Guze Sambor

Kołowrocki Krzysztof
Maritime University, Gdynia, Poland

Reliability analysis of multi-state ageing consecutive „*k* out of *n*: F" systems

Keywords

multi-state system, ageing system, consecutive "k out of n: F" system, reliability

Abstract

A multi-state approach to reliability analysis of systems composed of ageing components is introduced and basic reliability characteristics for such systems are defined. Further, a multi-state consecutive "k out of n: F" system composed of ageing components is defined and the recurrent formulae for its reliability function are proposed. Moreover, the application of the proposed reliability characteristics and formulae to reliability evaluation of the steel cover composed of ageing sheets is illustrated.

1. Introduction

Taking into account the importance of the safety and operating process effectiveness of technical systems it seems reasonable to expand the two-state approach to multi-state approach in their reliability analysis. The assumption that the systems are composed of multi-state components with reliability states degrading in time [4]-[5], [10] gives the possibility for more precise analysis and diagnosis of their reliability and operational processes' effectiveness. This assumption allows us to distinguish a system reliability critical state to exceed which is either dangerous for the environment or does not assure the necessary level of its operational process effectiveness. Then, an important system safety characteristic is the time to the moment of exceeding the system reliability critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state reliability function that is a basic characteristic of the multi-state system. The main results determining the multi-state reliability functions and the risk functions of typical series, parallel, series-parallel, parallel-series, series-"*k* out of *n*" and "*k* out of *n*"- series systems with ageing components are given in [4]-[5]. The paper is devoted to transmitting these results on the multi-state ageing consecutive "k out of n: F" systems [1], [2]-[3], [6], [7]-[8], [9].

2. Multi-state system with ageing components

In the multi-state reliability analysis to define systems with degrading components we assume that [4]-[5], [10]:

- E_i, $i = 1,2,...,n,$ are components of a system,
- all components and a system under consideration have the reliability state set $\{0,1,...,z\}$, $z \geq 1$,
- the state indexes are ordered, the state 0 is the worst and the state *z* is the best,
- $T_i(u)$, $i = 1,2,...,n,$ are independent random variables representing the lifetimes of components E_i in the state subset $\{u,u+1,...,z\}$, while they were in the state *z* at the moment $t = 0$,
- $T_i(u)$, is a random variable representing the lifetime of a system in the state subset $\{u,u+1,...,z\}$ while it was in the state *z* at the moment $t = 0$,
- the system state degrades with time *t* without repair,
- $e_i(t)$ is a component E_i state at the moment *t*, $t \geq 0$,
- $s(t)$ is a system state at the moment *t*, $t \geq 0$.

The above assumptions mean that the reliability states of the system with degrading components may be changed in time only from better to worse. The way in which the components and the system reliability states change is illustrated in *Figure 1*.

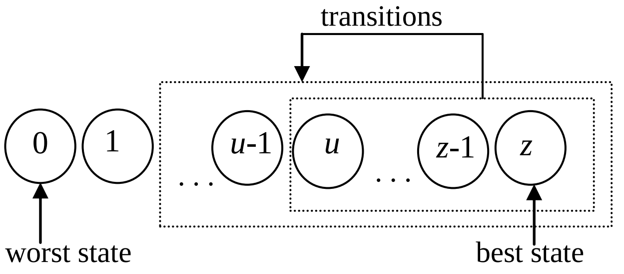

Figure 1. Illustration of reliability states changing in system with ageing components

The basis of our further consideration is a system component reliability function defined as follows.

Definition 1. A vector

$$R_j(t,\cdot) = [R_i(t,0), R_i(t,1), ..., R_i(t,z)],\ t \geq 0,$$

where

$$R_i(t,u) = P(e_i(t) \geq u \mid e_i(0) = z) = P(T_i(u) > t)$$

for $t \geq 0$, $u = 0,1,...,z$, $i = 1,2,...,n$, is the probability that the component E_i is in the reliability state subset $\{u, u+1, ..., z\}$ at the moment t, $t \geq 0$, while it was in the reliability state z at the moment $t = 0$, is called the multi-state reliability function of a component E_i.

Similarly, we can define a multi-state system reliability function.

Definition 2. A vector

$$\boldsymbol{R}_n(t,\cdot) = [1, \boldsymbol{R}_n(t,0), \boldsymbol{R}_n(t,1), ..., \boldsymbol{R}_n(t,z)],\ t \geq 0,$$

where

$$\boldsymbol{R}_n(t,u) = P(s(t) \geq u \mid s(0) = z) = P(T(u) > t),$$

for $t \geq 0$, $u = 0,1,...,z$, is the probability that the system is in the reliability state subset $\{u, u+1, ..., z\}$ at the moment t, $t \geq 0$, while it was in the reliability state z at the moment $t = 0$, is called the multi-state reliability function of a system.

Under this definition we have

$$\boldsymbol{R}_n(t,0) \geq \boldsymbol{R}_n(t,1) \geq \ldots \geq \boldsymbol{R}_n(t,z),\ t \geq 0,$$

and if

$$p(t) = [p(t,0), p(t,1), ..., p(t,z)],\ t \geq 0,$$

where

$$p(t,u) = P(s(t) = u \mid s(0) = z),$$

for $t \geq 0$, $u = 0,1,...,z$, is the probability that the system is in the state u at the moment t, $t \geq 0$, while it was in the state z at the moment $t = 0$, then

$$\boldsymbol{R}_n(t,0) = 1,\ \boldsymbol{R}_n(t,z) = p(t,z),\ t \geq 0, \tag{1}$$

and

$$p(t,u) = \boldsymbol{R}_n(t,u) - \boldsymbol{R}_n(t,u+1),\ u = 0,1,...,z-1,\ t \geq 0. \tag{2}$$

Moreover, if

$$\boldsymbol{R}_n(t,u) = 1 \text{ for } t < 0,\ u = 1,2,...,z,$$

then

$$M(u) = E[T(u)] = \int_0^\infty \boldsymbol{R}_n(t,u)dt,\ u = 1,2,...,z, \tag{3}$$

is the mean lifetime of the system in the state subset $\{u, u+1, ..., z\}$,

$$\sigma(u) = \sqrt{D[T(u)]} = \sqrt{N(u) - [M(u)]^2},\ u = 1,2,...,z, \tag{4}$$

where

$$N(u) = 2\int_0^\infty t\,\boldsymbol{R}_n(t,u)dt,\ u = 1,2,...,z, \tag{5}$$

is the standard deviation of the system lifetime in the state subset $\{u, u+1, ..., z\}$ and moreover

$$\overline{M}(u) = \int_0^\infty p(t,u)dt,\ u = 1,2,...,z, \tag{6}$$

is the mean lifetime of the system in the state u while the integrals (3), (4) and (5) are convergent.
Additionally, according to (1), (2), (3) and (6), we get the following relationships

$$\overline{M}(u) = M(u) - M(u+1),\ u = 1,2,...,z-1, \tag{7}$$

$$\overline{M}(z) = M(z).$$

Close to the multi-state system reliability function its basic characteristic is the system risk function defined as follows.

Definition 3. A probability

$$\boldsymbol{r}(t) = P(s(t) < r \mid s(0) = z) = P(T(r) \le t),\ t \ge 0,$$

that the system is in the subset of states worse than the critical state r, $r \in \{1,...,z\}$ while it was in the reliability state z at the moment $t = 0$ is called a risk function of the multi-state system.

Considering *Definition 3* and *Definition 2*, we have

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R_n}(t, r),\ t \ge 0, \qquad (8)$$

and if τ is the moment when the system risk function exceeds a permitted level δ, then

$$\tau = \boldsymbol{r}^{-1}(\delta), \qquad (9)$$

where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}(t)$.

3. Reliability of a multi-state ageing consecutive „*k* out of *n*: F" system

Definition 4. A multi-state system is called an ageing consecutive "k out of n: F" system if it is out of the reliability state subset $\{u,u+1,...,z\}$ if and only if at least its k neighbouring components out of n its components arranged in a sequence of E_1, E_2, ..., E_n, are out of this reliability state subset.

In our further analysis, we denote by $s_{k,n}(t)$ the reliability state of the ageing consecutive "k out of n: F" system at the moment t, $t \in <0,\infty)$, and by $T_{k,n}(u)$ the lifetime of this system in the reliability subset $\{u,u+1,...,z\}$. Moreover, we denote by

$$\boldsymbol{R}_{k,n}(t,u) = P(s_{k,n}(t) \ge u \mid s(0) = z) = P(T_{k,n}(u) > t)$$

for $t \ge 0$, $u = 0,1,...,z$, the probability that the ageing consecutive "k out of n: F" system is in the reliability state subset $\{u, u+1,...,z\}$ at the moment t, $t \ge 0$, while it was in the reliability state z at the moment $t = 0$ and by

$$\boldsymbol{F}_{k,n}(t,u) = 1 - \boldsymbol{R}_{k,n}(t,u) = P(T_{k,n}(u) \le t)$$

for $t \ge 0$, $u = 0,1,...,z$, the distribution function of the lifetime $T_{k,n}(u)$ of this system in the reliability state subset $\{u,u+1,...,z\}$ while it was in the state z at the moment $t = 0$.

Theorem 1. The reliability function of the ageing consecutive "k out of n: F" system composed of components with independent failures is given by the following recurrent formula

$$\boldsymbol{R}_{k,n}(t,\cdot) = [1, \boldsymbol{R}_{k,n}(t,1), \boldsymbol{R}_{k,n}(t,2), ..., \boldsymbol{R}_{k,n}(t,z)],$$

where

$$\boldsymbol{R}_{k,n}(t,u) = \begin{cases} 1 & \text{for } n < k, \\ 1 - \prod F_j(t,u) & \text{for } n = k, \\ R_n(t,u)\boldsymbol{R}_{k,n-1}(t,u) & \\ + \sum_{i=1}^{k-1} R_{n-i}(t,u)\boldsymbol{R}_{k,n-i-1}(t,u) & \\ \cdot \prod_{j=n-i+1}^{n} F_j(t,u) & \text{for } n > k, \end{cases} \qquad (10)$$

for $t \in <0,\infty>$, $u = 1,2,...,z$.

Motivation. Since for each fixed u, $u = 1,2,...,z$, the assumptions of this theorem as the same as the assumptions of *Theorem 2* proved in [2] and the formula (10) is equivalent with the formula (12) from [2], then after considering *Definition 4*, we conclude that this theorem is valid.

From the above theorem, as a particular case for the system composed of components with identical reliability, we immediately get the following corollary.

Corollary 1. If components of the ageing consecutive "k out of n: F" system are independent and have identical reliability functions, i.e.

$$R_i(t,u) = R(t,u),\ F_i(t,u) = F(t,u) \text{ for } t \in <0,\infty),$$
$$u = 1,2,...,z,\ i = 1,2,...,n,$$

then the reliability function of this system is given by

$$\boldsymbol{R}_{k,n}(t,\cdot) = [1, \boldsymbol{R}_{k,n}(t,1), \boldsymbol{R}_{k,n}(t,2), ..., \boldsymbol{R}_{k,n}(t,z)],$$

where

$$\boldsymbol{R}_{k,n}(t,u) = \begin{cases} 1 & \text{for } n < k, \\ 1-[F(t,u)]^n & \text{for } n = k, \\ R(t,u)\boldsymbol{R}_{k,n-1}(t,u) & \\ + R(t,u)\sum_{i=1}^{k-1} F^i(t,u) & \\ \cdot \boldsymbol{R}_{k,n-i-1}(t,u) & \text{for } n > k, \end{cases} \quad (11)$$

for $t \in <0,\infty)$, $u = 1,2,...,z$.

From *Corollary 1*, in a particular case, substituting $k = 2$ in (11), we get:

- for $n = 1$

$$\boldsymbol{R}_{2,1}(t,\cdot) = [1, \boldsymbol{R}_{2,1}(t,1), \boldsymbol{R}_{2,1}(t,2), ..., \boldsymbol{R}_{2,1}(t,z)], \quad (12)$$

where

$$\boldsymbol{R}_{2,1}(t,u) = 1 \text{ for } t \in <0,\infty), \ u = 1,2,...,z, \quad (13)$$

- for $n = 2$

$$\boldsymbol{R}_{2,2}(t,\cdot) = [1, \boldsymbol{R}_{2,2}(t,1), \boldsymbol{R}_{2,2}(t,2), ..., \boldsymbol{R}_{2,2}(t,z)], \quad (14)$$

where

$$\boldsymbol{R}_{2,2}(t,u) = 1 - F^2(t,u) \text{ for } t \in <0,\infty), \quad (15)$$
$$u = 1,2,...,z,$$

- for $n \geq 3$

$$\boldsymbol{R}_{2,n}(t,\cdot) = [1, \boldsymbol{R}_{2,n}(t,1), \boldsymbol{R}_{2,n}(t,2), ..., \boldsymbol{R}_{2,n}(t,z)], \quad (16)$$

where

$$\boldsymbol{R}_{2,n}(t,u) = R(t,u)\ \boldsymbol{R}_{2,n-1}(t,u)$$

$$+ R(t,u)F(t,u)\ \boldsymbol{R}_{2,n-2}(t,u) \text{ for } t \in <0,\infty), \quad (17)$$
$$u = 1,2,...,z.$$

4. Application

Example 1. Let us consider the steel cover composed of $n = 24$ arranged identical sheets $E_1, E_2, ..., E_{24}$. We assume that $z = 4$, i.e. the cover and the sheets it is composed of may be in the one of the reliability states from the set $\{0,1,2,3,4\}$. The cover is out of the reliability state subset $\{u, u+1, ..., 4\}$ if at least $k = 2$ of its neighbouring sheets is out of this reliability state subset. If the considered steel cover critical reliability state is $r = 2$, then this steel cover is failed if at least 2 neighbouring sheets from 24 sheets are out of the reliability state subset $\{2,3,4\}$. Thus, the considered steel cover is a five-state ageing consecutive "2 out of 24: F" system, and according to (16)-(17), its the reliability function is given by

$$\boldsymbol{R}_{2,24}(t,\cdot) =$$

$$[1, \boldsymbol{R}_{2,24}(t,1), \boldsymbol{R}_{2,24}(t,2), \boldsymbol{R}_{2,24}(t,3), \boldsymbol{R}_{2,24}(t,4)], \quad (18)$$

where

$$\boldsymbol{R}_{2,24}(t,u) = R(t,u)\ \boldsymbol{R}_{2,23}(t,u)$$

$$+ R(t,u)F(t,u)\ \boldsymbol{R}_{2,22}(t,u) \text{ for } t \in <0,\infty), \quad (19)$$
$$u = 1,2,3,4.$$

In the particular case when the lifetimes $T_i(u)$, $u = 1,2,3,4,$ of the sheets E_i, $i = 1,2,3,4,5,$ in the reliability state subsets have Weibull distributions of the form

$$F(t,u) = 1 - e^{-\lambda(u)t^2} \text{ for } t \geq 0, \ u = 1,2,3,4,$$

where

$$\lambda(1) = 0.01, \ \lambda(2) = 0.02, \ \lambda(3) = 0.05, \ \lambda(4) = 0.10,$$

i.e. if the reliability function of the sheets E_i, $i = 1,2,3,4,5,$ is given by

$$R(t,\cdot) = [1, R(t,1), R(t,2), R(t,3), R(t,4)], \ t \in <0,\infty),$$

where

$$R(t,1) = e^{-0.01t^2}, \ R(t,2) = e^{-0.02t^2}, \ R(t,3) = e^{-0.05t^2},$$

$$R(t,4) = e^{-0.10t^2} \text{ for } t \geq 0,$$

considering (12)-(19), we get the following recurrent formula for the cover reliability

$$\boldsymbol{R}_{2,24}(t,\cdot) =$$

$$[1, \boldsymbol{R}_{2,24}(t,1), \boldsymbol{R}_{2,24}(t,2), \boldsymbol{R}_{2,24}(t,3), \boldsymbol{R}_{2,24}(t,4)], \quad (20)$$

where

- $\boldsymbol{R}_{2,24}(t,1)$ is determined by the formulae

$$\boldsymbol{R}_{2,1}(t,1) = 1 \text{ for } t \in <0,\infty), \quad (21)$$

$$\boldsymbol{R}_{2,2}(t,1) = 1 - [1 - e^{-0.01t^2}]^2 \text{ for } t \in <0,\infty), \quad (22)$$

$$\boldsymbol{R}_{2,n}(t,1) = e^{-0.01t^2}\ \boldsymbol{R}_{2,n-1}(t,1)$$

$$+ e^{-0.01t^2}\ [1 - e^{-0.01t^2}]\ \boldsymbol{R}_{2,n-2}(t,1) \text{ for } t \in <0,\infty), \quad (23)$$

$n = 3,4,\ldots,24,$

- $\boldsymbol{R}_{2,24}(t,2)$ is determined by the formulae

$$\boldsymbol{R}_{2,1}(t,2) = 1 \text{ for } t \in <0,\infty), \quad (24)$$

$$\boldsymbol{R}_{2,2}(t,2) = 1 - [1 - e^{-0.02t^2}]^2 \text{ for } t \in <0,\infty), \quad (25)$$

$$\boldsymbol{R}_{2,n}(t,2) = e^{-0.02t^2}\ \boldsymbol{R}_{2,n-1}(t,2)$$

$$+ e^{-0.02t^2}\ [1 - e^{-0.02t^2}]\ \boldsymbol{R}_{2,n-2}(t,2) \text{ for } t \in <0,\infty), \quad (26)$$

$n = 3,4,\ldots,24,$

- $\boldsymbol{R}_{2,24}(t,3)$ is determined by the formulae

$$\boldsymbol{R}_{2,1}(t,3) = 1 \text{ for } t \in <0,\infty), \quad (27)$$

$$\boldsymbol{R}_{2,2}(t,3) = 1 - [1 - e^{-0.05t^2}]^2 \text{ for } t \in <0,\infty), \quad (28)$$

$$\boldsymbol{R}_{2,n}(t,3) = e^{-0.05t^2}\ \boldsymbol{R}_{2,n-1}(t,3)$$

$$+ e^{-0.05t^2}\ [1 - e^{-0.05t^2}]\ \boldsymbol{R}_{2,n-2}(t,3) \text{ for } t \in <0,\infty), \quad (29)$$

$n = 3,4,\ldots,24,$

- $\boldsymbol{R}_{2,24}(t,4)$ is determined by the formulae

$$\boldsymbol{R}_{2,1}(t,4) = 1 \text{ for } t \in <0,\infty), \quad (30)$$

$$\boldsymbol{R}_{2,2}(t,4) = 1 - [1 - e^{-0.10t^2}]^2 \text{ for } t \in <0,\infty), \quad (31)$$

$$\boldsymbol{R}_{2,n}(t,4) = e^{-0.10t^2}\ \boldsymbol{R}_{2,n-1}(t,4)$$

$$+ e^{-0.10t^2}\ [1 - e^{-0.10t^2}]\ \boldsymbol{R}_{2,n-2}(t,4) \text{ for } t \in <0,\infty), \quad (32)$$

$n = 3,4,\ldots,24.$

The values of the particular vector components of the multi-state reliability function of the steel cover given by (20), calculated by the computer programme based on the formulae (21)-(32), are presented in the *Tables 1-4* and illustrated in *Figure 1*. As earlier we have assumed that $r = 2$ is the cover critical reliability state, then according to (8) and (26) its risk function is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{R}_{2,24}(t,2) = 1 - e^{-0.02t^2}\ \boldsymbol{R}_{2,23}(t,2)$$

$$- e^{-0.02t^2}\ [1 - e^{-0.02t^2}]\ \boldsymbol{R}_{2,22}(t,2) \text{ for } t \in <0,\infty). \quad (33)$$

The values of the steel cover risk function are given in *Table 5* and illustrated in *Figure 2*.

Table 1. The values of the steel cover multi-state reliability function vector component $u = 1$

t	$\boldsymbol{R}_{2,24}(t,1)$	$2t\ \boldsymbol{R}_{2,24}(t,1)$
0.0	1.0000	0.0000
1.0	0.9978	1.9955
2.0	0.9664	3.8657
3.0	0.8531	5.1183
4.0	0.6362	5.0889
5.0	0.3750	3.7499
6.0	0.1664	1.9957
7.0	0.0538	0.7534
8.0	0.0125	0.2001
9.0	0.0021	0.0374
10.0	0.0002	0.0049

Table 2. The values of the steel cover multi-state reliability function vector component $u = 2$

t	$\boldsymbol{R}_{2,24}(t,2)$	$2t\ \boldsymbol{R}_{2,24}(t,2)$
0.0	1.0000	0.0000
0.5	0.9994	0.9994
1.0	0.9912	1.9824
1.5	0.9580	2.8742
2.0	0.8802	3.5207
2.5	0.7479	3.7398
3.0	0.5731	3.4388
3.5	0.3876	2.7131
4.0	0.2275	1.8200
4.5	0.1145	1.0307
5.0	0.0491	0.4905
5.5	0.0178	0.1958
6.0	0.0055	0.0655
6.5	0.0014	0.0184
7.0	0.0003	0.0044

Table 3 The values of the steel cover multi-state reliability function vector component $u = 3$

t	$\boldsymbol{R}_{2,24}(t,3)$	$2t\ \boldsymbol{R}_{2,24}(t,3)$
0.0	1,0000	0,0000
0.2	0.9999	0.3999
0.4	0.9986	0.7988
0.6	0.9928	1.1914
0.8	0.9781	1.5649
1.0	0.9489	1.8978
1.2	0.9005	2.1613
1.4	0.8302	2.3246
1.6	0.7385	2.3632
1.8	0.6299	2.2675
2.0	0.5122	2.0489
2.2	0.3953	1.7392
2.4	0.2883	1.3837
2.6	0.1980	1.0298
2.8	0.1278	0.7158
3.0	0.0774	0.4642
3.2	0.0438	0.2806
3.4	0.0233	0.1581
3.6	0.0115	0.0830
3.8	0.0053	0.0406
4.0	0.0023	0.0185

Table 4. The values of the steel cover multi-state reliability function vector component $u = 4$

t	$\boldsymbol{R}_{2,24}(t,4)$	$2t\ \boldsymbol{R}_{2,24}(t,4)$
0.0	1.0000	0.0000
0.1	0.9999	0.0399
0.2	0.9996	0.1599
0.3	0.9982	0.3593
0.4	0.9943	0.6364
0.5	0.9864	0.9864
0.6	0.9725	1.4004
0.7	0.9508	1.8636
0.8	0.9195	2.3540
0.9	0.8775	2.8433
1.0	0.8244	3.2975
1.1	0.7605	1.6731
1.2	0.6875	1.6499
1.3	0.6076	1.5799
1.4	0.5242	1.4677
1.5	0.4406	1.3217
1.6	0.3602	1.1528
1.7	0.2862	0.9731
1.8	0.2207	0.7944
1.9	0.1650	0.6269
2.0	0.1195	0.4779
2.1	0.0838	0.3519
2.2	0.0569	0.2502
2.3	0.0373	0.1718
2.4	0.0237	0.1138
2.5	0.0146	0.0728
2.6	0.0086	0.0450
2.7	0.0050	0.0268
2.8	0.0028	0.0154
2.9	0.0015	0.0086
3.0	0.0008	0.0046

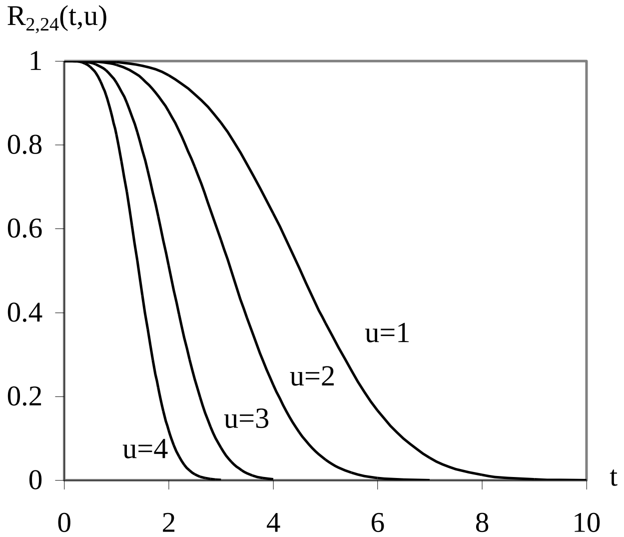

Figure 1. The graphs of the steel cover multi-state reliability function vector components

Table 5. The values of the steel cover multi-state reliability function vector component $u = 2$ and its risk function

t	$\boldsymbol{R}_{2,24}(t,2)$	$\boldsymbol{r}(t)$ = 1- $\boldsymbol{R}_{2,24}(t, 2)$
0.0	1.0000	0.0000
0.5	0.9994	0.0006
1.0	0.9912	0.0088
1.5	0.9581	0.0419
2.0	0.8802	0.1198
2.5	0.7480	0.2520
3.0	0.5731	0.4269
3.5	0.3876	0.6124
4.0	0.2275	0.7725
4.5	0.1145	0.8855
5.0	0.0490	0.9510
5.5	0.0178	0.9822
6.0	0.0055	0.9945
6.5	0.0014	0.9986
7.0	0.0003	0.9997

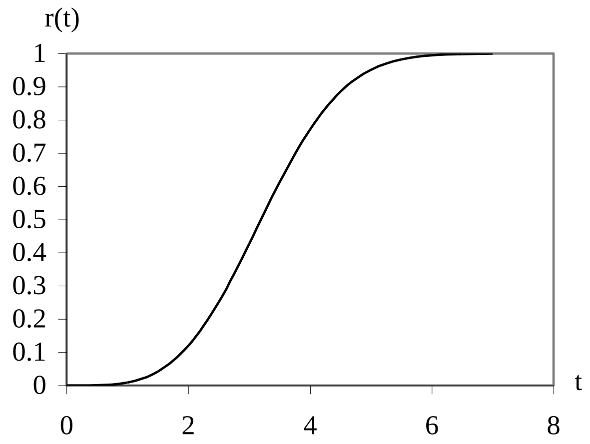

Figure 2. The graphs of the steel cover risk function

Using the values given in these *Tables 1-4*, the formulae (3)-(7) and numerical integration we find:
- the mean values of the cover lifetimes in the reliability state subsets

$$M(1) = E[T_{2,24}(1)] = \int_0^\infty \boldsymbol{R}_{2,24}(t,1)dt \cong 4.5634,$$

$$M(2) = E[T_{2,24}(2)] = \int_0^\infty \boldsymbol{R}_{2,24}(t,2)dt \cong 3.2268,$$

$$M(3) = E[T_{2,24}(3)] = \int_0^\infty \boldsymbol{R}_{2,24}(t,3)dt \cong 2.0408,$$

$$M(4) = E[T_{2,24}(4)] = \int_0^\infty \boldsymbol{R}_{2,24}(t,4)dt \cong 1.4431,$$

- the second ordinary moments of the cover lifetimes in the reliability state subsets

$$N(1) = E[T_{2,24}^2(1)] = 2\int_0^\infty t\,\boldsymbol{R}_{2,24}(t,1)dt \cong 22.9715,$$

$$N(2) = E[T_{2,24}^2(2)] = 2\int_0^\infty t\,\boldsymbol{R}_{2,24}(t,2)dt \cong 11.4879,$$

$$N(3) = E[T_{2,24}^2(3)] = 2\int_0^\infty t\,\boldsymbol{R}_{2,24}(t,3)dt \cong 4.5944,$$

$$N(4) = E[T_{2,24}^2(4)] = 2\int_0^\infty t\,\boldsymbol{R}_{2,24}(t,4)dt \cong 2.2967,$$

- the standard deviations of the cover lifetimes in the reliability state subsets

$$\sigma(1) = \sqrt{N(1) - [M(1)]^2} \cong 1.4651,$$

$$\sigma(2) = \sqrt{N(2) - [M(2)]^2} \cong 1.0370,$$

$$\sigma(3) = \sqrt{N(3) - [M(3)]^2} \cong 0.6553,$$

$$\sigma(4) = \sqrt{N(4) - [M(4)]^2} \cong 0.4628,$$

- the mean values of the cover lifetimes in the reliability particular states

$$\overline{M}(1) = M(1) - M(2) \cong 4.5634 - 3.2268 = 1.3366,$$

$$\overline{M}(2) = M(2) - M(3) \cong 3.2268 - 2.0408 = 1.1860,$$

$$\overline{M}(3) = M(3) - M(4) \cong 2.0408 - 1.4431 = 0.5977,$$

$$\overline{M}(4) = M(4) \cong 1.4431.$$

Using the values given in these *Tables 5* and the formula (9) we find the approximate value of the moment when the system risk function exceeds an exemplary permitted level $\delta = 0.05$, namely

$$\tau = \boldsymbol{r}^{-1}(0.05) \cong 1.58.$$

5. Conclusion

Two recurrent formulae for multi-state reliability functions, a general one for non-homogeneous and its simplified form for homogeneous multi-state consecutive "k out of n: F" systems composed of ageing components have been proposed. The formulae for multi-state reliability function of a homogeneous multi-state consecutive "k out of n: F" system has been applied to reliability evaluation of the steel cover composed of ageing components. The considered steel cover was a five-state ageing consecutive "2 out of 24: F" system composed of components with Weibull reliability functions. On the basis of the recurrent formula for steel cover multi-state reliability function the approximate values of its vector components have been calculated and presented in tables and illustrated graphically. On the basis of these vales the mean values and standard deviations of the steel cover lifetimes in the reliability state subsets and the mean values of the steel cover lifetimes in particular reliability states have been estimated. Moreover, the cover risk function and the moment when the risk function exceeds the permitted risk level have been determined.
The input structural and reliability data of the considered steel cover have been assumed arbitrarily and therefore the obtained its reliability characteristics evaluations should be only treated as an illustration of the possibilities of the proposed methods and solutions. The proposed methods and solutions and the software are general and they may be applied to any multi-state consecutive "*k* out of *n*: F" system of ageing components.

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