SPEED OF A SOUND IN NONIDEAL GAS

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The general formulas for the basic thermodynamic parameters are revisited (entropy, free energy and others) form point of view of the speed of a sound determining dependence on pressure and temperatures in nonideal environments. As an example a behaviour of speed of a sound is investigated as functions of temperature and pressure for Van der Waals gas. A comparison of the results of evaluation with the available experimental data is carried out. The analysis has shown, that received formulas describe the velocity of a sound in a wide range of pressures and temperatures.

INTRODUCTION

The problem of a calculation of the speed of a sound in gases has a long history. The first theoretical calculation of speed of a sound in gas was made by Newton which used the hypothesis about isothermal process in the gas disturbance. However, calculated by him value of speed of a sound for the air under normal conditions appeared approximately on 20% less than the valid value. The adiabatic process of a disturbance induced by a sound in gas which for the first time has specified Laplace, results at calculation of speed of a sound in ideal gas in the formula:

$$c_L = \sqrt{\frac{\gamma P_0}{\rho_0}} \qquad (1)$$

Which for the sound velocity in the air under normal conditions gives the value C_L =330 m/s that better fit to experiment. As follows from the formula (1) the speed of a sound in ideal gas does not depend on ambient pressure, but depends on the temperature. However for real gases dependence of sound speed on parameters of environment looks more complicated, in particular, character of the distribution begins to depend on a length of a wave. When the length of ultrasonic waves becomes very small, it is necessary to consider a molecular nature

of gas. All this has demanded the profound development of the mathematical part of a question connected to a conclusion the appropriate differential equations describing process of sound waves propagation in a gas in view of thermal effects arising at it.

Exact measurements of speed of ultrasound in gases have resulted in opening very interesting features of the phenomena. It was revealed, that in multinuclear gases at enough high ultrasonic frequencies speed of ultrasound undergoes changes, for such gases the dispersion of ultrasound takes place. Simultaneously with increase of an ultrasound speed his absorption [1,2,3] is increased. To explain observable effects it is possible only taking into account molecular structure of particles of gas. Calculations of the speed of a sound for real gases show [4, 5] that the value in the certain range of temperatures and pressure is better: it will better correspondent to experiment, than for ideal gas. However, the formulas obtained in [4, 5], are fair only at small pressure. Ours results as shows comparison with experiment, are fair up to pressure in a wider range, up to about hundred of bars. Expanding our formulas for background fields in a series of small pressure reproduce the results [4, 5]. A square of adiabatic speeds of a sound is general

$$c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s . \qquad (2)$$

1. BASIC IDEA

The formula (2) is inconvenient for calculations at the any equations conditions of environment as the derivative in the right part is calculated at the constant entropy. It demands, as a matter of fact, explicit integration of the equation dS = 0. However, it is possible to consider the alternative approach. For this purpose, from reasons of convenience, let's proceed in the ratio (2) to molar to volume $V_1 = V / \nu = M / \rho$, where V - volume, and ν - molar number.

$$c^2 = -\frac{V_1^2}{M}\left(\frac{\partial P}{\partial V_1}\right)_s \qquad (3)$$

Here M - molar weight of the substance. Having taken advantage of a Jacobian method, it is possible to proceed from variables (V_1, S) to variable (V_1, T).

$$\left(\frac{\partial P}{\partial V_1}\right)_s = \left(\frac{\partial P}{\partial V_1}\right)_T - \frac{T}{cV_1}\left(\frac{\partial P}{\partial T}\right)_{V_1}\left(\frac{\partial S}{\partial V_1}\right)_T$$

Further, using the Maxwell relation, we obtain for the square adiabatic speed:

$$c^2 = \frac{V_1^2}{M}\left[\frac{T}{cV_1}\left(\frac{\partial P}{\partial T}\right)_{V_1}^2 - \left(\frac{\partial P}{\partial V_1}\right)_T\right] \qquad (4)$$

Use of the equation (4) allows to calculate a sound speed dependence in the explicit form on the temperature and pressure for the real gases plugging various equations of a state. Deriving by (4) the explicit formula for the speed of a sound contains temperature and molecular volume which are not independent as are connected by the thermal equation conditions P = P(V_1, T). For the dependence of speed of a sound on temperature or pressure, the molecular volume V_1 should be excluded from obvious expressions c = c(T, V_1) with the

help of the specified equation of state, which in sets with (4) can be considered, as dependence c(P) (at T $=T_0$) or c(T) (at P $=P_0$) in the parametrical form.

2. VAN DER WAALS GAS

In contrary to the ideal gas, the speed of a sound in Van der Waals gas depends not only on temperature, but also on volume. Using the equation of a state of the Van der Waals gas:

$$p = \frac{R_0}{V_1 - b} - \frac{a}{V_1^2} \qquad (5)$$

Having substituted from (5) the temperature T in the expression (6), it is possible to obtain the dependence of the speed of a sound in Van der Waals gas on the pressure. For a square of a sound speed from (4) it is obtained:

$$c^2 = \frac{\gamma R_0 V_1 P}{M(V_1 - b)} - \frac{a}{M}\left(\frac{\gamma}{V_1 - b} - \frac{2}{V_1}\right) \qquad (7)$$

3. COMPARISON OF THE THEORETICAL AND EXPERIMENTAL DATA

Diagrams of dependence of speed of a sound on the pressure for various gases are below given with use of the equation of state (7):

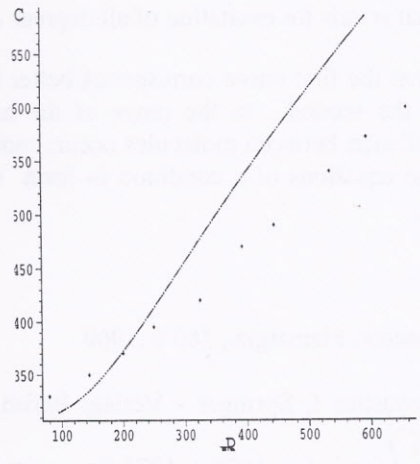

Fig.1. Dependence of speed of a sound on pressure for argon (speed of a sound c it is calculated in m/s, and pressure P in atm.), at $T_0 = 273K$;

__ - the diagrams constructed under the formula (7),

. - experimental data [3].

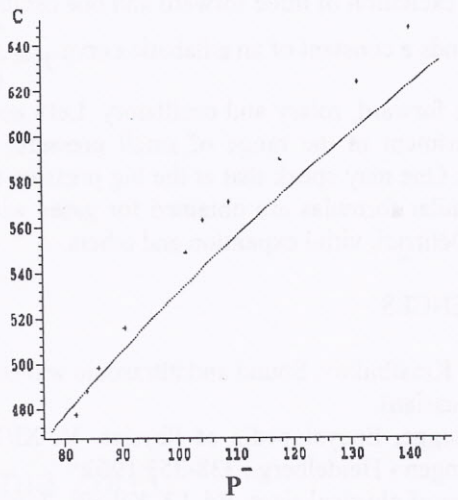

Fig.2. Dependence of speed of a sound on pressure for ethane (speed of a sound c is calculated in m/s, and pressure P in atm.), at $T_0 = 273K$;

__ - the diagrams constructed by the formula (7),

. - experimental data [3].

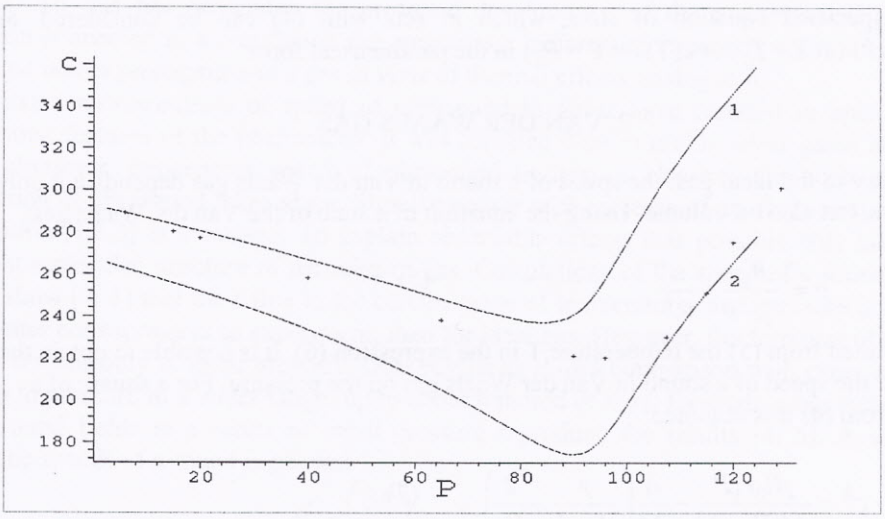

Fig.3. Dependence of the speed of a sound on pressure for carbonic gas (speed of a sound c it is calculated in m/s, and pressure P in atm.), at $T_0 = 273K$;

__ - the diagrams constructed under the formula (7),
. - experimental data [3].

On fig. 3 first curves there corresponds a constant of an adiabatic curve $\gamma = \frac{7}{5}$, that answers excitation of three forward and one oscillatory degrees of freedom. The second curve corresponds a constant of an adiabatic curve $\gamma = \frac{7}{6}$, that stands for excitation of all degrees of freedom: forward, rotary and oscillatory. Let's note, that the first curve corresponds better to the experiment in the range of small pressure, and the second - in the range of the big pressure. One may speak that at the big pressure of collision between molecules occurs more often.Similar formulas are obtained for gases with the equations of a condition in form of Bertlo, Detirrici, virial expantion and others.

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