Contribution of New Parameter Inverses to Robustification of Fractional-Order Perfect Control for LTI MIMO Discrete-Time State-Space Systems

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Abstract:
The paper presents results of application of various right inverses to fractional-order discrete-time perfect control in terms of improving its stability and robustness. For that reason the newly introduced σ-inverse and H-inverse are applied finally to obtain the mentioned above control strategy strictly dedicated to LTI MIMO nonsquare systems described by state-space framework. It is highlighted that parameter σ-inverse and H-inverse with different so called 'degrees of freedom' outperform the typical minimum-norm right T-inverse. Moreover, this new approach deals with the same class of problems concerning integer-order systems. The simulation studies performed in Matlab/Simulink environment confirm high potential of proposed here method.

Keywords: *LTI MIMO fractional-order systems, perfect control, inverses of nonsquare matrices, stability*

1. Introduction

The issues concerning the stability and robustness of LTI MIMO integer-order discrete-time systems in state-space domain are subject to more and more intensive scientific exploration [2, 4, 5, 9, 10]. The search of various methods to increase robustness of perfect control for such systems is being conducted in particular in order to obtain new classes of right inverses [4, 5]. Until now, the mostly used and widely presented in the literature is minimum-norm right T-inverse [2 , 3, 6, 7, 8]. T-inverse considered, however, as the "minimum-energy" inverse, cannot be used to wide class of cases where the control inputs remain unstable under perfect control law. The powerful tools that can be applied to mentioned class of problems are recently introduced σ-inverse and H-inverse with different types of so-called degrees of freedom [2,4,5,9]. It is interesting that the first inverse constitutes an extension of minimum-norm right T-inverse in which degrees of freedom are organized as the separate matrices being both in parameter as well as polynomial form. On the other hand, the second mentioned inverse based on well-known SVD factorization. That approach gives the opportunity to impact on the perfect control strategy by influencing matrices obtained from SVD factorization in form of parameter or polynomial degrees of freedom as well. In the paper mentioned above tools previously employed to integer-order systems are transferred to fractional-order perfect control for LTI MIMO discrete-time systems. It is remarkable that the concept of engaging the inverses of parameter matrices to increasing the robustness of discrete-time fractional-order perfect control structures in state-space is a new and original authors' idea, not presented so far.

The paper is, therefore, an attempt to synthesis of discrete-time fractional-order perfect control with regard of its stability and robustness. To obtain that objective, the recently devised fractional-order multivariable discrete-time perfect control algorithm, in particular dedicated to the so-called nonsquare systems, i.e. systems with different numbers of input and output variables, is used [1]. The simulation studies of perfect control involve parameter σ-inverse [4, 9] and parameter H-inverse [5, 11]. The results of research conducted in Matlab/Simulink environment show that utilization of appropriate inverse machinery may determine stability and, as a consequence, minimum phase property of the non-integer systems. The paradigm adapted from integer-order calculus, still need to be proved, which as is worth of scientific effort, become the aim future research.

The paper is organized in following manner. In Section 2 the fractional-order system representation is presented. Next section outlines the notion of fractional-order perfect control strictly dedicated to LTI MIMO discrete-time systems defined in state-space framework. The newly introduced nonunique σ-inverse and H-inverse are shortly presented in Section 4. An application of touched in previous section parameter right inverses into fractional-order perfect control algorithm is shown in Section 5. Simulation studies in Section 6 indicate the contribution of the new idea to increasing non-integer order perfect control robustness property in terms of stability. Final conclusions are given in the last section of the paper.

2. System Representation

Consider a linear time-invariant (LTI) discrete fractional-order system $S(\mathbf{A_d}, \mathbf{B}, \mathbf{C})$ with n_u-inputs $\mathbf{u}(k)$, n_y-outputs $\mathbf{y}(k)$ and state vector $\mathbf{x}(k)$ described by the following equations

$$\begin{cases} \Delta^{\alpha}\mathbf{x}(k+1) = \mathbf{A_d}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k);\ \mathbf{x}(0) = \mathbf{x_0} \\ \mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) \end{cases}, \quad (1)$$

where k represents the discrete time, Δ^{α} is the Grünwald–Letnikov fractional difference operator of order α (with $0<\alpha<2$) in form of

$$\Delta^{\alpha}\mathbf{x}(k) = \sum_{j=0}^{k}(-1)^j \binom{\alpha}{j} \boldsymbol{x}(k-j), \tag{2}$$

whilst

$$\binom{\alpha}{j} = \begin{cases} 1 & j=0 \\ \frac{\alpha(\alpha-1)\dots(\alpha-j+1)}{j!} & j>0 \end{cases}. \tag{3}$$

Note that $\mathbf{A_d} = \mathbf{A} - \mathbf{I_n}$, where $\mathbf{A}$ is the $n \times n$-state matrix of integer-order system and $\mathbf{I_n}$ denotes the identity n-matrix.

3. Fractional-Order Perfect Control Algorithm

Let us consider discrete-time fractional-order system described by formulas (1–3). After minimizing the performance index

$$J(\mathbf{u}(k)) = [\mathbf{y}(k+1) - \mathbf{y_{ref}}(k+1)]^{\mathrm{T}} \times [\mathbf{y}(k+1) - \mathbf{y_{ref}}(k+1)], \tag{4}$$

where $\mathbf{y}(k+1)$ and $\mathbf{y_{ref}}(k+1)$ are the one-step deterministic output predictor and the output reference/setpoint, respectively, we obtain the fractional-order perfect control law [1]

$$\mathbf{u}(k) = (\mathbf{CB})^{\#} \Big[\mathbf{y}_{\boldsymbol{ref}}(k+1) - \mathbf{CA_d x}(k) + \mathbf{C}\sum_{j=1}^{k+1}(-1)^j \binom{\alpha}{j} \mathbf{x}(k-j-1)\Big]. \tag{5}$$

In further study we will touch only the case where product of **CB** is right-invertible. Thus, index '#' in Eqn. (5) is replaced by index 'R', which designates any nonunique right inverse (including the unique minimum-norm right inverse also known as *T*-inverse [2, 3]). Therefore in the next section the crucial nonunique parameter σ-inverse and *H*-inverse are presented in order to obtain a robust fractional-order perfect control algorithm.

4. Inverses of Nonsquare (Parameter) Matrices

Let start our consideration with definitions of the classical minimum-norm *T*-inverses $\underline{\mathbf{B}}(q^{-1})$ (in the backward shift operator domain) being of full rank or non-full rank with dimension $n_y \times n_u$, respectively.

Definition 1 ([2]). Let the polynomial matrix $\underline{\mathbf{B}}(q^{-1}) = \mathbf{b_0} + \mathbf{b_1}q^{-1} + \dots + \mathbf{b_m}q^{-m}$ be of full rank n_y (or n_u). The (unique) minimum-norm right (or least square left) *T*-inverse of $\underline{\mathbf{B}}(q^{-1})$ is defined as $\underline{\mathbf{B}}_0^{\mathrm{R}}(q^{-1}) = \underline{\mathbf{B}}^{\mathrm{T}}(q^{-1})[\,\underline{\mathbf{B}}(q^{-1})\underline{\mathbf{B}}^{\mathrm{T}}(q^{-1})]^{-1}$ (or $\underline{\mathbf{B}}_0^{\mathrm{L}} = [\underline{\mathbf{B}}(q^{-1})\underline{\mathbf{B}}^{\mathrm{T}}(q^{-1})]^{-1}\underline{\mathbf{B}}^{\mathrm{T}}(q^{-1})$).

Definition 2 ([2]). Let the polynomial matrix $\underline{\mathbf{B}}(q^{-1}) = \underline{\mathbf{b}}_\mathbf{0} + \underline{\mathbf{b}}_\mathbf{1}q^{-1} + \dots + \underline{\mathbf{b}}_\mathbf{m}q^{-m}$ of non-full normal rank r be skeleton-factorized as $\underline{\mathbf{B}}(q^{-1}) = \underline{\mathbf{C}}(q^{-1})\underline{\mathbf{D}}(q^{-1})$, where $dim[\underline{\mathbf{B}}(q^{-1})] = n_y \times n_u$, $dim[\underline{\mathbf{C}}(q^{-1})] = n_y \times r$, $dim[\underline{\mathbf{D}}(q^{-1})] = r \times n_u$. The (unique) Moore-Penrose *T*-inverse of $\underline{\mathbf{B}}(q^{-1})$ is defined as $\underline{\mathbf{B}}_0^{\#} = \underline{\mathbf{D}}_0^{\mathrm{R}}(q^{-1})\underline{\mathbf{C}}_0^{\mathrm{L}}(q^{-1})$, where $\underline{\mathbf{D}}_0^{\mathrm{R}}(q^{-1}) = \underline{\mathbf{D}}^{\mathrm{T}}(q^{-1})[\underline{\mathbf{D}}(q^{-1})\underline{\mathbf{D}}^{\mathrm{T}}(q^{-1})]^{-1}$ and $\underline{\mathbf{C}}_0^{\mathrm{L}}(q^{-1}) = [\underline{\mathbf{C}}(q^{-1})\underline{\mathbf{C}}^{\mathrm{T}}(q^{-1})]^{-1}\underline{\mathbf{C}}^{\mathrm{T}}(q^{-1})$.

Remark 1. The system taken into account in our considerations is parameter system and constitutes a special case of polynomial one i.e. for $\underline{\mathbf{B}}(q^{-1}) = \underline{\mathbf{b}}_\mathbf{0}$.

An extension of mentioned above unique right *T*-inverse of full normal rank is nonunique right σ-inverse including degrees of freedom. The newest definition of σ-inverse can be find in Ref. [9] in form of Corollary 1.

Corollary 1 ([9]). Let the polynomial $\underline{\mathbf{B}}(q^{-1}) = \underline{\mathbf{b}}_\mathbf{0} + \underline{\mathbf{b}}_\mathbf{1}q^{-1} + \dots + \underline{\mathbf{b}}_\mathbf{m}q^{-m}$ be of full normal rank n_y (or n_u) and let $z^s\underline{\boldsymbol{\beta}}(z^{-1}) = \underline{\boldsymbol{\beta}}(z) \in \mathbb{R}^{n_y \times n_u}[z]$ of full normal rank n_y (or n_u) be arbitrary order s. Assume additionally that the product $\underline{\mathbf{B}}(q^{-1})\underline{\boldsymbol{\beta}}^{\mathrm{T}}(q^{-1})$ is of full normal rank n_y (or n_u). Then an old form of σ-inverse (see [2], p. 57, Eqn. (5.3)) can be equivalently redefined as $\underline{\mathbf{B}}(q^{-1}) = \underline{\boldsymbol{\beta}}^{\mathrm{T}}(q^{-1})[\,\underline{\mathbf{B}}(q^{-1})\underline{\boldsymbol{\beta}}^{\mathrm{T}}(q^{-1})]^{-1}$.

Remark 2. Naturally, the presented polynomial matrix right σ-inverse reduces to the parameter one (for $\underline{\boldsymbol{B}}(q^{-1}) = \underline{\mathbf{b}}_\mathbf{0}$) as follows

$$\underline{\mathbf{B}}^{\mathrm{R}}(q^{-1}) = \boldsymbol{\beta}^{\mathrm{T}}[\,\underline{\mathbf{B}}(q^{-1})\boldsymbol{\beta}^{\mathrm{T}}]^{-1}, \tag{6}$$

with parameter degrees of freedom $\boldsymbol{\beta}$. Some issues concerning $\boldsymbol{\beta}$ having polynomial matrices can be found in Ref. [9].

Remark 3. For $\underline{\boldsymbol{\beta}}^{\mathrm{T}}(q^{-1}) = \underline{\mathbf{B}}(q^{-1})$ σ-inverse specializes to *T*-inverse.

Remark 4. It is clear now that in Eqn. (6) it should rather be written $\mathbf{B}^{\mathrm{R}}$ instead of $\underline{\mathbf{B}}^{\mathrm{R}}(q^{-1})$, as well as $\mathbf{B}$ instead of $\underline{\mathbf{B}}(q^{-1})$.

Finally, the recently introduced non-unique parameter right *H*-inverse based on SVD factorization is shown below.

Theorem 1 ([11]). Consider an $m \times n$ matrix $\mathbf{A}$ being of full rank m under the SVD decomposition $\mathbf{A} = \mathbf{U\Sigma V}^{\mathrm{T}}$, where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices and $\boldsymbol{\Sigma} = [diag(\sigma_1, \dots, \sigma_m), \mathbf{0}_{m \times (n-m)}]$ includes eigenvalues of $\mathbf{A}$. Then the right matrix *H*-inverse of $\mathbf{A}$ can be given as

$$\mathbf{A}^{\mathrm{R}} = (\mathbf{V}^{\mathrm{T}})^{-1}\boldsymbol{\Sigma}^{\mathrm{R}}\mathbf{U}^{-1}, \tag{7}$$

where $\boldsymbol{\Sigma}^{\mathrm{R}} = \begin{bmatrix} \boldsymbol{\Sigma}_{\mathrm{m}}^{-1} \\ \mathbf{L} \end{bmatrix}$, $\boldsymbol{\Sigma}_{\mathrm{m}}^{-1} = diag(1/\sigma_1, \dots, 1/\sigma_m)$, whilst $\mathbf{L} \in \mathbb{R}^{(n-m) \times m}(q^{-1})$ is an arbitrary matrix polynomial in the backward shift operator domain.

Remark 5. As it is in preceding cases, the presented polynomial matrix $\mathbf{L}(q^{-1})$, including degrees of freedom, can be reduced to the parameter one $\mathbf{L}$.

Having the needed notion of polynomial inverses and their parameter representations, we are able to proceed with the implementation of T-, σ- and H-inverses into the fractional-order perfect control law as in Eqn. (5), which is a subject of subsequent section.

5. An Application of Parameter Matrix Right Inverses into Fractional-Order Perfect Control Law

Starting to consider the implementation outlined in former section right inverses we have to assume that the product of **CB** is of full rank. For **CB** being of non-full rank the perfect control does not exist.

Firstly, we take into account the classical minimum-norm T-inverse. It is important to note, that minimum-norm T-inverse stand for a special case of more general one related to σ-inverse (see Remark 3). Therefore, regarding Eqn. (5), the fractional-order perfect control formula for LTI MIMO discrete-time systems in state-space can be presented in following form

$$\mathbf{u}(k) = (\mathbf{CB})^{\mathrm{T}}[\mathbf{CB}(\mathbf{CB})^{\mathrm{T}}]^{-1}\Big[\mathbf{y_{ref}}(k+1) - \mathbf{CA_d x}(k) + \mathbf{C}\sum_{j=1}^{k+1}(-1)^j\binom{\alpha}{j}\mathbf{x}(k-j-1)\Big]. \quad (8)$$

Secondly, to implement σ-inverse, we rewrite the Eqn. (5) according to Corollary 1 with corresponding remarks as follows

$$\mathbf{u}(k) = \boldsymbol{\beta}^{\mathrm{T}}[\mathbf{CB}\boldsymbol{\beta}^{\mathrm{T}}]^{-1}[\mathbf{CB}(\mathbf{CB})^{\mathrm{T}}]^{-1}\Big[\mathbf{y_{ref}}(k+1) - \mathbf{CA_d x}(k) + \mathbf{C}\sum_{j=1}^{k+1}(-1)^j\binom{\alpha}{j}\mathbf{x}(k-j-1)\Big], \quad (9)$$

with parameter matrix $\boldsymbol{\beta}$ (of $n_y \times n_u$ dimension).

Eventually, the SVD-based H-inverse formula defined in Theorem 1 is applied to Eqn. (5). Therefore, we obtain related to H-inverse the fractional-order perfect control law in succeeding form

$$\mathbf{u}(k) = \left(\mathbf{V_{CB}^{T}}\right)^{-1}\mathbf{\Sigma_{CB}^{R}}\mathbf{U_{CB}^{-1}}\Big[\mathbf{y_{ref}}(k+1) - \mathbf{CA_d x}(k) + \mathbf{C}\sum_{j=1}^{k+1}(-1)^j\binom{\alpha}{j}\mathbf{x}(k-j-1)\Big]. \quad (10)$$

Remark 6. According to the issues mentioned in Section 3 the product of **CB** has to be right invertible. For left invertible systems, i.e. systems comprising more output than input variables, the fractional-order perfect-control cannot be established.

Note that used in non-integer order perfect control problems the minimum-norm T-inverse in many cases may make the whole control system unstable. Therefore, in perfect control robustness research study (in terms of stability) there is a need to employ other non-unique inverses, for example said σ-inverse or H-inverse implemented in formulas (9) and (10). In fact, the application of the other right inverses could be connected with the problem of control zeros placement [2, 3] and finally with the issue of selecting the appropriate right inverse degrees of freedom. The problem of stability of multivariable fractional-order perfect control systems is illustrated in the next section by the complex simulation examples related to parameter σ-inverse and H-inverse.

6. Simulation Example

Consider an LTI MIMO discrete-time fractional-order system with three-inputs and two-outputs described by Eqns. (1–3) as follows

$$\mathbf{A_d} = \begin{bmatrix} -0.68 & -0.60 & 0.76 \\ -0.66 & -1.61 & -0.06 \\ -0.44 & -0.35 & -1.19 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} -0.64 & 0.69 & 0.75 \\ 0.94 & 0.23 & 0.57 \\ -0.19 & -0.25 & -0.07 \end{bmatrix}, \quad (11)$$

$$\mathbf{C} = \begin{bmatrix} 0.63 & -0.14 & 0.19 \\ 0.80 & -0.33 & 0.80 \end{bmatrix},$$

and initial state vector $\mathbf{x_0^T} = [4.04 \ -2.45 \ 4.70]$.

Employing the unique T-inverse to product of **CB** ($\boldsymbol{\beta} = \mathbf{CB}$ in Eqn. (6)) for α = 0.5, we obtain, not presented here, unstable fractional-order perfect control described by Eqn. (8). Now the drawback of T-inverse due to the lack of possibility of influencing the non-integer order prefect control stability by choosing degrees of freedom can be corrected through the application of said σ-inverse or H-inverse to system (11). This intriguing property of aforementioned inverses is presented below.

6.1. Parameter σ-inverse

After application of special selected degrees of freedom in form of parameter matrix $\beta_{\mathrm{stab}} = \begin{bmatrix} -1.33 & -2.14 & 2.67 \\ 7.68 & -6.42 & 2.48 \end{bmatrix}$ we obtain asymptotically stable non-integer perfect control (see Eqn. (9)) depicted in Fig. 1. It should be noted that degrees of freedom can be chosen by using of some criterion for example minimum-energy gauge, which will be the subject of future research. Asymptotically stable state vector is illustrated in Fig. 2.

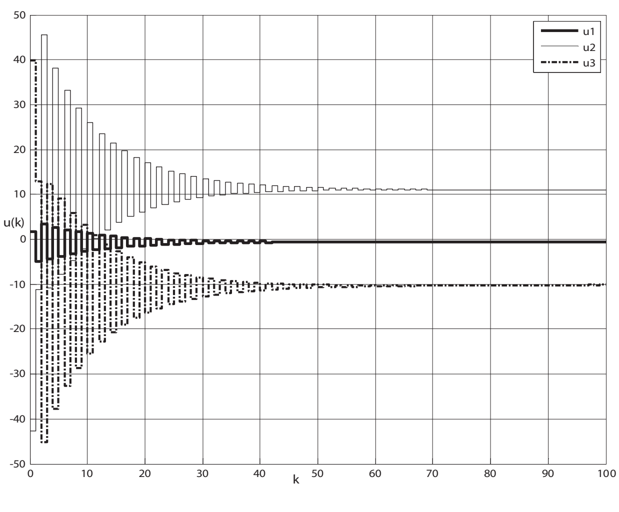

Fig. 1. Fractional-order perfect control: asymptotically stable u(k) (case: σ-inverse with β_{stab})

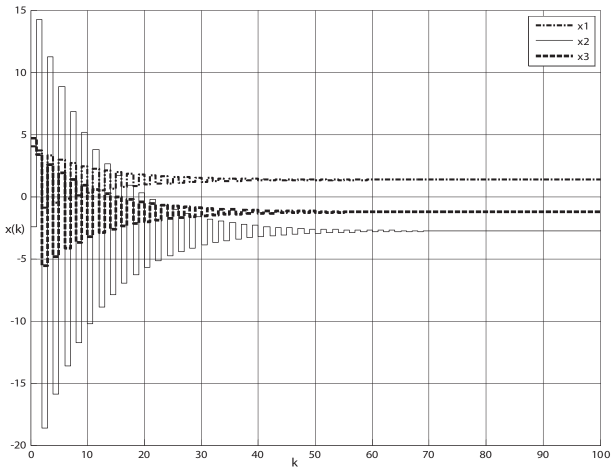

Fig. 2. Fractional-order perfect control: asymptotically stable* x*(k) (case: σ-inverse with* β*_{stab})

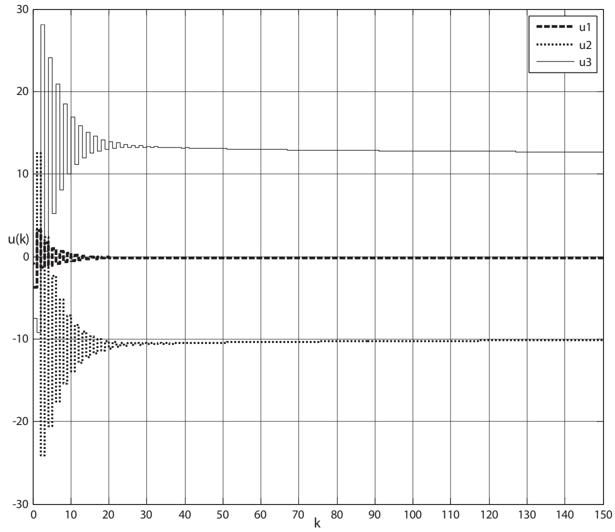

Fig. 3. Fractional-order perfect control: asymptotically stable* u*(k) (case: H-inverse with* L*stab)

6.2. Parameter *H*-inverse

Finally, let us introduce our new parameter *H*-inverse into the fractional-order perfect control. For that reason it is necessary to use Eqn. (10) with degrees of freedom, selected as in previous subsection 6.1, contained in parameter matrix $\mathbf{L}_{\mathbf{stab}}$. Thus, for

$$\mathrm{U}_{\mathrm{CB}} = \begin{bmatrix} -0.5778 & -0.8161 \\ -0.8161 & 0.5778 \end{bmatrix},$$

$$\Sigma_{\mathrm{CB}} = \begin{bmatrix} 1.3079 & 0 & 0 \\ 0 & 0.1928 & 0 \end{bmatrix},$$

$$\mathrm{V}_{\mathrm{CB}} = \begin{bmatrix} 0.8601 & -0.5028 & 0.0856 \\ -0.3291 & -0.6754 & -0.6600 \\ -0.3897 & -0.5395 & -0.7464 \end{bmatrix},$$

and $\mathbf{L}_{\mathbf{stab}}$=[–7.0141 –4.8498] we obtain the asymptotically stable fractional-order perfect control $\mathbf{u}(k)$ and asymptotically stable state vector $\mathbf{x}(k)$ shown in Figs. 3 and 4, respectively.

It is obvious that in all cases, i.e. stable and unstable ones, after time delay $k \geq \mathrm{d} = 1$, the output remains at the reference/setpoint $\mathbf{y}_{\mathbf{ref}}=[1\ 1]^{\mathrm{T}}$, which is the main property of MV/prefect control strategies (see Fig. 5).

7. Conclusions

In the paper the application of σ-inverse and *H*-inverse into the problem of fractional-order perfect control for systems comprising different numbers of inputs and output variables has been given. The non-unique inverses have been employed here to design the robust structures of mentioned control strategy, in particular in terms of their stability. It has been shown that σ-inverse and *H*-inverse are able to stabilize fractional-order perfect control in the cases where the application of the classical minimum-norm right *T*-inverse lead to the unstable control systems. Simulation examples conducted in Matlab/Simulink environment under the authors' complex procedure, confirm the significant potential of whole presented

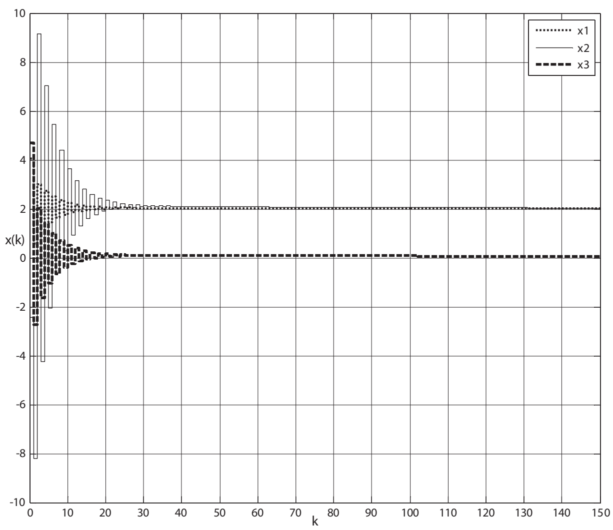

Fig. 4. Fractional-order perfect control: asymptotically stable* x*(k) (case: H-inverse with* L*stab)

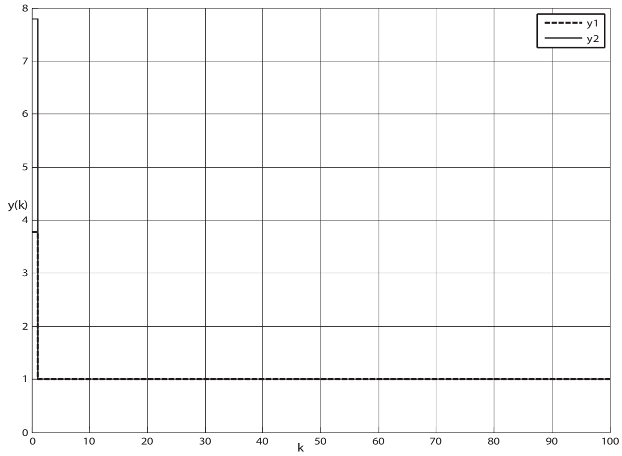

Fig. 5. Fractional-order perfect control: stable* y*(k) (case: T-inverse, σ-inverse with* β*stab and H-inverse with* L*stab)

in the paper inverse machinery. The future study will be focused on search of the analytical proofs of carried out simulation tasks.

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