The methods of solving the multiscenario model of joint balancing energy and power reserves

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Article concerns a joint model of balancing energy and power reserves for the energy system which takes into account power and regulation reserves. Admission a varied situations of using energy from the system power reserves causes, that characterizes this model with a vast complexity of calculation. The aim of this paper is to present practical ways to solve this problem using several proposed heuristic algorithms of reduction scenarios. Their quality was presented at the sample data.

Keywords and phrases: linear programming, load flow analysis, power generation dispatch, power reserves, multi-scenario model.

Introduction

The basic commodity which is produced, transmitted, received, and by which is being held a trade in the energy market is the electricity. Beside it, in the power system, there are other goods and services needed for smooth and safe operation. An important element in every energy system (ES) are system services and their constituent power reserve system. They serve to ensure safety work of ES and to protect consumers in ES of the unknown real need for the energy. With the existence of reserve capacity it is possible to react during the real-time balancing in situations of changed demand of consumers in relation to a forecast demand and respond as a result of failure of network elements.

This article concerns an integrated model of balancing energy and reserve capacity, taking into account the safety of the ES related to the use of reserve capacity by the consumers. The model was first formulated in [1]. This model takes into account: interrelation of energy and reserve capacity, a limited number of resources in the system, the specificity of power flows, the ability to deliver a combination of energy and reserves, the security constraints. The formulated model is an extension of the classical OPF model [2]. This paper concerns the practical ways of solving this model.

The structure of the article is as follows. In section 2 is presented a model of the joint energy and reserves balancing, which ensures the ability to deliver power to customers from the reserves from the suppliers. The model is characterized by high computational complexity, so in section 3 are described the heuristic algorithms to reduce the size of the problem. For the proposed algorithms in section 4 are presented the results of computational experiments. Paragraphs 5 and 6 concern to the following methods of solving the problem using the algorithm presented in section 3. Paragraph 7 provides a summary.

The multiscenario model of balancing power and reserves

In the presented multiscenario model demand for energy and reserves is a parameter in the system nodes, which are consumers of those goods. To ensure the ability to deliver energy and any combination of reserves under the power flows, the model takes into account the projected demands of consumers (so-called *nominal load flow*) as well as situations of power flows by using variable power reserves by the consumers. From the perspective of fulfilling power constraints are essential maximum power flows by the lines. They can occur for up to a diversified combination of maximum reduced or maximum increased energy from the reserves in the consumers. This causes that “the extreme flows” for sure (maximum flow through at least one transmission line) contains the model which takes into account all possible combinations of maximum using of up and down reserves in particular nodes in the system. A single combination of maximum use of reserves in the system nodes is called *scenario*. The total number of scenarios is 2^r, where r is the number of nodes, where there is a demand for reserves. Scenarios will be marked by the letter $f, f \in F^0$, where F^0 is the set of all scenarios.

In the specific scenario, demand for the energy in the consumer's node is a parameter. Scenario in which we assume no use of reserves by the consumers is called *the nominal scenario*.

The multiscenario model of balancing energy and regulation reserves based on the standard DC OPF problem can be written as the following linear programming problem (model RES_SC):

$$\min \sum_{a \in N} [\sum_{f \in F^0} \alpha_f K_a(p_a^f) + K_a^+(p_a^+) + K_a^-(p_a^-)] \qquad (1)$$

$$\sum_{b \in N} P_{ab}^f - p_a^f + D_a^f = 0 \quad \forall a \in N, \forall f \in F^0 \qquad (2)$$

$$p_a^f - p_a^{f_0} \leq p_a^+ \quad \forall a \in N, \forall f \in F^0 \qquad (3)$$

$$p_a^{f_0} - p_a^f \leq p_a^- \quad \forall a \in N, \forall f \in F^0 \qquad (4)$$

$$P_{ab}^f = V_a V_b Y_{ab}(\theta_a^f - \theta_b^f) \ \forall (a,b) \in E, \forall f \in F^0 \qquad (5)$$

$$\sum_{a=1}^{N} s_a \theta_a^f = 0 \quad \forall f \in F^0 \qquad (6)$$

$$p_a^- \leq p_a^{f_0} \leq \overline{P_a} - p_a^+ \quad \forall a \in N \qquad (7)$$

$$0 \leq p_a^- \leq \overline{P_a^-} \quad \forall a \in N \qquad (8)$$

$$0 \leq p_a^+ \leq \overline{P_a^+} \quad \forall a \in N \qquad (9)$$

$$-S_{ab} \leq P_{ab}^f \leq S_{ab} \quad \forall (a,b) \in E, \forall f \in F^0 \qquad (10)$$

Variables occurring in the model: p_a^f — the power fed into node a in f scenario, $p_a^{f_0}$ — the power fed into node a in the nominal scenario, P_{ab}^f — power flow through transmission line between nodes a and b in scenario f, p_a^k — amount of approved reserves in node a of type k ($k \in \{+,-\}$, where "+" is the up reserve, "–" is the down reserve), θ_a^f — voltage phase angle in node a in scenario f relative to the reference node.

The model contains the following parameters: D_a^f — demand for energy in node a in scenario f, P_a — the maximum amount of power offered at node a, P_a^+ — the maximum amount of up reserve offered in node a, P_a^- — the maximum amount of down reserve offered in node a, s_a — the value 1 of a position a is an index of the reference node (values on the remaining positions are zero), V_a — voltage in node a, Y_{ab} — admittance of transmission line between nodes a and b, S_{ab} — capacity of the transmission line between nodes a and b, N — set of nodes, E — set of transmission lines.

The objective function (1) minimizes the cost of system balancing. It means purchasing the required amount of energy and powers reserves. It consists of the components: $K_a(p_a^f)$ — the cost of buying energy at node a in scenario f, $\sum_{f \in F^0} \alpha_f = 1$; $K_a^+(K_a^-)$ cost of purchasing the up (down) reserve in node a.

Constraint (2) designates the balance of energy for node a in scenario f. Use of reserves from the suppliers may change with a change of scenario, so it is needed to buy such a quantity of reserves to be able to provide it regardless of the configuration needs of customers. Therefore, constraints (3) and (4) determine the amount of bought reserves (up and down) in node a as a maximum value among all scenarios. Constraint (5) defines power flow in transmission line in scenario f. Constraint (6) sets the voltage phase angle in the selected node to zero. Constraint (7) defines the relationship between the maximum amount of power placed to the ES at node a and values of power reserves available at node a. This limitation must be met for the nominal scenario. Constraints (8) and (9) determine the amount of up and down reserves available in node a. Constraint (10) limits the capacity of transmission lines between nodes a and b in every scenario.

The RES_SC model ensures that in the ES it will be able to deliver additional energy from the up reserves or decrease demand with using the down reserves. Unfortunately, the number of scenarios $|F^0|$ depends exponentially on r, where r is the number of nodes, where customers need to provide reserves. The result is that the model is difficult to solve for the real transmission network.

The remainder of this article will be provided methods of reducing the number of scenarios and thus the quality of obtained solutions.

Heuristic algorithms of reduction the set of scenarios

The simplest way to reduce the full set of scenarios is to assume to calculations by using the RES_SC model only three scenarios: with the nominal demands (D_a^0),with the minimal demands on all the nodes (D_a, denoted as $f_{\min}$) and with the maximum demands in all nodes (D_a, denoted as $f_{\max}$) (model SC_3). Unfortunately, in many cases, such simplification may be too strong. Much better results can apply any of these heuristic algorithms of reduction the set of scenarios.

Presented algorithms of reduction the computational complexity of the RES_SC model are based on the designation of a subset of scenarios denoted as F^T, with which is then solved the RES_SC model (instead of the set of all scenarios). In each of the algorithms a single scenario is created for a single transmission line. Designated in this way, the scenarios do not need to be extreme scenarios. A subset of scenarios contains only unique scenarios. Detailed information about the algorithms can be found in [3, 4].

The DC_MF algorithm

It is assumed the unlimited capacity of the current transmission line, while the capacity of other transmission lines are consistent with the input data. For a single transmission line scenario is determined by the demands of customers D_a^f, where $D_a^f = d_a$ is the optimal solution of the auxiliary power flow model which maximizes the power flow through the current transmission line. After solving of the auxiliary model on the basis of directions of wages by the consumers (in the range set by the required amount of up and down reserves) in relation to the nominal demand is created a scenario. If obtained in a single node power consumption is less than the nominal one is taken to use the down reserve, but in the opposite case — the use of the up reserves.

The DC_ND algorithm

It is assumed the unlimited capacity of the current transmission line, while the capacity of other transmission lines are consistent with the input data. For a single transmission line algorithm for the scenario creation consists of *r* iterations. In iteration *k* is tested how power flow changes in the analyzed line depends on the use in the node *a* up or down reserve. For this calculation is used the reduced RES_SC model with four scenarios: f_0, $f_{\min}$, $f_{\max}$ and modified nominal (with the changed demand for reserve in node *a*). If greater flow is caused by the use of down reserves p_a^- in node *a*, then $D_a^f = D_a = D_a^0 - p_a^-$, in the opposite case $D_a^f = D_a = D_a^0 + p_a^+$.

The DC_FT algorithm

To construct a scenario is used the location of the analyzed line and direction of power flow in relation to nodes, which are receiving energy from the reserves. Lines in the network has an interpretation of the undirected arc so there is needed knowledge of power flows so that it would be possible establishment of a referral line. To this solution is used the reduced RES_SC model consisting of three scenarios: f_0, $f_{\min}$, $f_{\max}$. The main goal of the algorithm is to maximum load (through the use of up reserves) nodes, to which the power flows through the given transmission line and to minimize the load on the nodes (through the use of down reserves), through which flows the power reaching to the given node. For the analysis of power flow is used the power flow tracing method [5].

Computational experiments

Tests conducted on the algorithms are presented on five network topologies with different number of nodes, transmission lines, customers and suppliers. Adopted data are the test sets from the Reliability Test System — 1996 (RTS-96) derived from [6]. Sets are supplemented by consumer demand for reserves and reserve offers of suppliers. Parameters of test data are presented in Table 1.

Table 1. Parameters of test sets.

No.	No. of nodes	No. of lines	No. of suppliers	No. of consumers
W1	24	38	10	11
W2	24	38	10	12
W3	48	79	20	13
W4	48	79	20	14
W5	73	120	30	15

Solutions obtained by using the proposed algorithms will be compared with the solutions designated by the RES_SC model with all scenarios (the full model) and with the solutions obtained using the RES_SC model with three scenarios: f_0, $f_{\min}$ and $f_{\max}$ (the SC_3 model). Calculation experiments were conducted by using CPLEX 9.1 package.

The results of the presented algorithms are formulated in terms of: time of determining the optimal solution (*T [s]*), the minimum directional efficiency ($\rho^{\min}$), the average directional efficiency (ρ^{av}), the probability that the available generation and the reserves will not be able to cover the demands of consumers (*LOLP*, loss-of-load probability), the number of scenarios (LS), which contains the subset F^T.

The term *directional efficiency* is a measure of the maximum possible deviation Δ in the direction $h = (D_a^f - D_a^0)$, caused by a desire to change the amount of received energy from the nominal demand (D_a^0) to demand (D_a^f) in scenario *f*.

The results of calculation experiments for the algorithm DC_MF are shown in Table 2 (FP — model with a set of scenarios F^0).

DC_MF algorithm represents a compromise between the solution of the model with three scenarios and the full model. During the few seconds it is able to determine the solution, which admittedly is a bit worse than the solution obtained using the full model, but looks much better compared to a model with three scenarios. Average directional efficiency in all cases is at least 0.99%, while the minimum directional efficiency in most cases exceeds 0.9%. The resulting solutions are characterized by a value of LOLP equals a maximum few hundredths, while more than half of the cases is less than 0.01.

Results obtained for the algorithm DC_ND are presented in Table 3.

Table 2. Results obtained with the algorithm DC_MF.

No.	SC_3			RES_SC					FP
	ρ^{av}	$\rho^{\min}$	*LOLP*	ρ^{av}	$\rho^{\min}$	*LOLP*	*LS*	*T*	*T*
W1	0.923543	0.6332	0.2196	0.997981	0.9285	0.0620	28	1.26	241
W2	0.936112	0.7708	0.1795	0.999722	0.9475	0.0144	24	1.11	988
W3	0.917069	0.7094	0.1306	0.999846	0.9154	0.0082	27	4.79	>10h
W4	0.943281	0.6264	0.2045	0.999928	0.9265	0.0046	36	4.95	>10h
W5	0.939254	0.6572	0.1425	0.999994	0.9598	0.0004	33	9.92	>10h

Table 3. Results obtained with the algorithm DC_ND.

No.	SC_3			RES_SC					FP
	ρ^{av}	$\rho^{\min}$	*LOLP*	ρ^{av}	$\rho^{\min}$	*LOLP*	*LS*	*T*	*T*
W1	0.923543	0.6332	0.2196	0.999149	0.9384	0.0376	26	21	241
W2	0.936112	0.7708	0.1795	0.999998	0.9902	0.0002	24	21	988
W3	0.917069	0.7094	0.1306	0.999960	0.9141	0.0016	41	98	>10h
W4	0.943281	0.6264	0.2045	0.999928	0.9265	0.0046	48	106	>10h
W5	0.939254	0.6572	0.1425	0.999995	0.9562	0.0003	66	305	>10h

Table 4. Results obtained with the algorithm DC_FT.

No.	SC_3			RES_SC					MP
	ρ^{av}	$\rho^{\min}$	*LOLP*	ρ^{av}	$\rho^{\min}$	*LOLP*	*LS*	*T*	*T*
W1	0.923543	0.6332	0.2196	0.998223	0.9375	0.0640	27	0.67	241
W2	0.936112	0.7708	0.1795	0.999067	0.9152	0.0269	30	0.73	988
W3	0.917069	0.7094	0.1306	0.999972	0.9140	0.0005	32	2.42	>10h
W4	0.943281	0.6264	0.2045	0.999994	0.9287	0.0002	33	2.53	>10h
W5	0.939254	0.6572	0.1425	1	0.9885	0	47	6.34	>10h

Table 5. Results obtained with the improvement algorithm.

Starting algorithm	ρ^{av}	$\rho^{\min}$	*LOLP*	*LS*	*T*	*Iter*
SC_3	0.993871	0.8315	0.1191	30	2.38	3
DC_MF	0.999212	0.9398	0.0347	59	6.59	4
DC_ND	0.999195	0.9398	0.0366	55	26	4
DC_FT	0.998321	0.9383	0.0615	59	5.79	4

The average directional efficiency for each solution is at least 0.999%, while in four cases is more than 0.9999%. With large values ρ^{av} are related small values of LOLP. Time determining the optimal solution by the algorithm DC_ND depends mainly on the number of transmission lines in the network and lesser extent on the number of nodes in which there is demand for reserves. This time is longer than for the algorithm DC_MF since solved many simple problems, but still shorter compared to the full model.

Results obtained for the DC_FT algorithm can be found in Table 4.

The maximum time set by the algorithm DC_FT was slightly more than 6 seconds, which means that it is almost a half faster than the fastest of the presented algorithms so far. The advantage of the algorithm is also the accuracy of obtained with its help solutions. In all cases, ρ^{av} equals at least 0.99. For a set W5 it can not fully provide the energy in only one out of 32 768 scenarios. For this set of LOLP = 0.000030518. The advantage of the DC_FT algorithm is also need to solve in preparation of the scenarios just one simple problem.

The improvements algorithm

Algorithm to improve the accuracy of obtained solutions starts with some initial solution and is performed iteratively until a stop condition.

The starting point to improve the algorithm is a subset of the initial set of scenarios F^T e.g., as a result of any of the algorithms (DC_MF, DC_ND, DC_FT) or as a result of scenarios: f_0, $f_{\min}$ and $f_{\max}$. For a designated subset of scenarios the RES_SC model is solved with a set of scenarios F^T. Then, for each transmission line is solved an auxiliary model in which for the accepted bids for energy and reserves is maximized the absolute value of power flow through a transmission line which is analyzed. On the basis of directions of variation in demand for nominal demand is created scenario. If scenario created in this way does not occur in the set F^T

then it is added to it. The stop criterion of the algorithm is the situation where the result of solving the auxiliary model could not get even one new scenario.

The proposed improvement algorithm can be treated as enhancing the results already obtained as well as an algorithm by which a solution can be obtained not earlier using any other algorithm.

Table 5 shows the operation with the improvement algorithm of a set W1 on the assumption that the starting point is the set of scenarios F^T including: f_0, $f_{\min}$ and $f_{\max}$ or obtained by algorithm DC_MF, DC_ND, DC_FT.

In the *LS* column is given number of scenarios included in the set F^T in the last iteration. Column *T* is the total time served since the beginning of the algorithm, the column *Iter* indicates the number of algorithm iterations.

An attempt to improve the results obtained by the algorithms DC_ND, DC_MF did not lead to better solutions. The reason is a very good quality of solutions obtained by using this algorithms. In case of solutions obtained by algorithm DC_FT after four iterations was reached exactly the same solution as the one in which the initial set of scenarios contains only scenarios: f_0, $f_{\min}$ and $f_{\max}$. In this case it was easier to improve the solution, because the effectiveness of directional were less than one in 238 scenarios.

The sum of sets of scenarios

Each of the algorithms works on a different principle, while the common feature is that each of them generates scenarios for particular transmission lines. Thus it is possible combination with each other sets of scenarios generated by particular algorithms and solving the problem RES_SC with the merged set of scenarios.

After the junction sets of scenarios improvement of the solution quality in relation to the results of the algorithm DC_FT, DC_MF DC_ND occurred for a set W1. For the remaining sets the obtained solutions are like the best of the solutions obtained for the algorithm DC_FT, DC_MF or DC_ND.

Time of getting the optimal solution varies in the range of tens of seconds for the simplest of sets up to several minutes for the most complicated networks. The biggest share at this time is the time needed to fix the set of scenarios by the algorithm DC_ND. Despite the extended time of calculations, it is competitive in relation to time getting the optimal solution for the full model.

Summary

In the paper was presented the problem of integrated balancing energy and reserves in the ES. The model is characterized by the high computational complexity, and therefore were presented some heuristic reduction algorithms. The results indicate the practical ability to solve the problem quickly while maintaining high accuracy and effectiveness of the obtained solutions.

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