ANNUAL OF NAVIGATION 23/2016

DOI: 10.1515/aon-2016-0015

PAWEŁ ZALEWSKI
Maritime University of Szczecin, Poland

METHODS OF THRUST ALLOCATION IN A DP SIMULATION SYSTEM OF MARITIME UNIVERSITY OF SZCZECIN

ABSTRACT

Vessels conducting dynamic positioning (DP) operations are usually equipped with thruster configurations that enable generation of resultant force and moment in any direction. These configurations are deliberately redundant in order to reduce the consequences of thruster failures and increase the safety. On such vessels a thrust allocation system must be used to distribute the control actions determined by the DP controller among the thrusters. The optimal allocation of thrusters' settings in DP systems is a problem that can be solved by several convex optimization methods depending on criteria and constraints used. The paper presents linear programming (LP) and quadratic programming (QP) methods adopted in DP control model which is being developed in Maritime University of Szczecin for ship simulation purposes.

Keywords:
dynamic positioning thrust allocation, convex optimization.

INTRODUCTION

A convex optimization problem is one of the form [Boyd, Vandenberghe, 2009]:

$$\begin{aligned} &\text{minimize } \ f_0(x) \\ &\text{subject to } \ f_i(x) \le b_i,\ i = 1, \ldots, m, \end{aligned} \tag{1}$$

where the functions $f_0, \ldots, f_m : \mathbb{R}^n \to \mathbb{R}$ are convex, i.e., satisfy:

$$f_i(\alpha x + \beta y) \le \alpha f_i(x) + \beta f_i(y)$$

for all $x, y \in \mathbb{R}^n$ and all $\alpha, \beta \in \mathbb{R}$ with $\alpha+\beta = 1$, $\alpha \ge 0$, $\beta \ge 0$. There is in general no analytical formula for the solution of convex optimization problems, but there are very effective methods for solving them, like: the least-squares in quadratic programming (QP), linear programming (LP) or interior-point methods used in second-order cone programming (SOCP) or geometric programming (GP) [Boyd, Vandenberghe, 2009].

Dynamic Positioning (DP) system can be defined as a system which automatically controls a vessel, influenced by external excitations, to maintain her position and heading exclusively by means of active thrust. DP system divides forces among ship's thrusters to achieve resultant force and momentum equal to the one set by the control system. Optimization of thrust allocation is based on minimization of energy usage (need for power or fuel if feasible), additionally taking into account limitations like forbidden zones for thrusters' settings (individual and relative to each other — opposing thruster pairs).

The optimal allocation of forces generated by thrusters in DP systems is a problem that can be solved by several convex optimization methods depending on criteria and constraints used. A survey of these methods, including direct allocation by QP has been presented by Johansen and Fossen [Johansen, Fossen, 2013]. This paper presents linear programming (LP) and quadratic programming (QP) methods adopted in DP control model developed in Maritime University of Szczecin (MUS) for ship simulation purposes.

GENERATION OF FORCES WITH THRUSTERS

For a DP control, similarly to ship simulation, a ship's hull can be treated as a rigid body with the center of mass at origin $p = 0 \in \mathbb{R}^2$. Measurements of the position of the vessel are compared with the required position. The difference is fed into an Extended Kalman Filter (EKF) and PID-controller to convert this to the resultant force and momentum required to correct the position. The allocation unit controls the thrusters, which must generate the component forces of the resultant one. Therefore the model of thrust allocation used for a vessel with i^{th} — number of azimuth thrusters can be built according to geometrical relations presented in Figure. 1.

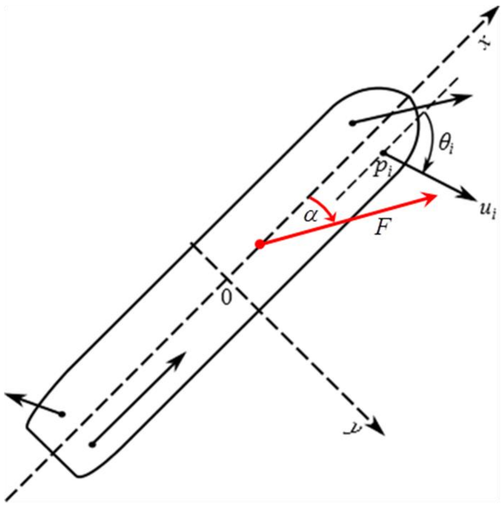


Fig. 1. Thrust forces acting on a vessel with i^{th} number of azimuth thrusters

The assumptions of the model are:

- the vessel's position is stabilized at low speed (less than 2 knots or 1 m/s), and the centre of mass (force reference origin) is the fixed rotation centre;
- there are n component forces with magnitude u_i [kN] or [tf], acting at $p_i = (p_{ix}, p_{iy})$ [m, m], in direction θ_i [°], $i=1, 2, \ldots, n$;
- the resultant force [kN] or [tf]:

$$F = \sqrt{F_x^2 + F_y^2}\ ; \tag{2}$$

- the resultant longitudinal force (horizontal in ship-body frame) [kN] or [tf]:

$$F_x = \sum_{i=1}^{n} u_i \cos\theta_i\ ; \tag{3}$$

- the resultant transverse force (vertical in ship-body frame) [kN] or [tf]:

$$F_y = \sum_{i=1}^{n} u_i \sin\theta_i\ ; \tag{4}$$

- the resultant torque (moment of the resultant force) [kNm] or [tfm]:

$$M_z = \sum_{i=1}^{n} (p_{iy} u_i \cos\theta_i - p_{ix} u_i \sin\theta_i)\ ; \tag{5}$$

- the force limits [kN] or [tf]:

$$0 \le u_i \le u_{\max}; \tag{6}$$

– the thruster angle limits [rad] or [°]:

$$0 \le \theta_i < 2\pi \; ; \tag{7}$$

– the energy or fuel usage is strictly dependent on u_i and in the case of the same type of thrusters the total energy is assumed to be linearly correlated to:

$$\sum_{i=1}^{n} u_i = u_1 + u_2 + \ldots + u_n. \tag{8}$$

The problem to solve is: find u_i and θ_i that yield desired resultant force and moment and minimize the fuel or energy usage. Note that the problem is considered to be 3-DOF (degrees of freedom) or solved in 2-dimensional space. In fact, any movement in the z-direction (up/down) or around x- and y-axis is ignored because common actuators in offshore vessels do not have the ability to produce thrust in these directions. This clearly reduces the complexity of the problem.

LP PROBLEM SOLUTION

For the thruster allocation problem with variables of u_i and θ_i the formulation of the objective function and constraints in the form of constrained LP can be given in matrix notation as:

$$\begin{aligned} &\text{minimize} \;\; \mathbf{1}^T u \\ &\text{subject to} \;\; f u^T = F^T \\ &\qquad 0 \le u_i \le u_{\max},\; 0 \le \theta_i < 2\pi,\; i = 1,\ldots,n \end{aligned} \; , \tag{9}$$

where:

$\mathbf{1} = [1, 1, 1, \ldots, 1]$ of size n,

$u = [u_1, u_2, \ldots, u_n]$,

$$f = \begin{bmatrix} \cos\theta_1 & \ldots & \cos\theta_n \\ \sin\theta_1 & \ldots & \sin\theta_n \\ p_{1y}\cos\theta_1 - p_{1x}\sin\theta_1 & \ldots & p_{ny}\cos\theta_n - p_{nx}\sin\theta_n \end{bmatrix}, \tag{10}$$

$F = [F_x, F_y, M_z]$,

F_x, F_y, M — designated longitudinal, transverse forces and moment.

QP PROBLEM SOLUTION

For the thruster allocation problem with variables u_i and θ_i transformed to f_{xi} and f_{yi} (longitudinal and transverse components of forces u_i) the formulation of the objective function and constraints in the form of constrained QP can be given in matrix notation as:

$$\begin{aligned} &\text{minimize } \mathbf{1}^T\left(f_x^2 + f_y^2\right) \\ &\text{subject to } F_x = \mathbf{1}^T f_x \\ &\qquad F_y = \mathbf{1}^T f_y \\ &\qquad M_z = \mathbf{1}^T\left(p_x \bullet f_y - p_y \bullet f_x\right) \\ &\qquad \max\left(f_x^2 + f_y^2\right) \le f_{\max}^2 \end{aligned}, \tag{11}$$

where:

$$\begin{aligned} f_x &= [f_{x1}, f_{x2}, \ldots, f_{xn}] \\ f_y &= [f_{y1}, f_{y2}, \ldots, f_{yn}]' \end{aligned} \tag{12}$$

$$\begin{aligned} f_{xi} &= u_i \cos\theta_i \\ f_{yi} &= u_i \sin\theta_i \\ u_i^2 &= f_{xi}^2 + f_{yi}^2 \end{aligned} \tag{13}$$

and F_x, F_y, M_z — designated longitudinal, transverse forces and moment constraints in analogy to LP problem. If the final constraints worked out by EKF and PID are in the form of (see Fig. 1):

F — resultant force,
α — orientation of the resultant force,
M_z — resultant momentum,
f_{max} — maximum individual thruster force,

then:

$$\begin{aligned} F_x &= F\cos\alpha \\ F_y &= F\sin\alpha \end{aligned} \tag{14}$$

and the ordinates of the application point of the resultant force F can be calculated as:

$$P_x = \mathbf{1}^T (p_x \bullet f_y) / F_y$$
$$P_y = \mathbf{1}^T (p_y \bullet f_x) / F_x \quad . \qquad (15)$$

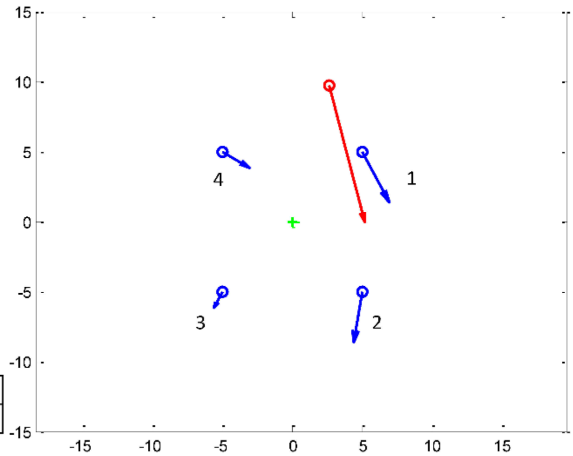
The formula (11) can be father extended by additional constraints on thrusters' work sectors (limits of θ_i or ratio f_{yi}/f_{xi}).

IMPLEMENTATION IN DP SIMULATION SYSTEM

The algorithms, solving (9) and (11) by applying Newton's method to a sequence of equality constrained problems, have been developed in Matlab with Optimization or CVX Toolbox [Boyd, Vandenberghe, 2009] and afterwards translated to C#. The tests proved that though generally solution efficiency of LP is better than efficiency of QP (which formally is a further generalization of LP) in case of the thruster allocation algorithm more robust and faster was QP. The main reasons of this have been nonlinearities in constraints (10) where trigonometric functions are directly applied. The problem elaborated as (11) avoids non convexity as all constraints are strictly convex, and it can be classified into linearly constrained QP. Because f_x^2 and f_y^2 are convex functions then their pointwise maximum f_{max}^2 is also convex and it can be re-expressed as two separate inequalities using a standard LP formulation. The route of QP is also followed by most of the authors dealing with thrust allocation in shipborne DP systems [van Daalen, Cozijn et al., 2011], [Gierusz, Tomera, 2006], [Lindfors, 1993], [Rindarøy, 2013], [Ruth, 2008], [Johansen, Fossen, 2013].

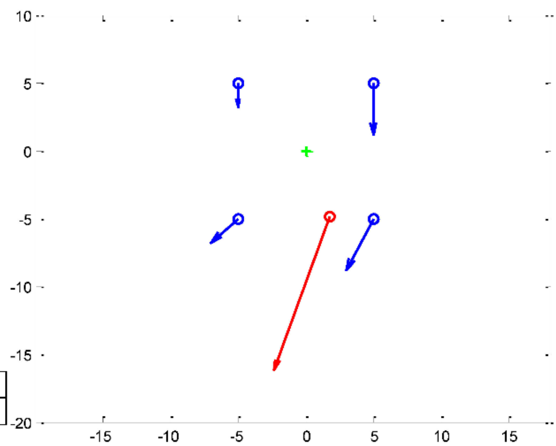
The examples of thrust allocation to four azimuth thrusters, calculated by the model adopted in DP simulation system of Maritime University of Szczecin, with the resultant force in various four quadrants of ship-body fixed co-ordinate system, are presented in the Figures 2, 3, 4, 5 and Tables 1, 2, 3, 4 (corresponding to the Figure 1: angles 360° clockwise, *x*-axis up, *y*-axis right, ordinates in [m] from the centre of gravity marked as green cross, the resultant force in red, the component forces in blue).

The co-ordinates p_{xi} [m] and p_{yi} [m] of azimuth thrusters in the Figures 2, 3, 4 and 5 have been set for visualisation purposes as: p_x = [5; –5; –5; 5]; p_y = [5; 5; –5; –5]; and f_{max} = 5tf.



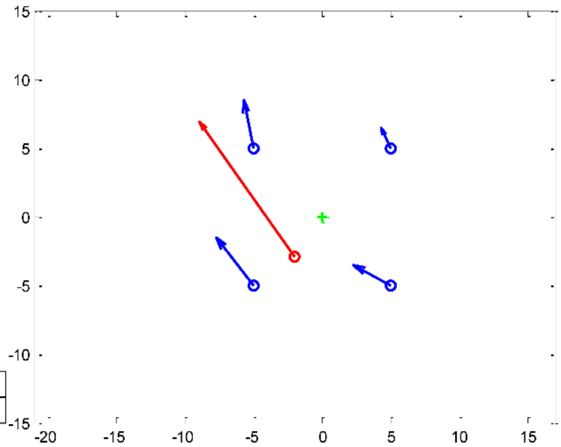
F [tf]	α [°]	M_z [tfm]	P_x [m]	P_y [m]
10.00	165.0	50.00	9.66	2.59
i	u_i	ϑ_i		
1	4.12	152.63		
2	3.71	189.34		
3	1.31	207.37		
4	2.23	121.55		

Fig. 2 and Tab. 1. Example of thrust forces allocation with the resultant force in 1st quadrant of ship-body fixed co-ordinate system



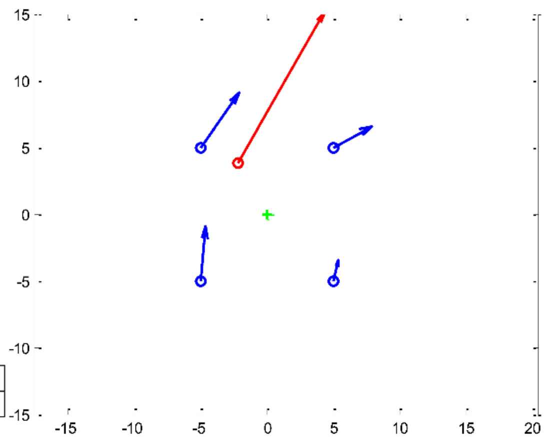
F [tf]	α [°]	M_z [tfm]	P_x [m]	P_y [m]
12.00	200.0	40.00	-4.87	1.77
i	u_i	ϑ_i		
1	3.82	180.39		
2	4.32	207.95		
3	2.72	228.08		
4	1.82	180.82		

Fig. 3 and Tab. 2. Example of thrust forces allocation with the resultant force in 2nd quadrant of ship-body fixed co-ordinate system



F [tf]	α [°]	M_z [tfm]	P_x [m]	P_y [m]
12.00	325.0	40.00	-2.91	-2.03
i	u_i	ϑ_i		
1	1.63	333.69		
2	3.09	298.18		
3	4.40	321.80		
4	3.53	348.22		

Fig. 4 and Tab. 3. Example of thrust forces allocation with the resultant force in 3rd quadrant of ship-body fixed co-ordinate system



F [tf]	α [°]	M_z [tfm]	P_x [m]	P_y [m]
13.00	30.0	50.00	3.85	-2.22
i	u_i	θ_i		
1	3.27	61.45		
2	1.61	13.48		
3	4.08	5.27		
4	4.98	35.27		

Fig. 5 and Tab. 4. Example of thrust forces allocation with the resultant force in 4th quadrant of ship-body fixed co-ordinate system

The Figure 6 presents the visualisation of marine environment within the MPSV (multi-purpose supply vessel) controlled by three azimuth thrusters in the DP Simulation System programmed in MUS by the staff of Marine Traffic Engineering Centre.

Fig. 6. DP simulation system with thrusters' allocation and vision prepared in MUS

CONCLUSIONS

A thrust allocation system must be used to distribute the control actions determined by the DP controller among the thrusters. The allocation problem can be translated to a constrained optimization problem. The quadratic programming (QP) method has been developed for this purpose in the DP ship simulation model implemented in ship simulator of MUS. The tests proved that the optimization algorithm translated into C# programming language worked efficiently using interior-point methods [Boyd, Vandenberghe, 2009] to solve the problem by applying Newton's method to a sequence of equality constrained problems. The further development will include extra constraints like limits to the thrusters' work sector (forbidden zones) and non-azimuth thrusters.

REFERENCES

[1] Boyd S., Vandenberghe L., Convex Optimization, Cambridge University Press, 7th printing, New York, 2009.

[2] Gierusz W., Tomera M., Logic thrust allocation applied to multivariable control of the training ship, 'In Control Engineering Practice', 2006, Vol. 14, p. 511–524.

[3] Johansen Tor A., Fossen Thor I., Control allocation — a survey, 'In Automatica', 2013, Vol. 49, p. 1087–1103.

[4] Lindfors I., Thrust allocation method for the dynamic positioning system, 10th International Ship Control Systems Symposium, Ottawa 1993, p. 3.93–3.106.

[5] Rindarøy M., Fuel Optimal Thrust Allocation in Dynamic Positioning, NTNU Norwegian University of Science and Technology, M.Sc. Thesis, 2013.

[6] Ruth E., Propulsion control and thrust allocation on marine vessels, 'Doctoral Theses at NTNU', 2008:203, Norwegian University of Science and Technology.

[7] Van Daalen E. F. G., Cozijn J. L., Loussouarn C., Hemker P. W., A Generic Optimization Algorithm for the Allocation of DP Actuators, Proceedings of the ASME 2011 30th International Conference on Ocean, Offshore and Arctic Engineering OMAE, Rotterdam, 2011.

[8] Wit C. D., Optimal Thrust Allocation Methods for Dynamic Positioning of Ships, Delft University of Technology, Netherlands, M.Sc. Thesis, 2009.

Received September 2016
Reviewed December 2016

PAWEŁ ZALEWSKI
Maritime University of Szczecin
Wały Chrobrego 1-2 Str., 70-500 Szczecin, Poland
e-mail: p.zalewski@am.szczecin.pl

STRESZCZENIE

Statki pozycjonowane dynamicznie (DP) mają konfiguracje pędników umożliwiające wytwarzanie wypadkowej siły i momentu w dowolnym kierunku. Konfiguracje te są zazwyczaj nadmiarowe w celu zmniejszenia skutków awarii pojedynczych pędników, a tym samym zwiększenia poziomu bezpieczeństwa wykonywanej pracy. Na takich jednostkach musi być stosowany system alokacji wyliczający i przydzielający nastawy do poszczególnych pędników. Optymalna alokacja nastaw pędników w systemach DP jest problemem, który można rozwiązać, w zależności od zadanych kryteriów i ograniczeń, kilkoma metodami optymalizacji wypukłej. W artykule przedstawiono metody programowania liniowego (LP) i kwadratowego (QP) zastosowane w modelu sterowania DP w symulatorach nawigacyjno-manewrowych Akademii Morskiej w Szczecinie.