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COMPARISON OF ESTIMATION METHODS OF RELIABILITY CHARACTERISTICS: MTTF AND MTTR IN A MODEL DESCRIBED BY EXPONENTIAL DISTRIBUTIONS

Abstract: In the paper a classical model of failures is considered in that successive failure-free times are supposed to have exponential distributions and are followed by exponentially distributed times of repairs. Three methods of reliability characteristics estimation are compared: Maximum Likelihood method, empirical moments method and method based on renewal theory approach. Mean Time To Failure and Mean Time of Repair based on historical data about failure-free times and repair times are estimated. In the paper a numerical example is given.

1. Introduction

In the paper reliability characteristics: Mean Time To Failure (MTTF) and Mean Time of Repair (MTTR) are predicted. Three methods of reliability characteristics estimation are compared: Maximum Likelihood method (MLM), empirical moments method (EMM) and method based on renewal theory approach (RTA). Information about a number of sample and failure-free times in successive periods of times is input data to the first and second methods, only the number of sample in successive periods of times is the input data to the third method.The objective is to compare the three methods with various complexity of input data and to predict the MTTF to generate a predictive schedule.The cycle time of operation predicted to be disturbed is increased by MTTR.

2. A production scheduling model of failures

Let we assume that a failure-free times $X_{i,1},...,X_{i,N_i}$ in the *i*th period $[(i-1)T, iT),\ i=1,...,m+1$ have exponential distribution with parameter $\mu_i > 0$, and a repair times $Y_{i,1},...,Y_{i,N_i}$ are also exponentially distributed with parameter $\lambda_i > 0$. The evolution of the system can be observed on successive cycles $Z_{i,k} = X_{i,k} + Y_{i,k},\ i=1,...,m+1,\ k=1,...,N_i$ which are independent random variables with PDFs defined as follows [1]:

$$h(t)=\int_0^t f(t-y)g(y)dy=\int_0^t \mu_i e^{-\mu_i(t-y)}\lambda_i e^{-\lambda_i y}dy=\frac{\lambda_i\mu_i}{\mu_i-\lambda_i}\left(e^{-\lambda_i t}-e^{-\mu_i t}\right), t>0 \tag{1}$$

where λ_i is a known parameter of exponential time of repair. Parameters λ_i and μ_i, in general, are different in different scheduling periods.

3. Estimation of unknown parameter with three methods

Basing on information about the number of failures and failure-free times in a number of periods of the same duration in the past, three different methods of unknown parameters of the model estimation are proposed: MLM, EMM and RTA.

According to the MLM to estimate unknown parameter μ_i for Exponential distribution, for the i*th* scheduling period we solve the equation [3]:

$$\tilde{\mu}_i=\frac{n_i}{\sum_{k=1}^{n_i}x_{i,k}} \tag{2}$$

where n_i - number of failures in i*th* period.

According to the RTA to estimate unknown parameter μ_i for Exponential distribution, for the i*th* scheduling period we solve the equation [1]:

$$\frac{\lambda_i\mu_i}{(\lambda_i+\mu_i)^2}\left(e^{-(\lambda_i+\mu_i)T}-1+(\lambda_i+\mu_i)T\right)=\bar{x}_i, \tag{3}$$

where $T=100$ [hours] (is a duration time of the scheduling period).
Substituting known values of : λ_i and $\bar{x}_i$ we estimate μ_i for successive scheduling periods.

According to the EMM to estimate unknown parameter μ_i for Exponential distribution, for the i*th* scheduling period we solve the equation (2). Suppose that we have given sample values $x_{1,1}$, $x_{1,2}$, ..., x_{1,n_1} of failure-free times observed for period $[0,T)$. Introduce sample mean and theoretical moment as follows:

$$m_1(x_1)=\bar{x}_1=\frac{1}{n_1}\sum_{k=1}^{n_1}x_{1,k} \text{ and } m_1(x_1)=\frac{1}{\lambda_1} \tag{4,5}$$

Comparing empirical moment (5) to theoretical one (4) we obtain:

$$\frac{1}{\mu_1}=\frac{1}{n_1}\sum_{k=1}^{n_1}x_{1,k}, \tag{6}$$

and we also get (2).

4. A model of production machine failures

There are 7 scheduling periods "time windows", each takes 100 hours, $T=100$. 3 jobs have to be executed on 7 machines.

The operations' times $a_{v_j,w}$ $(v_j=1,2,3, w=1,2,...,7)$ are described in minutes in MOT. The processes routes are described in MPR. In the matrices MOT(7) and MPR(8) a number of row represents a number of job *j*, a number of column states as a number of machine *w*. The dead lines and butch sizes of jobs are described in VDD (9) and VBS (10). In vectors, a number of column states as a number of job *j*. The machines start work at time $t=0$. Historical data of the number of failures of machine j are presented in Tab.1.

$$MOT=\begin{bmatrix}3,4,5,1,3,5,2\\3,2,1,2,0,0,0\\4,0,1,1,2,2,0\end{bmatrix}, MPR=\begin{bmatrix}2,1,5,3,4,6,7\\2,4,1,3,0,0,0\\2,0,1,3,4,5,0\end{bmatrix}, \quad (7,8)$$

$$VDD=\lfloor 450,\ 550,\ 650\rfloor,\ VBS=\lfloor 40,\ 50,\ 60\rfloor. \quad (9,10)$$

Table 1. Data of the number of failures of parallel machine j

		The number of scheduling period *i*						
		1	**2**	**3**	**4**	**5**	**6**	**7**
		Duration of the scheduling period *i* [in hours]						
		[0,100)	[100,200)	[200,300)	[300,400)	[400,500)	[500,600)	[600,700)
		The number of failures of machine j						
j	**1**	4	5	7	9	9	12	11
	2	3	6	7	7	8	10	11
	3	2	6	6	8	9	7	10
	4	3	4	6	7	9	10	9
	5	3	3	7	6	7	11	10
$\bar{x}$		3	4.8	6.6	7.4	8.4	10	10.2

In Tab. 2 historical data of the failure-free times of machine j and are presented. Repairing times [in hours] are exponentially distributed with parameters λ_i presented also in Tab.2.

Table 2. The failure-free times of machine j an λ_i in the scheduling period i

		No. of failure																								
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
j=1														*j*=2												
	1	20	25	25	25									**1**	25	30	30									
	2	10	20	20	22	25								**2**	10	13	15	20	17	23						
	3	10	13	13	13	15	15	15						**3**	10	12	12	13	15	15	15					
i	**4**	5	8	8	8	10	10	10	13	15				**4**	5	7	7	10	15	20	25					
	5	6	6	7	7	9	9	9	12	14				**5**	6	8	8	14	14	15	16	15				
	6	3	3	5	6	8	8	8	11	9	11	11	10	**6**	3	5	5	5	8	11	12	12	15	15		
	7	2	2	4	5	7	8	10	10	10	11	12		**7**	2	4	4	4	7	8	10	11	12	14	12	
j=3														*j*=4												
	1	25	40											**1**	20	25	40									
	2	9	12	17	18	19	22							**2**	15	20	25	30								
	3	10	13	13	15	15	18							**3**	11	14	14	16	16	19						
i	**4**	5	8	8	10	15	16	18	18					**4**	6	10	10	12	13	16	22					
	5	6	9	9	10	12	13	14	15	10				**5**	7	8	9	10	10	11	12	15	10			
	6	9	11	11	11	14	17	18						**6**	6	6	7	8	9	9	10	11	11	15		
	7	8	8	9	9	10	9	10	10	11	11			**7**	5	6	8	9	11	10	10	14	15			
j=5														λ_i												
	1	20	23	42										**1**	1.5											
	2	22	25	38										**2**	1.2											
	3	8	11	12	13	14	16	20						**3**	0.8											
i	**4**	10	12	12	15	18	18							**4**	0.7											
	5	10	10	11	13	13	15	15						**5**	0.5											
	6	3	3	5	6	6	7	9	10	11	13	11		**6**	0.4											
	7	7	7	8	9	9	10	11	11	12	11			**7**	0.3											

5. The renewal theory approach

In the RTA having values of : λ_i and $\overline{x}_i$ we estimate μ_i for successive periods, for example, for the first scheduling period $i=1$, we have [2]:

$$\frac{1.5\mu_1}{(1.5+\mu_1)^2}\left(e^{-(1.5+\mu_1)100}-1+(1.5+\mu_1)\cdot 100\right)=3. \qquad (11)$$

In successive periods we have: $\mu_1 = 0.0308177$, $\mu_2 = 0.05042$, $\mu_3 = 0.0728442$, $\mu_4 = 0.083945$, $\mu_5 = 0.103015$, $\mu_6 = 0.13673$, $\mu_7 = 0.159811$. In successive periods mean failure-free times of the machine equal: $EX_1 = 32.4489$, $EX_2 = 19.8334$, $EX_3 = 13.7279$, $EX_4 = 11.9126$, $EX_5 = 9.70732$, $EX_6 = 7.31368$, $EX_7 = 6.25739$.

Having parameters μ_i for the periods $i=1,\ldots,7$ we use the classical regression to predict a parameter for period $i=8$, and we have $\mu_8 = 0.148847$. The regression function is as follows: $\mu_i = 0.00683031 + 0.017752\cdot i$. The MTTF in period $i=8$ equals $EX_8 = 6.71832$.

6. Maximum Likelihood and empirical moments approaches

To estimate unknown parameter μ_i for Exponential distribution, for the successive periods we solve the equation (2). In periods $i = 1,\dots,7$ for machine j we have μ_i described in Tab.3.

Table 3. The parameter μ_i in scheduling period i and for the bottle neck $j = 6$

μ_i		J					The regression function	μ_i
		1	**2**	**3**	**4**	**5**		for the bottle neck ***j=6***
i	**1**	0,040	0,035	0,031	0,035	0,035	=0,001·36-0,008·6+0,047	0,035
	2	0,052	0,061	0,062	0,042	0,035	=-0,003·36+0,017·6+0,039	0,033
	3	0,071	0,071	0,070	0,065	0,075	=0,001·36-0,007·6+0,078	0,072
	4	0,099	0,074	0,080	0,079	0,071	=0,01·LN(6)+0,094	0,111918
	5	0,100	0,082	0,091	0,098	0,080	=0·LN(6)+0,096	0,096
	6	0,120	0,112	0,079	0,101	0,126	=0,008·36-0,052·6+0,168	0,144
	7	0,124	0,121	0,104	0,098	0,101	=0,001·36-0,016+0,141	0,161

The bottleneck is one from a set of parallel machines therefore we use all data to predict the MTTF. Let we assume that we don't know which machine was the bottle neck in the i*th* scheduling periods, therefore we introduced artificial variable of the bottle neck $j = 6$. First we predict μ_i for the i*th* scheduling periods for the bottle neck.

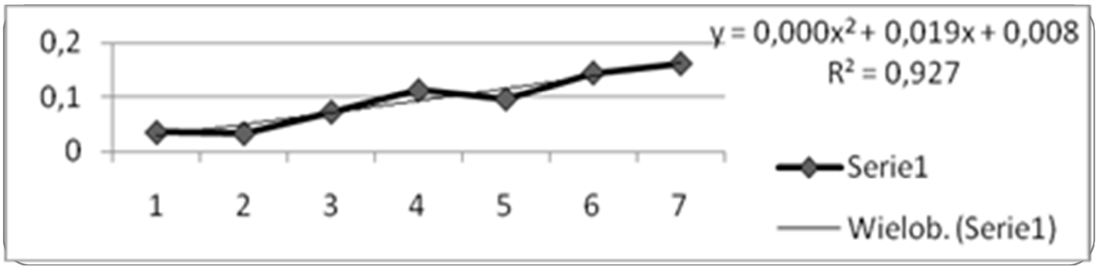

Figure1.The prediction of the parameter μ_8 for scheduling period i=8 for the bottle neck $j = 6$

Having parameters μ_i for the periods $i = 1,\dots,7$ we use the classical regression to predict parameter μ_8 for period $i = 8$ (Fig. 1). The regression function is described in Fig. 1. We have $\mu_8 = 0.144$. In the period $i = 8$ The MTTF equals $EX_8 = 6.94$ hours.

Having parameters λ_i for the periods $i = 1,\dots,7$ we use the classical regression to predict parameters for period $i = 8$, and we have $\lambda_8 = 0.235952$. The function of the regression is as follows: $\lambda_i = -0.63\ln(i) + 1.546$. In the period $i = 8$ the MTTR equals $EY_8 = 0.235952$ hours.

7. Modeling and simulation

The production system described by MOT, MPP, *VDD* and *VBS* is modeled in the

Enterprise Dynamics (ED) [2]. After doing the first simulation for RANDOM priority rule, the "bottle neck" is identified. MTBF and MTTR are defined for the "bottle neck" - the machine 1.

Criteria: Makespan $C_{\max}$, total tardiness of production jobs T are used to evaluate schedules. $C_{\max}$ and T of predicted schedules generated using FIFO, LIFO, RANDOM rules are presented in Tab. 4. Values of the scalar function for weights of the criteria: $w1 = 0.4$ i $w2 = 0.6$ are presented in Tab. 4. The best predictive schedule is selected according to the minimum value of the scalar objective function $f(x)$ and is obtained for FIFO rule [2].

Table 4. $C_{\max}$, T and $f(x)$ of predicted schedules generated using FIFO, LIFO, RANDOM rules [2]

	$C_{\max}$	T	$f(x)$
LIFO	610	8	$= (8/8)\cdot 0.6 + (610/610)\cdot 0.4 = 1$
FIFO	610	0	$= (0/0)\cdot 0.6 + (610/610)\cdot 0.4 = 0.4$
RANDOM	610	4	$= (4/8)\cdot 0.6 + (610/610)\cdot 0.4 = 0.7$

8. Summary

Using the TRM we have $\mu_8 = 0.148847$ and the MTTF equals $EX_8 = 6.71832$. Using the MLM or EMM we have $\mu_8 = 0.144$ and the MTTF equals $EX_8 = 6.94$. Although the level of complexity of input data to the three methods is various: a number of failures in the RTM and the number of failures and failure-free times in the MLM or EMM the results are very similar. The technical survey of the bottle neck is scheduled after 6.71 hours of work.

References

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