

THE APPLICATION OF GENETIC ALGORITHM FOR WAREHOUSE LOCATION IN LOGISTIC NETWORK

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Abstract

The paper presents a certain approach to solving the problem of warehouse locations based on the application of a genetic algorithm. The main objective is to indicate a location of warehouses from among those that already exist, which are most likely to assure the best value of the adopted criterion function, concurrently meeting recipients' needs. A formal notation was presented of the mathematical model, allowing for the indispensable data, decision-related variables of limitations and the criterion function. The problem is to determine the location of warehouses taking into account minimising costs of transport and storage of forwarded cargo between determined facilities within the network. To allow solving the problem related to warehouse location within the logistics network of a manufacturing enterprise the use of a genetic algorithm was proposed. The structure of the algorithm was adapted to the mathematical model. A genetic algorithm was used to determine the volume of cargo stream flows between particular facilities in the network. To form a genetic algorithm it is advisable to define the chromosome structure, the adaptation function, cross-linking process and mutation. The location problem is solved in such a way that if for any day within the analysed period flows take place from a given warehouse or to a given warehouse, such a logistics facility should constitute an element of the target logistics network. If there are no such flows, no warehouse is necessary in the given location.

Keywords: *warehouse location, genetic algorithms, logistics network*

1. Introduction

The problem of warehouse location is an issue that is commonly known and extensively defined in the context of decision-making problems [2, 3, 8, 16]. The classical issue of warehouse location is defined in literature as the "capacitated warehouse location problem" CWLP [1, 3]). In this class of problems, the structure of the transport and logistics network consists of warehouse

facilities and customers. The main objective is to find such a location of warehouse facilities that the costs arising from forwarding of a specific volume of goods to the customers are as low as possible. Limitations arise from satisfying the needs of all customers and the capacitive limit for the dispatch of goods from the given warehouse facility.

The problem of warehouse situation may be generally reviewed from two basic viewpoints. The first one concerns selecting the place where a new warehouse is to be erected [1]. Basically in such a case potential locations for the construction of a new warehouse are determined. The second aspect consists in a review of the selection of warehouse locations from the already existing facilities, which points to a reconfiguration of the logistic network for various types of enterprises [6, 7].

A review of literature sources [2-4, 6] allows the presumption that main factors that determine the selection of warehouse location comprise the following: labour costs in areas of warehouse location, storage and transport costs, required time for order implementation (service level), existing infrastructure (paved roads, railway lines, inland water routes, airports), distance from supply markets and customers, local taxes and stimuli for regional development, possibilities of acquiring building structures or other land property, construction of parking squares etc.

The presented factors emphasise the multi-criteria aspect of issues connected with warehouse location and impose the necessity of adopting appropriate algorithms of multi-criteria in decision-making assistance. Multi-criteria decision-making assistance [14, 17] is aimed at providing required tools to the decision maker to enable solving complex decision-related problems allowing for numerous viewpoints, which are as a rule contradictory, such as the MAJA algorithm [9]. This algorithm is used to solve single-facility location problems, which make it possible to define a set of admissible variants. The appraisal of location variants may take into consideration numerous criteria of a fixed weight.

The paper proposed a genetic algorithm that solves the problem of warehouse facilities in the transport and logistic network of a manufacturing plant. The structure of the algorithm has been adapted to the sample mathematical model that determines the location of warehouses with view to minimum transport and storage costs of the cargo stream flowing between particular facilities in the network. Genetic algorithms are a known optimising tool in literature, which is successfully used in difficult decision-making problems, e.g. for the determination of routes for vehicles [5], [11] or location of reloading points [15].

2. Mathematical model of warehouse facility location

The analysis comprises the logistic network of the manufacturing plant. The network structure comprises suppliers, warehouses and manufacturing plants. Suppliers can supply raw materials directly or indirectly through warehouses to manufacturing plants. The main objective is to indicate a location of warehouses from among those that already exist, which are most likely to assure the best value of the adopted criterion function, concurrently meeting recipients' needs. In the article, the function of the criterion minimises the cost of transport and passage of raw materials via a warehouse. Selection of warehouses location is carried out based on the existing facilities with known location within the network.

The following definitions have been formulated for the needs of establishing a location model:

- $V = \{v: v = 1, 2, \dots, v', \dots, V\}$ – set of numbers of spot elements of the logistical network: suppliers, warehouses, manufacturing plants,
- $T = \{t: t = 1, 2, \dots, t', \dots, T\}$ – set of numbers of working days,
- $DS = \{v: \alpha(v) = 0 \text{ for } v \in V\}$ – set of numbers of suppliers,
- $MS = \{v: \alpha(v) = 1 \text{ for } v \in V\}$ – set of numbers of warehouses,
- $P = \{v: \alpha(v) = 2 \text{ for } v \in V\}$ – set of numbers of manufacturing plants,
- $D1 = [d1(v, v'): d1(v, v') \in \mathbf{R}^+, v \in DS, v' \in MS]$ – distance matrices in relations: suppliers –

warehouses,

- $\mathbf{D2} = [d2(v, v') : d2(v, v') \in \mathbf{R}^+, v \in \mathbf{DS}, v' \in \mathbf{P}]$ – distance matrices in relations: suppliers – enterprises,
- $\mathbf{D3} = [d3(v, v') : d3(v, v') \in \mathbf{R}^+, v \in \mathbf{MS}, v' \in \mathbf{P}]$ – distance matrices in relations: warehouses – enterprises,
- $\mathbf{Q1} = [q1(v)]$ – vector of the volume of deliveries from suppliers,
- $\mathbf{Q2} = [q2(v, t)]$ – matrix of the demand volume of enterprises on particular working days in palette loading units,
- $\mathbf{Q3} = [q3(v)]$ – vector of total demand of enterprises in palette loading units,
- $\mathbf{POJ} = [poj(v)]$ – vector of warehouse capacity,
- $\mathbf{K} = [k(v)]$ – vector of passage costs of a load unit through the warehouse facilities,
- $\mathbf{C} = [c(v, v')]$ – matrix of transport costs of load unit per distance unit between particular facilities of the network.

Variables formulated in the form of matrix $\mathbf{X1}$ (relation: suppliers – warehouses), $\mathbf{X2}$ (relation: suppliers – manufacturing plant), $\mathbf{X3}$ (relation: warehouse – manufacturing plant) related to the interpretation of the volume of raw materials carried between network points on a given working day acquires the following form:

$$\mathbf{X1} = [x1(v, v', t) : x1(v, v', t) \in \mathbf{R}^+ \cup \{0\}, v \in \mathbf{DS}, v' \in \mathbf{MS}, t \in \mathbf{T}], \quad (1)$$

$$\mathbf{X2} = [x2(v, v', t) : x2(v, v', t) \in \mathbf{R}^+ \cup \{0\}, v \in \mathbf{DS}, v' \in \mathbf{P}, t \in \mathbf{T}], \quad (2)$$

$$\mathbf{X3} = [x3(v, v', t) : x3(v, v', t) \in \mathbf{R}^+ \cup \{0\}, v \in \mathbf{MS}, v' \in \mathbf{P}, t \in \mathbf{T}]. \quad (3)$$

Principal limitations of the analysed model have the following form:

- Warehouses tend to have a limited capacity, and so the total of the volume of cargo supplied to each warehouse may not exceed its admissible capacity.

$$v' \in \mathbf{MS}, t \in \mathbf{T},$$

$$\sum_{v \in \mathbf{DS}} x1(v, v', t) - \sum_{v \in \mathbf{P}} x3(v', v, t) + \sum_{t'=1}^{t-1} \sum_{v \in \mathbf{DS}} x1(v, v', t') - \sum_{t'=1}^{t-1} \sum_{v \in \mathbf{P}} x3(v', v, t') \leq poj(v'). \quad (4)$$

- Maintaining the raw material flow stream through the warehouse. Volume of raw material coming out from the given warehouse on a given working day may not exceed the current state of the warehouse on that day:

$$v' \in \mathbf{MS}, t \in \mathbf{T}$$

$$\sum_{v \in \mathbf{DS}} x1(v, v', t) + \sum_{t'=1}^{t-1} \sum_{v \in \mathbf{DS}} x1(v, v', t') - \sum_{t'=1}^{t-1} \sum_{v \in \mathbf{P}} x3(v', v, t') \geq \sum_{v \in \mathbf{P}} x3(v', v, t). \quad (5)$$

- The entire cargo has to be collected from the supplier on a given working day. The suppliers do not store the raw material. The limitation eliminates a situation in which a manufacturing plant does not use warehouses and picks up raw materials from the suppliers:

$$v \in \mathbf{DS}, t \in \mathbf{T} \quad \sum_{v' \in \mathbf{MS}} x1(v, v', t) + \sum_{v' \in \mathbf{P}} x2(v, v', t) - q1(v) = 0. \quad (6)$$

- The demand of manufacturing plants in a given working day has to be met:

$$v' \in \mathbf{P}, t \in \mathbf{T} \quad \sum_{v \in \mathbf{DS}} x2(v, v', t) + \sum_{v \in \mathbf{MS}} x3(v, v', t) = q2(v', t). \quad (7)$$

- Total demand of manufacturing plants has to be met:

$$v' \in P \quad \sum_{t \in T} \sum_{v \in DS} x2(v, v', t) + \sum_{t \in T} \sum_{v \in MS} x3(v, v', t) = q3(v'). \quad (8)$$

The function for minimising costs of transport and storage acquires the following form:

$$F1(\mathbf{X1}, \mathbf{X2}, \mathbf{X3}) = \sum_{v \in DS} \sum_{v' \in MS} \sum_{t \in T} x1(v, v', t) \cdot d1(v, v') \cdot c(v, v') + \sum_{v \in DS} \sum_{v' \in P} \sum_{t \in T} x2(v, v', t) \cdot d2(v, v') \cdot c(v, v') + \\ + \sum_{v \in MS} \sum_{v' \in P} \sum_{t \in T} x3(v, v', t) \cdot d3(v, v') \cdot c(v, v') + \sum_{v \in DS} \sum_{v' \in MS} \sum_{t \in T} x1(v, v', t) \cdot k(v') \longrightarrow \min. \quad (9)$$

3. Structure of genetic algorithm

A genetic algorithm was used to determine the volume of cargo stream flows between particular facilities in the network. To form a genetic algorithm it is advisable to define the chromosome structure, the adaptation function, cross-linking process and mutation.

Subsequent steps of formulating an algorithm are as follows:

- Step 1: determination of the structure of input data. The structure of input data was presented as matrices $\mathbf{M}(t)$, which present the flow of material goods between particular elements of the logistics network. Lines and columns of those matrices define facilities of the logistic network structure. Matrix cells are located in the following sequence: suppliers, warehouses and manufacturing plants. The graphical representation of the matrix structure $\mathbf{M}(t)$ with sample stream volumes was shown on Fig. 1 (**DS** – suppliers, **MS** – warehouses, **P** – recipients).

	DS1	DS2	MS1	MS2	MS3	P1	P2
DS1	0	0	7	5	10	10	5
DS2	0	0	5	3	15	5	5
MS1	0	0	0	0	0	6	6
MS2	0	0	0	0	0	4	4
MS3	0	0	0	0	0	10	15
P1	0	0	0	0	0	0	0
P2	0	0	0	0	0	0	0

Fig. 1. Structure of input data of a genetic algorithm for selected day

- Step 2: definition of the adaptation function. To search for the minimum cost the function of adaptation Fp_n for the n -th structure of the logistics network acquires the following form:

$$Fp_n = C - \mathbf{KPS}_n, \quad (10)$$

where:

C – value higher than the value of costs of materials flow in the network,

\mathbf{KPS}_n – cost of material flow in the n -th structure of the logistics network, formula (9).

- Step 3: determination of the cross-linking operator. The cross-linking operator is adequate to the adopted matrix structure. To implement the cross-linking process for each day (t), two matrices are developed: $\mathbf{DIV}(t)$ which comprises rounded up average values from both parents, and matrix $\mathbf{REM}(t)$ containing information whether the rounding up was indeed necessary.

Assuming that the value of matrices $\mathbf{M1}(t)$ and $\mathbf{M2}(t)$ (parents) in all cells assume determination $m^1_{v,v',t}$, $m^2_{v,v',t}$ values of elements of matrices $\mathbf{DIV}(t)$ and $\mathbf{REM}(t)$ are calculated from the following dependencies:

$$dim_{v,v',t} = \left\lceil (m^1_{v,v',t} + m^2_{v,v',t}) / 2 \right\rceil, \quad (11)$$

$$rem_{v,v',t} = (m^1_{v,v',t} + m^2_{v,v',t}) / mod 2. \quad (12)$$

The full description of the cross-linking process was presented in [18], and presented in

a graphical way to Fig. 2. The applied cross-linking operator guarantees the correctness of individuals following a completed cross-linking process, without the necessity of using repair algorithms.

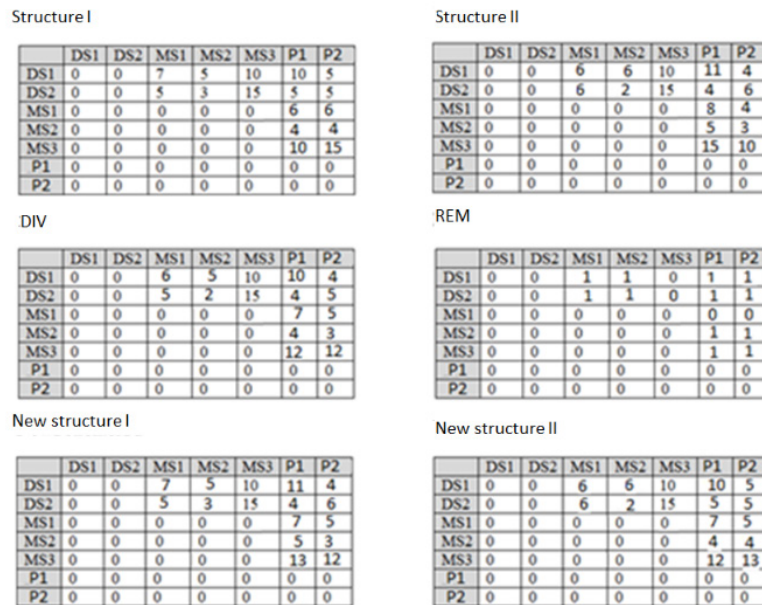


Fig. 2. Cross-linking of matrix structure for selected day

- Step 4: determination of mutation operator. The operation rule of mutation operator consists in sampling of two figures p and q from the range: $2 \leq p \leq k$ and $2 \leq q \leq n$, which determine the number of lines and columns of a sub-matrix with dimensions $p \times q$ (p – number of lines in the main matrix (processed by the algorithm), q – number of columns in this matrix). The generated matrix is modified in such a way that the total value in columns and lines before and after the modification process is not changed. The detailed mutation process has been outlined in [13], and in a graphical way, it was presented on Fig. 3.

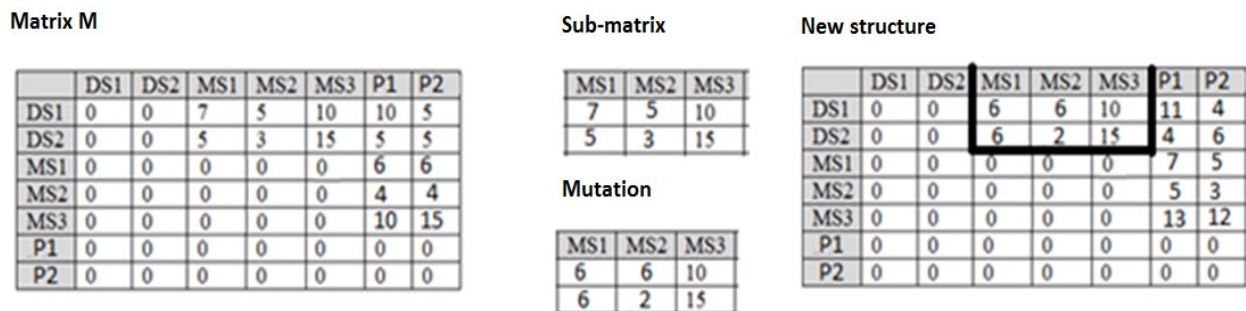


Fig. 3. Mutation of the matrix structure for selected day

Steps 2-4 of the algorithm are reiterated a given number of times, until the stop condition has been achieved. A condition for stop in the developed algorithm is the fixed iterations number. In the selection process, the roulette method was adopted, while the process of cross-linking and mutation occurs with a defined likelihood set at the beginning of functioning of an algorithm.

The problem issue in the analysed case is the determination of the initial population processed by the algorithm that meets limitations of the demand of manufacturing enterprises, possibilities offered by suppliers, capacity of warehouses. The algorithm for determination of initial population was presented on Fig. 4.

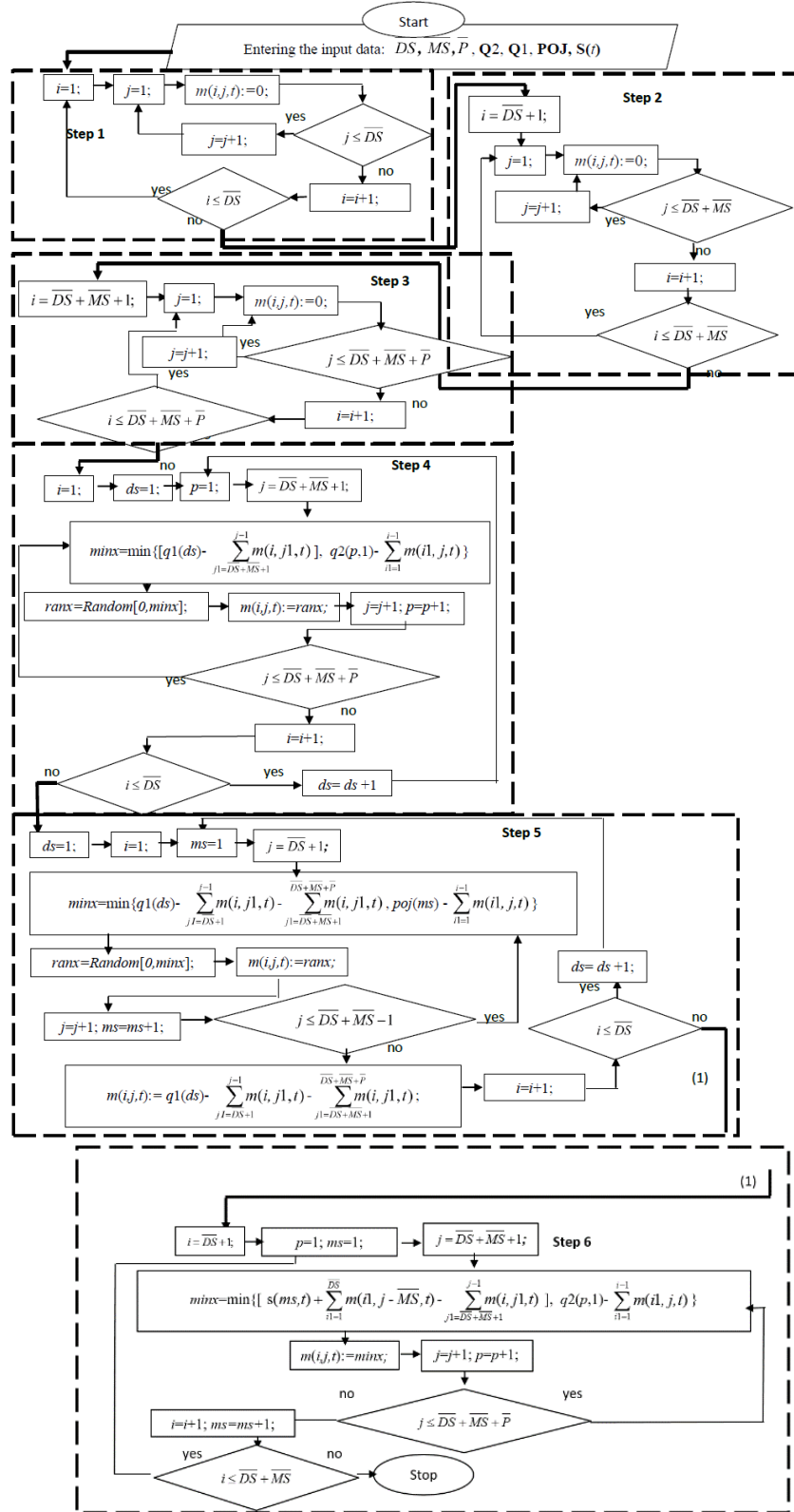


Fig. 4. Algorithm for selection of initial population in the location problem

This algorithm delimited the matrix specifying the chromosome structure processed by a genetic algorithm $\mathbf{M}(t) = [m(i, j, t)]$, where $\mathbf{I} = \{1, \dots, \bar{I}\}$ – set of numbers of lines in matrix $\mathbf{M}(t)$, $\mathbf{J} = \{1, \dots, \bar{J}\}$ set of numbers of columns in the matrix $\mathbf{M}(t)$, where \bar{I}, \bar{J} assume values $\bar{I} = \overline{DS} + \overline{MS} + \overline{P}$, $\bar{I} = \bar{J}$, \overline{DS} – numerical amount of set \mathbf{DS} , \overline{MS} – numerical amount of set \mathbf{MS} ,

\bar{P} – numerical amount of set P and matrix that defines the state of a warehouse from the preceding day $S(t) = [s(ms, t)]$ where ms – subsequent warehouse, t – next working day of manufacturing plant. Ways of filling out matrix cells for the given day were described in the following steps:

- Step 1: entering the 0 value in relations: suppliers – suppliers; no raw material forwarding between suppliers,
- Step 2: entering the 0 value in relations: warehouses – suppliers and warehouses – warehouses; no raw material forwarding between warehouses and suppliers and between warehouses,
- Step 3: entering the 0 value in relations: enterprises – suppliers, enterprises – warehouses and enterprises – enterprises,
- Step 4: determination of random flows of raw materials between suppliers and enterprises. The $minx$ variable determines the minimum value from among two values: the present volume of supplies from the given supplier and the current demand of enterprises, where ds – next supplier from the set DS , p – next enterprise from the set P . The $ranx$ variable is a random value from the range of $[0, minx]$ that determines random flows of raw materials between suppliers and enterprises. Those flows meet limitations arising from manufacturing capacity of suppliers and the demand of recipients,
- Step 5: determination of random raw material flows between the suppliers and warehouses. The random value of that flow has to satisfy the warehouse capacity limitation and limited possibilities of supplies to be provided by each supplier. This step took into consideration the fact that the entire cargo is collected from the supplies and carried to warehouses or to manufacturing plants. In this step, the $minx$ variable defines the minimum value of between two values: the present value of supplies from the given suppliers and the present warehouse capacity. The $ranx$ variable defines the random value of stream flow between suppliers and warehouses,
- Step 6: determination of flows between warehouses and enterprises. In this step, the $minx$ variable determines the minimum value of the following two values: current demand of an enterprise for raw materials and the current state of the warehouse and indicates the flow between the warehouse and the enterprise. It should be kept in mind that the current warehouse stock depends on the warehouse stock on the preceding day. Implementation of step 6 ensures that the demand of manufacturing plants is satisfied.

4. Conclusions

The selection of algorithms that solve the problem of warehouse location depends on the complexity of the logistic network, within which warehouses are allocated. As regards complex structures of a large number of elements in the network, such as suppliers, warehouses and recipients to solve the location problem, use is made of heuristic algorithms.

The presented matrix structure processed by the genetic algorithm refers to each working day of an enterprise. The location problem is solved in such a way that if for any day within the analysed period flows take place from a given warehouse or to a given warehouse, such a logistics facility should constitute an element of the target logistics network. If there are no such flows, no warehouse is necessary in the given location.

A further direction of research related to the presented genetic algorithm is its implementation in the form of a computer application. To allow an in-depth analysis of functioning of the algorithm research should be performed using various selection methods.

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