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AN ALGORITHM FOR GARBAGE TRUCK ROUTING IN CITIES WITH A FIXATION ON CONTAINER FILLING LEVEL

Summary. An algorithm for optimizing the routes of a set of vehicles used for the collection and removal of municipal solid waste in a metropolis is proposed. The algorithm eliminates the problem of applying heuristic methods for multi-agent optimization, which is NP non-deterministic polynomial-time-hard. The application of the algorithm leads to a guaranteed exact solution. Through the application of restrictions on the carrying capacity of vehicles, the size of the input matrix representing the transport network can be reduced to an adequate size. This process uses statistical information about the filling levels of container waste bins. The algorithm is applied to an example of two megacities. The shortest routes are built for different numbers of points (from 12 to 72) on the route. The dependence of the total mileage on the number of involved vehicles is studied.

1. INTRODUCTION

The intensity of the generation of household garbage in megacities grows with the expansion of their geographical size and population [1]. The requirements for waste processing also increase at the same time, which provides for their sorting, both at the stage of accumulation and during collection and disposal [2]. All these matters significantly complicate the use of motor transport, which is the main element in the communal system of cities. Garbage trucks must be used with maximum performance in the most economical modes, given that the requirements for environmental friendliness are also growing [3]. This situation means that there is an increasing need to carry out the maximum volume of transportation while providing the slightest shortest run on the road network of the metropolis. In this regard, the relevance of the problem of routing vehicles operating on combined routes is very high. However, two obstacles prevent these problems from being solved. First, garbage collection transport networks are very cumbersome. In some cities (e.g., Krakow), solid waste management is coordinated by municipal service companies. Solid waste management consists of the collection of seven types of segregated solid waste [4]. The number of container sites that vehicles need to visit in a city with a population of about 1 million people and a population density of about 116 people per km^2 ranges from 200 to 300. Of course, the number of such sites also grows proportionally with the scale of the city. In addition, one garbage container site consists of several containers according to the types of waste. The assortment of waste corresponds to the frequency of visits by vehicles to sites. The number of types of garbage is standardized in some countries. Other countries proceed from the manufacturability of its processing [5]. Thus, routing garbage trucks is one of the most significant tasks in organizing the work of vehicles serving a metropolis.

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On the other hand, the garbage accumulation process is stochastic in nature. The amount of waste produced by residents of cities and enterprises varies substantially. Thus, we have studied that the rate of filling containers in large cities of Ukraine (Kyiv, L'viv) ranges from 0.04-0.074 m^3/day according to the season of the year [3]. The annual coefficient of intensity unevenness reaches 1.87. There has been a trend of growth of this coefficient with the growth of the populations of cities. The overall unevenness of waste removal volumes in cities is slightly less than the unevenness of removal volumes on any given route for all garbage trucks, for each scheduled time period. Thus, the coefficient of unevenness in the amount of accumulated garbage in different container sites of the city for the same period in summer is 1.28-1.44, while it is 1.15-1.37 in winter [1]. Therefore, despite the possibility of prediction, the intensity of filling garbage containers is a random variable with high variance. At the same time, there is a significant unevenness in the input garbage flows, both in time and space. This significantly complicates the problem of the operational planning of the fleet of vehicles involved in transportation.

Planning is carried out mainly centrally by the municipal service of the city. However, there are known examples where the efforts of several independent carriers are concentrated on the territory of the city. In such cases, the waste collection sites are distributed among these transport enterprises, and there is competition between them regarding the quality of service. Thus, truck routing is reduced to the service area of one enterprise, but it concerns the optimization of several parallel routes with one origin and one destination. However, this reduces the dimension of routing problems and makes it possible to obtain exact solutions using known methods. On the other hand, this cuts off the domain of feasible solutions from the global optimal solution. There is also a contradiction in the methodology of organizing the utilization of municipal solid waste (MSW) in large metropolitan areas. Utilities are still guided by the norms of waste generation, depending on the city's population. Road freight carriers try to shorten the planned collecting routes of garbage trucks so that they do not cover extra mileage in the absence of reliable information about the actual accumulation of waste. As a rule, the frequency of garbage collection is greater than its actual accumulation in containers of a fixed volume. It is important not only to develop the shortest route but also to have a schedule for the implementation of garbage trucks taking into account the intensity of waste accumulation. Individual containers remain overflowing with waste as a result of the lack of clear schedules for trucks, which violates sanitation standards. In addition, public utilities cannot ensure the operation of trucks at close to the maximum actual load capacity while minimizing the total fleet mileage when guided by the available information on the intensity of filling garbage containers. The problem of public utilities is a rather high level of unevenness of transport tasks and a high level of uncertainty of route conditions. Therefore, this article proposes a method and algorithm for resolving problematic situations in the organization of prefabricated routes based on the general problem of multi-agent routing.

The problem is formulated as follows. A transport network is provided that represents a set of points–namely, the container sites of the city and the roads that connect these sites. The network and, accordingly, the task can be asymmetric in the general case (i.e., $a_{i.j} \neq a_{j.i}$) where $a_{i.j}$ is the distance between points i and j. Each container site has several containers, which may differ in waste grade. The level of filling of each container is a random variable Q_i, which depends on the intensity of its filling μ_i. The utility operates a specified number of container yards. A limited fleet of garbage trucks is used for this aim, the carrying capacities of which differ, ranging from 9 to 17 containers with a capacity of 1.1 m^3. Vehicles are sent to collection routes from one common depot, collect waste from the assigned territory, take it to processing plants, and return to the depot. Thus, the planned routes of trucks are circular, prefabricated, and cyclic. The fleet of trucks is planned cyclically. The number of containers scheduled for unloading in one cycle is variable and depends on the intensity of filling. The planning cycle is formed depending on the employment of the staff of the utility [2]. We assumed that the level of MSW filling of each container (as a percentage of the maximum capacity) is known at the beginning of each cycle with acceptable accuracy. It is necessary that trucks take the shortest routes possible to minimize the total mileage of all vehicles per cycle. Trucks are maximally loaded with waste at the end of the route. There should not be a single container that is 100% full in order to prevent improper garbage accumulation at the end of each cycle and the resultant unsanitary conditions. It should be noted that the task of such a plan is quite suitable for the formulation of the problem of multi-agent routing, with the exception of two restrictions: the level of filling trucks and the allowable waste storage rates at container sites. Therefore, an analog of the algorithm for solving the problem with a large amount of data was searched for among heuristic algorithms for solving the multiple vehicle routing problem (MVRP).

2. LITERATURE REVIEW

Garbage collection is organized in a certain way in L'viv. The city authorities determine where garbage sites are set up. These are sanitary areas that provide access to city residents on the one hand and free access and maneuvering of garbage trucks to load garbage from containers on the other hand. Moreover, the municipality determines the assortment of solid household waste, which must be observed by the residents of each district where a garbage site has been set up. Thus, plastic, glass, organic waste, paper, and other solid household waste are currently sorted separately. Furthermore, L'viv has organized public and private collection points for secondary raw materials—scrap metal (ferrous and non-ferrous metals) and wood—which deal with collecting, processing and transporting this waste separately. Each landfill is designed to serve a limited area of the district. Therefore, the number and density of each site depend on the number of city residents. At the same time, one municipal company and several private companies are engaged in the collection and transportation of solid household waste in L'viv. The number and duties of private enterprises are determined by the city council based on a competition. The conditions of the competition include, in particular, the sufficient transport capacity of the carrier's fleet and its ability to remove waste in a timely manner. Fixed waste sites are assigned to each carrier, the users of which form service agreements with the relevant company. Waste is taken to municipal processing plants or to a temporary city landfill by all enterprises. Thus, the authority of each carrier is responsible for designing the routes of garbage collection. The compilation of routes depends on the rate at which garbage containers are filled, the number and location of sites assigned to the carrier, the mode of operation of enterprises, and the intensity of traffic flows in the city by the hour of the day [1].

The problem of the optimal multiply routing of a fleet of vehicles under uncertainty has several aspects and several directions of resolution. Scientists are searching for effective algorithms and methods to solve this problem. This problem is Non-deterministic Polynomial-time (*NP*) hard and does not have an exact guaranteed solution that can be reached in the permissible time. Therefore, known modified heuristic algorithms are developed, which differ in terms of optimality [7]. However, all modern attempts to improve heuristic algorithms are the adaptations of known algorithms to changed initial conditions. The next most important problem, which has increased the relevance of multi-route construction, is the growth of the volume of input data. This growth is due to the globalization of the transport network, the growing number of vehicle fleets, and their cooperation [8].

We have not been able to note a successful attempt to significantly increase the efficiency of heuristics. Currently, MVRP tasks can be solved with acceptable accuracy in an acceptable time if their number does not exceed 200 and there are one or two agents. An increase in the number of agents multiplies the size of the task [9].

The MVRP is often found in various formulations for specific conditions. This means that the overall problem is subject to partial restrictions related to time windows [10], the composition of the vehicles fleet [1], restrictions on the route network [11, 12], etc. Considerable success has also been achieved by means of sequential (dynamic) routing, by which a general cumbersome long-term task is broken down into simple tasks in relation to a discrete input flow of data. In this regard, algorithms involving the clusterization of input data [13], methods of successive approximation of solutions, including a stepwise solution to the general problem [14], and linearization of the initial model [10] are considered promising.

In [4], it is noted that effective computer decision-making tools are not yet being used for garbage truck routing. The authors proposed solving the problem of routing motor vehicles (MVs) for the separate collection of solid waste using the large neighborhood search algorithm. It is shown that the task refers to those containing a large number of input data on a specific example of a metropolis in

Poland. Therefore, its solution is difficult, and sometimes impossible, to attain. However, the study did not take into account the actual intensity of garbage accumulation in garbage cans, which does not solve the formulated problem in principle.

The Consistent Vehicles Routing Problem (ConVRP) is a long-term problem related to meeting consistency requirements along with traditional vehicle capacity and route duration limitations. A few heuristics have been proposed, but there is no exact method for solving this problem. The latest accurate vehicle routing solution (VRP) method is column generation, which is applied to route-based formulations with columns generated by dynamic programming [15]. However, this method cannot be successfully extended to ConVRP since the linearization of route properties is not sufficient to adequately represent targets. An exact ConVRP method, which can solve medium-sized task instances with a five-day service scheduling period for up to 30 clients, was proposed for the first time in [16]. VRP is able to significantly improve the quality of the solution in the examples of standards in the literature compared to the most modern heuristic tools used as a stand-alone heuristic system. Other heuristic algorithms use modifications of taboo search [8, 13], nearest neighbor [17], ant colony [18], and annealing [19] methods. Hybrid, combined, and multi-stage algorithms based on a combination of simpler heuristics are also used. The routing problem is considered in [8], for example, when the carrier must fulfill an order for customers using a limited capacity for a discrete period. The goal is to minimize the total cost of delivering goods, including the cost of the carrier and the cost of consumers. The search for optimal routes by the taboo method and mathematical programming are combined in the mentioned work. The heuristic found only 192 (48%) optimal cases out of 402 possible instances with known optima and improved the top 125 estimates for problem sizes over 600 points [20]. Combined heuristics tested on 240 large instances (with 200 clients), for which optimal solutions were not known, improved the best solution for 220 (92%) of 240 instances. However, combined methods are still limited by their dimensions. The use of routing problems with known constraints reduces their dimensions but does not reduce the complexity of the search algorithm. Therefore, the optimization of the routes of vehicles based on the criterion of minimum total operating costs, for example, with a restriction on the number of vehicles, is a problem of O^3 complexity [21, 22]. The problem of the stochastic nature of the MVRP for the collection and removal of solid waste can be practically solved in two ways. The first way is through the long-term collection and processing of a-posteriori information and forecasting volumes for future periods [23-25]. The second way is based on state-of-the-art technology that allows utilities to use remote control during the process of filling garbage containers [26]. However, the use of automated accounting tools does not contribute to solving the entire problem. Current and retrospective information refines the forecast for the volume of garbage transportation. Available forecasts do not correspond to the method of drawing up the itineraries of the garbage trucks. In most cases, the transition of the municipal waste disposal system to an automated level is held back due to economic factors. The territorial communities of cities are financially often unable to retrofit all the garbage containers of megacities with telemetric means.

In summarizing the literature review, we note that all attempts to improve the heuristic algorithms to find the most profitable routes for a set of vehicles have aimed to obtain the best exact solution to a high-dimensional problem with a limited amount of reliable initial information. The most successful such algorithm was presented in [24]. An attempt was made by the authors to obtain an exact MVRP solution using an exact method. Well-known exact methods are branch-and-bound approaches and the lexical search approach. The lexical search approach appears to be better than the branching approach for the MVRP [27]. However, the authors verified the algorithm based on recursive procedures, which leads to an exponential increase, as is known to occur when large input data matrices are involved in the required amount of memory if computer technology is used [24]. Since the MVRP is a generalization of the traveling salesman problem, we propose a modified lexical search algorithm using restrictions on the number of points that a vehicle needs to visit. The purpose of such a modification is to obtain an exact MVRP solution and compare the proposed algorithm with existing exact methods in some instances of transport problems [18] involving a different number of trucks.

3. METHODOLOGY

The problem statement is as follows. The transport network of the city, which consists of n points (container sites), the point of departure of MVs (the depot), and the point of their final return, is given. It is assumed that the final point of garbage collection is combined with the depot. The transport points of the network are designated q_1, q_2, … q_n, where q_1 is the depot. The distances between any pair of network points are known, and they form a symmetric zero matrix $A_0=(a_{0.i.j})$. The problem is considered for each type of waste separately. The filling volume of containers is a random variable. It can be argued that, without the use of special telemetric means, by the beginning of the planned period of transportation, the volume of waste v_i in the i-th container does not exceed the numerical value of the filling limits:

$$v_{i.\min} \le v_i \le v_{i.\max}, \tag{1}$$

where $v_{i.\min}$ and $v_{i.\max}$ are a priori values determined with sufficient accuracy from previous transport cycles of trucks. Presently, we represent the value v_i as a part of the maximum physical content of the container (it was assumed that all containers are the same). Thus, the volume of cargo at each point for export can be given as a fixed discrete value (a fraction of a unit). The utility company uses m=1…k MV for garbage collection, which constitutes the determining capacity $V_{k.n}$, which was measured based on the number of unloaded containers. Garbage trucks must be used in such a way that they have loaded and removed the amount of garbage at least $V_{k.min}$ in one cycle, which we call the minimum load capacity MV. The problem involves developing routes that start at depot q_1, go through selected sections of the transport network, and end at the depot. The set of transport points, all of which must be included in m routes, was selected based on the criterion of the minimum total mileage of trucks:

$$L_\Sigma = \sum_{i=1}^{n} \sum_{j=1}^{n} x_{i.j} \cdot a_{0.i.j} \to \min, \tag{2}$$

where $x_{i.j}$ is a binary variable equal to 1 if the path $a_{i.j}$ is included in the planned routes or 0 otherwise.

This is subject to a restriction on the minimum effective actual carrying capacity:

$$V_{k.\min} \le \sum_i v_i \le V_{k.c} \tag{3}$$

where v_i is the volume of the i-th containers to be unloaded into one vehicle on the route.

The number of variables, $x_{i.j}$, corresponds to the size $n \times n$ of the matrix A_0 for m=1. However, if m>1, then the initial problem can be simplified. If m–1 fictitious vertices are introduced into the problem with n points and m>1 MV, then the problem can be considered a common single salesman routing problem [8]. Added points are taken as fictitious depots, and zero distances are applied between fictitious depots and other points. Sufficiently large numerical values are provided in order to limit the movement between fictitious points along the distance from depot to depot. The interpretation of the obtained results of the search for minimal routes for m>1 is shown in Fig. 1. There are nine network points, including the depot. Two vehicles are used.

1 → 7 → 10 → 2 → 5 → 4 → 3 → 6 → 9 → 8 → 11

Fig. 1. Interpretation of the MVRP for VRP transformation (selected vertices are depot vertices)

One shortest chain of graph vertices was found in the example of searching for routes with m=2, representing the desired routes for MV No. 1: 1–7–1, No. 2: 1–2–5–4–3–6–9–8–1. Thus, the MVRP is converted into a VRP. The distance matrix was also transformed in a suitable way: m–1 more columns and m–1 rows were added to the zero matrix A_0 to generate the matrix $A_1=(a_{1.i.j})$. The added elements of the matrix A_1 were defined as $a_{1.1.j}=+\infty$, $a_{1.j.1}=+\infty$, $a_{1.i.j}=a_{0.i.1}$, $a_{1.j.i}=a_{0.j.1}$, where $+\infty$ is a sufficiently large number, $j=n+1$, …, m–1. After the initial transformations were performed, the displacement procedure [27] was applied to the resulting matrix A_1. During this procedure, initially, the minimum column was selected from A_1 – $\{a_{1.1.\xi}, a_{1.2.\xi},\dots, a_{1.n+m-1.\xi}\}$, where ξ is the row number, for which $a_{1.i.\xi}$ = $\min\{a_{1.j.\xi}\}$. Thus, the minimum column contained the minimum number of elements in each row of the matrix A_1. After that, we looked for a new matrix $A_2=(a_{2.i.j})$, whose elements were the differences

$a_{2.i.j}=a_{1.i.j}-a_{1.i.\xi}$, where $a_{1.i.\xi}$ is the i-th element of the minimum column. Further, similarly to the minimum column, we looked for the minimum row of the matrix A_2. Each element of the minimum row was $a_{2.\zeta.j}=\min\{a_{1.i.\zeta}\}$. The final elements of the matrix A_2 were the differences $a_{2.i.j}=a_{1.i.j}-a_{1.\zeta.j}$, where $a_{1.\zeta.j}$ is the j element of the minimum row. The matrix A_2 is called a matrix without a bias. An example of a matrix without bias is shown in Table 1. The matrix was built for n=7 points, including the fictitious one, and the number of MV was m=2. The matrix bias was 15.7 km.

Table 1

Initial matrix without a bias

998.0	0.7	2.7	1.1	1.7	1.3	0.0	998.0
0.0	997.3	0.0	0.0	0.0	0.0	2.8	0.0
2.0	0.0	996.3	0.0	1.7	1.7	4.5	2.0
0.4	0.0	0.0	997.1	1.7	1.7	4.5	0.4
1.0	0.0	1.7	1.7	995.7	1.7	4.5	1.0
0.6	0.0	1.7	1.7	1.7	995.9	4.5	0.6
0.0	3.5	5.2	5.2	5.2	5.2	998.0	0.0
998.0	0.7	2.7	1.1	1.7	1.3	0.0	998.0

Bias = 15.7

An alphabetical table was then created, which is designated Q-A_2.

This square matrix of size $n+m-1\times n+m-1$ was created by arranging the elements of the matrix A_2 in ascending order of the elements $a_{2i.j}$ in rows and columns. The indexes of the table Q correspond to the numbers of the vertices. Table 2 is an example of an alphabetical table created for matrix A_2, where Q is the point and A_2 is its distance from the corresponding city in the first column.

Table 2

Alphabetical table

Point	Q-A	Q-A	Q-A	Q-A	Q-A	Q-A	Q-A	Q-A
1	7-0.0	2-0.7	4-1.1	6-1.3	5-1.7	3-2.7	1-998.0	8-998.0
2	1-0.0	5-0.0	6-0.0	8-0.0	3-0.0	4-0.0	7-2.8	2-997.3
3	2-0.0	4-0.0	6-1.7	5-1.7	1-2.0	8-2.0	7-4.5	3-996.3
4	2-0.0	3-0.0	1-0.4	8-0.4	5-1.7	6-1.7	7-4.5	4-997.1
5	2-0.0	1-1.0	8-1.0	4-1.7	3-1.7	6-1.7	7-4.5	5-995.7
6	2-0.0	1-0.6	8-0.6	3-1.7	5-1.7	4-1.7	7-4.5	6-995.9
7	1-0.0	8-0.0	2-3.5	3-5.2	4-5.2	5-5.2	6-5.2	7-998.0
8	7-0.0	2-0.7	4-1.1	6-1.3	5-1.7	3-2.7	1-998.0	8-998.0

The lexical search algorithm is effective for some classes of complex problems [25], where all possible solutions are organized, such as words in a dictionary, in such a way that a partial word denotes a block of words and is the leader of the block. Lower limits are computed for the objective function for each of these blocks compared to the current "best solution." If no solution (word) better than the current "best solution" is found in the current block under consideration, then it is necessary to move from the current block to the next block. However, if the bound shows the possibility of having the best solution in this block, then its sub-block must be considered, combining the current leader with the corresponding letter. The lower bound must then be calculated.

A partial word (incomplete route) containing some points is called a word block leader. In the current study, each city, including the fictitious depot, was treated as a letter of any alphabet for the MVRP. Therefore, all words (junctions) in the dictionary were divided into blocks. Calculating and finding a compact lower limit (*infimum*) for the block leader for the objective function of the MVRP is very difficult. Therefore, we considered the lower limit used for analogs [26], formulated as follows. Let the incomplete route be (q_0, q_1, q_2), and select point q_3 for connection. Before joining the q_3 next point to the route, we calculated the infinium for the block leader (q_0, q_1, q_2, q_3). To do this, we started the calculation from the second row. Then we continued the search to the last line of the alphabetic table. After that, we summed up the distances to the first accessible points, including point 1, in each row, except for q_1 and q_2 (since these points were already included in the route). The calculated total distance was the lower limit for this leader (q_0, q_1, q_2, q_3). Such an enumeration can significantly reduce the number of options considered when choosing the best one according to the criterion of total distance. Thus, if n points and m vehicles are given, then the number of possible route options that need to be considered under the condition of exhaustive search is $(n+m-2)!$. For example, with seven points and two vehicles, the number of options is 362,880. When using the alphabetical algorithm without additional restrictions, the number of options for enumeration depends on the values of the matrix A_2. The higher the coefficient of non-uniformity of the numerical values of the matrix, the fewer route options will have to be considered. However, if the elements $a_{2.i.j}$ have approximately the same numerical value, which is typical for the transport network of a metropolis, then the alphabetic algorithm does not provide a tangible improvement in the heuristic search [25]. Therefore, we used additional restrictions of type (3). Container fill level limits have a double effect. Firstly, the most filled containers are selected among the available points when drawing up the current plan, which directly affects the reduction of the entire route. Thus, the size of the initial matrix is reduced. Secondly, by sorting through the options of routes in the presence of a limit on the maximum actual carrying capacity of the vehicle, we will get shorter routes with a high probability. The reason is that, according to the $\min\{L_\Sigma\}$ criterion, the algorithm will lead to the selection of points where the loading of containers is maximal. Thus, the application of new restrictions leads to the following structure of the modified lexical algorithm. The procedures of the algorithm operate on an alphabetical table. The following quantities were introduced for use in the algorithm.

BS (better solution) – the value found through a comparison of options between different blocks from which the best of the available block leaders is selected. At the beginning of the algorithm, $BS=+\infty$. Subsequently, its numerical value is reduced in accordance with the obtained intermediate results and to the lower estimate.

Sol (solution) – the current solution, taking into account the length of the partial word (route) and the newly chosen path:

$$Sol = L_{1.i} + a_{i.j}, \tag{4}$$

where $L_{1.i}$ is the distance from the first to the current to the i-th point of the block leader.

The algorithm consists of 11 steps. Step 0 is initialization. All other steps are repeated cyclically.

Step 0. Calculate the offset of the given distance matrix A_1, build the reduced distance matrix A_2, and then build an alphabetical table based on the reduced matrix. Fix the best solution $BS=+\infty$ (very large number). Since point 1 of the transport network is a depot, the algorithm starts from the first row of the alphabetical table. Set $k=1$ as the value of the length of the current route $Sol:=0$, and then go to step 1.

Step 1. Go to the kth element of the first row of the alphabetical table (e.g., point q_j) with the distance to the current point $a_{2.k.j}$. If $Sol+a_{2.k.j}<BS$, then go to step 2; otherwise, go to step 9.

Step 2. If point q_j forms a cycle with point q_k or if the current point k and point j are both depots, leave point j, set $k=k+1$ and go to step 7; otherwise, go to step 3.

Step 3. If all given network points are visited, calculate *Sol* and go to step 4; otherwise, go to step 5.

Step 4. If $Sol \geq BS$, go to step 9; otherwise, perform $BS=Sol$ and go to step 9.

Step 5. Calculate the lower limit *LB* of the current leader for the objective function L_Σ and go to step 6.

Step 6. If $Sol+a_{2.k.j}+LB\geq BS$, or if condition (3) is not satisfied for any vehicle among those involved in transportation as a result of adding point j to the route, then bypass point j, set $k=k+1$, and go to step 7; otherwise, take point q_j on the route, calculate *Sol*, and go to step 8.

Step 7. If you have not selected all the waypoints yet, go to step 1 or step 9.

Step 8. Go to the sub-block (i.e., the qth row of the alphabetical table), take $k=1$, and go to step 1.

Step 9. Traverse the current block of items (word) by dropping item q_k and return to the previous item in the search procedure (e.g., p) (i.e., go to the p-th row of the alphabetical table and set $k:=k+1$, where k is the index of the last visited item in this row). If item $p=1$ and all items k have already been reviewed, go to step 10; otherwise, go to step 7.

Step 10. Now, *BS* is the optimal solution value for the reduced distance matrix, and *BS*=*BS*+*bias* is the optimal solution value for the original distance matrix; go to step 11.

Step 11. The current word is a good lookup sequence, given the modified distance matrix. Generate routes for each vehicle. Determine the volume of transportation.

The proposed algorithm was programmed in Delphi 10.4. The operation of the algorithm and the program was tested on data received from the municipal service of the city.

4. APPLICATION TO THE UTILITY SYSTEM OF A MEGAPOLIS

The developed algorithm and computer program were applied to determine the planning of routes for a fleet of MVs, depending on the rate of the filling of containers and the corresponding frequency of garbage collection. To do this, we used data from the garbage collection system in the metropolises of L'viv and Kyiv. Garbage sites in L'viv (Fig. 2) include one to 12 containers.

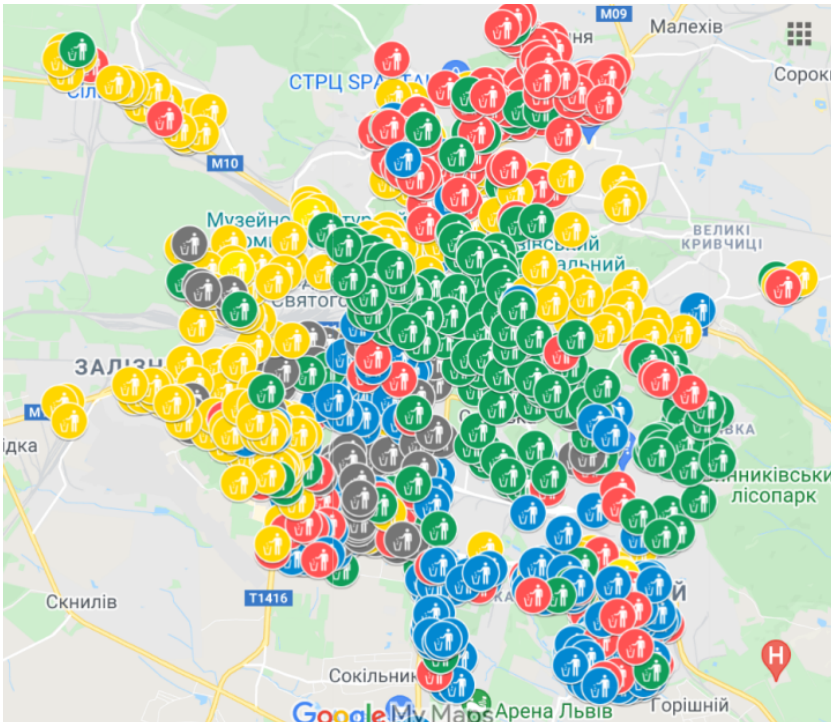

Fig. 2. Map of the locations of garbage sites in L'viv [28]

Waste sorting into two to four grades was used here: organic waste, glass, paper, plastic, and other solid garbage. The waste disposal system is set up in such a way that the collection and transportation of waste are mainly carried out by several private and public utilities (in Fig. 2, the sites assigned to them are marked with five different colors).

The work of the state communal enterprise was considered, which serves the garbage sites shown in yellow on the map. It is taken into account that traffic jams are possible in the metropolis and that, when they occur, the travel times between garbage sites will be prolonged. The truck park of the utility company is located within the city limits, and the organic waste processing plant, where the appropriate type of garbage is taken, is located in the suburban area. Transportation is carried out by specialized vehicles, namely MAZ-5907S2-310 garbage trucks, each of which technologically accommodates 16 m^3 of solid waste. According to the company, the average operating speed of a truck within the city limits is 22.5 km/h. Organic waste, as the main component of MSW, is not compressed in the garbage truck, so its volume of accumulation corresponds to the volume of removal.

The periodicity of garbage collection from each container varies from 13.5 to 24.5 hours. The average duration of work of a garbage truck per day ranges from 6.2 to 7.4 hours. At the same time, the vehicle performs one to two cycles of garbage collection and removal. The average duration of one cycle is 3.1-3.7 hours, depending on the configuration of the route and the filling level of the containers and the truck. The total average duration of filling operations of one garbage truck up to a volume of 16 m^3 ranges from 1.2-1.6 hours (without pressing), taking into account the duration and the movement of the vehicle. Thus, the total average length of all routes of garbage trucks ranges from 40.5-56.8 km, taking into account their operating number of six units and the average operating speed. The duration of the drivers' working hours is not taken into account in the model because the drivers working on the routes operate under production conditions while observing the norms of the maximum duration of the work shift, continuous driving, duration of rest between shifts, and lunch breaks.

Accurate information on the intensity of accumulation and removal of organic waste in the metropolis is not available. However, public utilities have a priori information indicating that, by the beginning of the next planning period, the filling level of one container or another will be approximately equal to 0.2, 0.3, ... 1.0 from $v_{i.\max}$. The garbage collection system is equipped with means of telemetric control over the location and movement of vehicles on the routes such that the real movement and flow of input data on the completion of the previous transport cycle by a group of garbage trucks is recorded online. There are no devices to control the amount of waste before disposal. The work of garbage trucks is planned based on standards for garbage collection, which are most often underestimated and do not correspond to reality. Garbage collection is scheduled at the enterprise on a weekly basis. One city utility serves an average of 240 container yards. The average intensity of garbage accumulation in containers is 0.65 m^3/day, with a non-uniformity coefficient of 1.87. The filling intensity ranges from 0.34-1.21 m^3/day.

Given this situation, the distribution of container filling intensity was calculated (Table 3).

Table 3

A priori distribution of container filling intensity

Intensity, m^3/day	less 0.2	0.2-0.4	0.4-0.6	0.6-0.8	0.8-1.0	1.0-1.2
Number of containers	118	55	37	27	16	9

The average intensity of garbage accumulation in the city in the summer months is 101.8 m^3/day. Considering that the minimum volume of a garbage truck used for SDW disposal is 27 m^3—taking into account the possibility of pressing (with the exception of organic waste) loaded trucks—this intensity corresponds to the carrying capacity of the utility vehicle fleet. The admissible term of accumulation of garbage in tanks according to sanitary norms depends on the category of garbage. For organic waste, for example, it should not exceed 24 hours, which 72 hours is permissible for other types of solid waste. Thus, the data in Table 3 make it possible to determine the number of containers that will be ready for unloading for each route planning period. Moreover, containers do not need to be 100% filled. When we applied the proposed algorithm, the need to use a distance matrix covering the entire transport network of the settlement (all 240 container sites and depots) disappeared. The shortest routes were constructed by choosing the points at which the a priori loading of containers was close to $v_{i.\max}$. Based on the data in Table 3 and the parameters of the garbage truck fleet, the initial size of the matrix A_0 varied from 12 to 72 points, with one to six vehicles involved in the process.

5. ANALYSIS OF THE RESULTS

As a result of the search, guaranteed optimal solutions were found for the configuration of routes with restrictions on their maximum carrying capacities. The influence of route organization on the achievement of indicators of utility company MVs based on the observation data was modeled for the period of April-May of the current year. It was established that, during this period, the intensity of MSW accumulation at container sites remained approximately constant. For the selected number of transport points, routing was performed with a variable number of vehicles. In this case, guaranteed solutions to the formulated problem were obtained. Fig. 3 shows examples of the results.

```
n=14 m=1
Roate: 1 – 7 – 8 – 2 – 4 – 13 – 11 – 12 – 3 – 5 – 6 – 9 – 10 – 14
Better Solution = 31.5 km
 Vehicles loading
Veh #1 - 30.90
Vehicle is overloaded
```

a

```
n=14 m=2
Roate:1 – 4 – 5 – 2 – 3 – 10 – 8 – 9 – 14 – 6 – 7 – 11 – 13 – 14 – 12 – 15
Better Solution = 27.2 km
 Vehicles loading
Veh #1 - 13.60
Veh #2 - 17.30
```

b

```
n=14 m=3
Roate:1 – 4 – 5 – 2 – 3 – 10 – 8 – 9 – 14 – 6 – 7 – 13 – 15 – 11 – 12 – 16
Better Solution = 30.5 km
 Vehicles loading
Veh #1 - 13.60
Veh #2 - 11.20
Veh #3 - 6.10
```

c

```
n=14 m=4
Roate: 1 – 4 – 5 – 2 – 3 – 10 – 8 –14 – 12 – 7 – 16 – 6 – 13 – 15 – 9 – 11 – 17
Better Solution = 30.3 km
 Vehicles loading
Veh #1 - 12.50
Veh #2 - 6.20
Veh #3 - 6.20
Veh #4 - 6.00
```

d

Fig. 3. Examples of the results of the search for the shortest routes over the network with n=12 points for different numbers of involved vehicles (m): a – 1; b – 2; c – 3; d – 4; *Better Solution* – the total length of the run, km; Vehicles loading – loading of a vehicle, containers (1 container = 1.1 m^3); depot in bold

Initially, it was known that there are 36 containers filled with different volumes at 12 container yards. In total, they contain 34 m^3 of solid waste. A volume of garbage of 30.9 m^3 must be collected and removed in one day to ensure sanitary conditions.. When starting the program, different numbers of MVs were set to collect such a volume of garbage. If the actual carrying capacity of a truck was exceeded as a result of the forced execution of the simulated route, then the program generated an error: *Vehicle is overloaded*. Otherwise, the load of each vehicle was calculated, routes were constructed, and the criterion was calculated.

As can be seen from the above example, involving different numbers of vehicles in transportation led to different results. The results for a two-month period were summarized in relation to the initially obtained data of the metropolis. Then, the following dependencies were constructed (Fig. 4).

As follows from the dependencies, vehicles cannot be rationally loaded for all route configurations (without overloads and without underloads). Thus, with n=12 or n=24, m=6 trucks cannot be used due to inefficient underloading; with n=72, at least m=4 MVs can accommodate the transportation volume.

It can also be seen that the involved amount of MV has an optimal value with a constant number of shipping points and the amount of garbage to be transported. Therefore, if there is an insufficient number of trucks, their total mileage will grow due to the great complexity of the configuration of routes in the metropolis, which, in turn, is associated with the transport scheme of the road network. If the number of trucks exceeds the optimal number, then the total mileage of vehicles will increase due to the large proportion of idle runs.

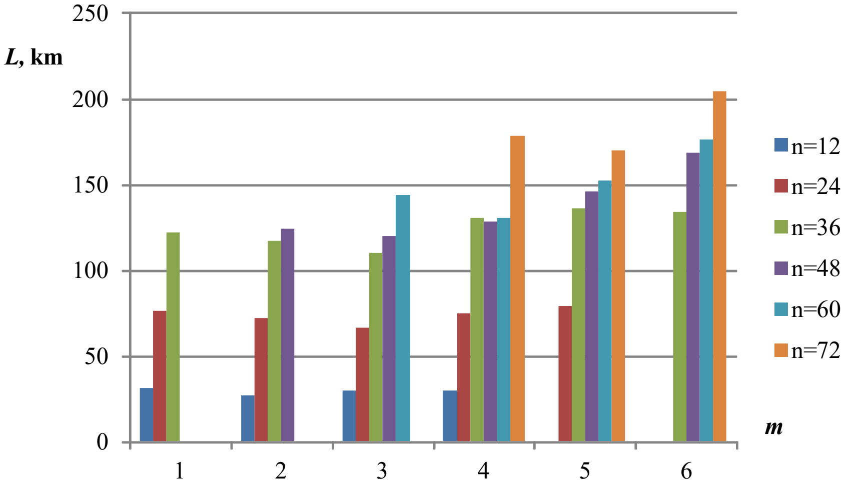

Fig. 4. Dependence of the total mileage of routes based on the number of involved vehicles (m) and the number of points on the routes

6. CONCLUSIONS

Based on the results of the research and the tests of the developed routing algorithm, we came to the following conclusions.

1. The alphabetic algorithm, which was applied to search for the shortest routes for a set of vehicles, makes it possible to find a guaranteed exact solution with an acceptable search duration with the help of a developed computer program. In this way, we managed to route four out of six available vehicles while serving the transport network, which consists of 240 points. The duration of calculations did not exceed a few seconds. The best solution to the problem made it possible to obtain the total average daily mileage on the routes within 27.31 km, which is one-third shorter than the current situation when heuristic routing methods are used. However, the alphabetic algorithm can lead to excessive consumption of computational resources if there are no restrictions on the input data and the field of possible results. Calculations are particularly complex for such input data (distance matrices) that are characterized by low unevenness.Thus, using a priori information about the likely level of filling of garbage containers, the routing of garbage collection from the specified 72 container sites with 240 containers in the city can be reduced to 12.
2. Applying restrictions on the maximum and minimum load in the routing algorithm also makes it possible to reduce the required initial volume of input data. Such restrictions also reduce the number of route selection options.
3. When a different number of vehicles are involved in the same volume of transport work to collect and dispose of solid waste, there is an optimal route scheme for which the total mileage on the road network of the metropolis will be minimal.

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