the sensors C1, C2, C3, C6 and C8.

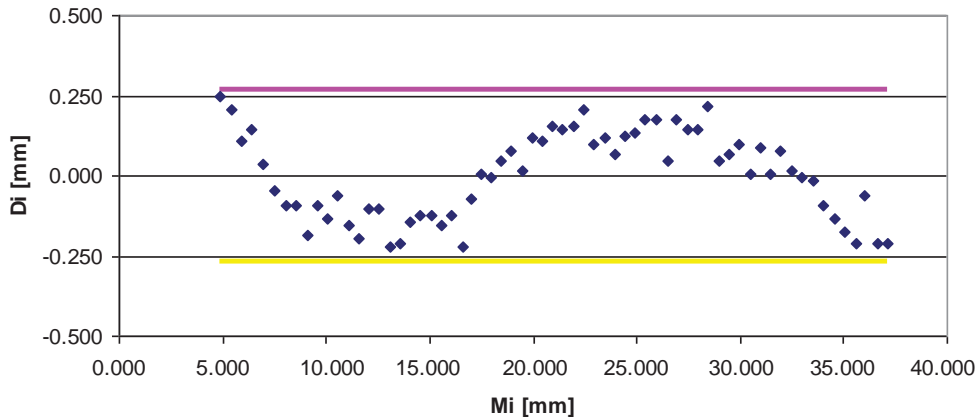


Fig. 2. The Bland–Altman test for the data from the micrometer and the C7 resistive sensor

The second stage of the Bland–Altman test is to verify if the expected value of the mean implementation difference of the estimators of the standard deviation of value measurement equals zero. This test is carried out depending on the quality of the measurement sample. In the case where the sample is derived from a normally distributed population, the Student's  $t$ -test can be successfully applied. This test is based on the following statistics:

$$t = \frac{\hat{x} - \mu_0}{\sigma} \sqrt{n-1} \quad (7)$$

where:

- $\sigma$  – standard deviation,
- $\mu_0$  – theoretical mean value ( $\mu_0 = 0$ ),
- $\hat{x}$  – estimator of mean value,
- $n$  – sample size.

If the measurement sample is not derived from a normally distributed population, the Wilcoxon test should be applied [7] for the paired observations. This test uses absolute values of the differences between the studied measuring methods ( $D_{ij}$ ), which then are subject to ranking.

The value of the statistics is calculated from the following formula:

$$W = \sum_{i=1}^n R_i \quad (8)$$

where:

$R_i$  – rank of the  $i$ -th difference,  
 $n$  – sample size.

Before commencing the second stage of the calculations, it is necessary to carry out an analysis aimed at determining whether the studied measurement samples are derived from a normally distributed population. A good test that can be used for this purpose, is the Shapiro–Wilk test [6]. In this test, test value of the statistics is calculated from the following formula:

$$W = \frac{\left( \sum_{i=1}^n a_i(n)(x_{(n-i+1)} - x_i) \right)^2}{\sum_{i=1}^n (x_i - \hat{x})^2} \quad (9)$$

where:

$i = 1, 2, \dots, n/2$ ,  
 $x_{(n-i+1)} - x_i$  – quasi-range of rank  $i$ ,  
 $a_i(n)$  – constants dependent on the sample size  $n$  and  $i$ .

A hypothesis of normality is rejected at the significance level when the value of the statistics, which is calculated from the non-grouped sample, falls outside the range  $\langle W(0,5a,n), W(1-0,5a,n) \rangle$ , whose ends constitute suitable quantiles of the  $W$  distribution. The results of the Shapiro–Wilk test have been presented in table 3, which contains the values of the statistics and the  $p$ -value. For the sensors C1, C5, C6 and C8, the  $p$ -value exceeded the value of 0.05. This means that it is possible to test the mean value hypothesis using the Student's  $t$ -test, because there are no grounds for rejecting the hypothesis of the estimators difference normal distribution. In the other cases, the  $p$ -value did not exceed the value of 0.05, which gives grounds for rejecting the hypothesis of the estimators difference normal distribution.

For the sensors C1, C5, C6 and C8, the Student's  $t$ -test was therefore carried out. A hypothesis with the mean value  $H: \mu = 0$  against the alternative hypothesis  $K: \mu \neq 0$  was studied. On the grounds of the calculations, the values of  $t$  statistics were obtained, with the assumed degrees of freedom. The next calculated parameter is the  $p$ -value. In all the cases, the  $p$ -value was greater than 0.05, which does not give grounds for rejecting the null hypothesis. All the parameters have been presented in table 4. Additionally, the table also contains the values of the mean and 95% confidence intervals.

**Table 3.** The results of the Shapiro–Wilk test for the distribution compliance of measurement differences with the normal distribution

Sensor	Value of statistics	<i>p</i> -value
C1	0.9703	0.131
C2	0.9464	0.012
C3	0.9600	0.044
C4	0.9459	0.008
C5	0.9862	0.741
C6	0.9853	0.653
C7	0.9496	0.010
C8	0.9750	0.228

**Table 4.** The results of the Student’s *t*-test for the sensors C1, C5, C6 and C8

Sensor	Value of statistics	Number of degrees of freedom	<i>p</i> -value	Mean	Confidence interval
C1	0.0953	62	0.924	0.002	(−0.032; 0.036)
C5	0.0000	58	1.000	0.000	(−0.028; 0.028)
C6	0.0000	62	1.000	0.000	(−0.030; 0.030)
C8	−0.1284	62	0.898	−0.002	(−0.034; 0.030)

The sensors which do not meet the hypothesis of the estimators difference normal distribution (sensors C2, C3, C4 and C7) were subjected to testing using the Wilcoxon test for paired observations. As in the case of applying the Student’s *t*-test, the hypothesis with the mean value  $H: \mu = 0$  against the alternative hypothesis  $K: \mu \neq 0$  was studied. As a result of the calculations, the *W* statistics values of the Wilcoxon test as well as the *p*-value were obtained. The calculation results have been presented in table 5. Similarly to applying the Student’s *t*-test, in all the cases the *p*-value was greater than 0.05, which does not give grounds for rejecting the null hypothesis.

**Table 5.** Wilcoxon test results for the sensors C2, C3, C4 and C7

Sensor	Value of statistics	<i>p</i> -value
C2	692	0.5159
C3	808	0.7158
C4	1006	0.9918
C7	1057	0.9219

On the grounds of the calculations and the statistical inference presented above, it can be stated that the results of the surveys with all the sensors in comparison with the micrometer, produce consistent results. The estimated accuracy of the resistive sensors can be evaluated at the level of a double standard deviation of the differences between the methods, for the 95% probability threshold. The values of double standard deviations have been shown in table 6.

**Table 6.** The values of double standard deviation of the differences between the resistive sensors and the micrometer

Sensor	$2\sigma_i$ [mm]
C1	0.264
C2	0.250
C3	0.250
C4	0.256
C5	0.218
C6	0.236
C7	0.270
C8	0.256

Additionally, table 7 presents in a cumulative form, the results of the tests conducted on particular data samples. This table facilitates the presentation of a selected analytical path for a given sensor. The “+” sign in the table indicates that a given test was carried out and the result was positive. The “-” sign indicates that a given test was carried out and the result was negative. The “0” sign indicates that a given test was not conducted.

**Table 7.** List of tests conducted on specific data samples

Sensor	Graph	Shapiro–Wilk test	<i>t</i> -Student test	Wilcoxon test
C1	+	+	+	0
C2	+	-	0	+
C3	+	-	0	+
C4	+	-	0	+
C5	+	+	+	0
C6	+	+	+	0
C7	+	-	0	+
C8	+	+	+	0

## 4. Summary

The described testing procedure allows to evaluate the suitability of the TELPOD SVP 45 resistive sensors to measure horizontal deformations. The obtained values of the double standard deviation of the differences between the resistive sensors and the micrometer can be regarded as satisfactory. Sensors of this type can successfully be used in mining areas to assess horizontal deformations [4], where the zero category of mining areas is characterized by horizontal deformations not greater than 0.3 mm/m [5]. A low price of the sensors allows us to hope that the monitoring systems based on them will be widely used in this type of surveys.



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## References

- [1] Altman D.G., Bland J.M.: *Measurment in medicine: the analysis of metod comparison studiem*. Statistican, Institute of Statisticians, 1983.
- [2] Ćmiel A.: *Statystyka matematyczna – materiały do wykładów*. AGH, Kraków 2008 (unpublished).
- [3] Górecki P.: *Rezystory. Część 1*. Elektronika dla Wszystkich, 1/1996, AVT – Korporacja sp. z o. o., Warszawa 1996.
- [4] Józwik M., Jaśkowski W.: *Rezystancyjny system pomiarów zmian długości pomiędzy znakami pomiarowymi*. The patent application, P-392572, 2010.
- [5] Popiołek E.: *Ochrona terenów górniczych*. Wydawnictwa AGH, Kraków 2009.
- [6] Shapiro S., Wilk M.B.: *An analysis of variance test for normality (complete samples)*. Biometrika, vol. 52 (3–4), Oxford University Press, Oxford 1965.
- [7] Wilcoxon F.: *Individual comparisons by ranking methods*. Biometrics, International Biometric Society, Arlington 1945.