Load by a Force Directed Towards a Positive Pole in the Aspect of Studies on Vibrations of a Column with Crack

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Abstract

In this paper the results of numerical studies on natural vibration frequency and stability of a slender supporting system loaded by external force directed towards a positive pole are presented. In the investigated structure the failure in the form of crack is present. The boundary problem is formulated on the basis of the principle of minimum action – Hamilton's principle. The results in the non-dimensional form are plotted as the characteristic curves in the external load – natural vibration frequency plane as well as the maximum loading capacity is discussed.

Keywords: column, crack, natural vibration, instability, characteristic curves

1. Introduction

The studies on cracks which can appear in the supporting systems are very important. The columns are classified as slender structures due to much greater length than cross section area. In slender systems the unwanted phenomena like flutter instability, buckling or non-axially applied load should be avoided. The presence of cracks reduces loading capacity and has an influence on dynamic behavior of the structure that is why an engineers must take care of this very dangerous problem before it is too late.

The investigations on cracks have been performed in recent years by inter alia Arif Gurel M. [1], Bergman [2], Binici [3], Chondros [4], Dimarogonas [5], Sokół [6] and Sokół and Uzny [7]. In the literature cracks are divided into always open and breathing cracks. In the first type is a linear problems - static deflection of the structure is much greater than an amplitude of vibrations while breathing cracks are the non-linear problem - crack opens and closes in time as vibration amplitude dependent. The simulation of cracks are mostly done as reduced cross section area or rotational springs. The studies presented in [4] and [1] show that the use of rotational spring leads to the good results accuracy (numerical simulation and experiment) despite their simplicity.

The presented in this paper slender supporting system is loaded by a force directed towards a positive pole (comp [8, 9 , 10]). This load is induced by a force with the line of action described by two points. The points are: loaded end of the column and a pole – point on the undeformed axis of the column. It is assumed that the positive pole is place below the loaded end. If the point is localized above the loaded end the pole is negative. The same nomenclature about the poles is used when the specific load introduced by

Tomski [11 – 14] is taken into account. Depending on the location of the pole (positive or negative) the different deflection angles of the loaded end can be obtained.

In this paper the results of numerical studies on a column subjected to external compressive load taking into account a defect in the form of a crack are presented. The discussed simulation data are concerned on external load – vibration frequency relationship, loading capacity, transom length and crack size.

2. Boundary problem formulation

The investigated slender system is shown in the figure 1. Structure is loaded by external force P which is placed on the free end of the column. The presented type of load is called the load with a force directed towards a positive pole. The column is composed of one element in which the crack is present. It is assumed that crack is open and the rotational spring C is used as a discreet element in the simulations. The presence of crack divides a structure into two elements as shown in the figure 1. In the common point the continuity of transversal as well as bending moments and shear forces are met by means of natural boundary conditions. The free end of a column is reinforced by a transom of length l_c. The donations shown in the figure 1 are as follows: E_i – Young's modulus, J_i – moment of inertia, A_i – cross section area, ρ_i – material density, C – rotational spring stiffness (crack size), P – external load, l_c – transom length, m – loading head mass. The total length of a column is $l = l_1+l_2$.

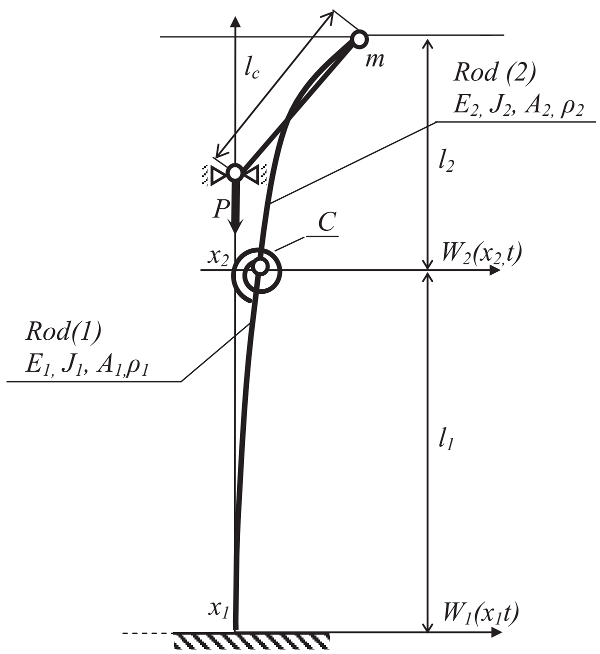


Figure 1. Investigated system

The boundary problem has been formulated on the basis of the Hamilton's principle

$$\delta\int_{t_1}^{t_2}(\mathrm{T}-V)dt=0 \tag{1}$$

according to which the kinetic T and potential V energies are described as follows:

$$T=\frac{1}{2}\sum_{i=1}^{2}\rho A_i\int_0^{l_i}\left(\frac{\partial W_i(x_i,t)}{\partial t}\right)^2dx+\frac{1}{2}m\left(\left.\frac{\partial W_2(x_2,t)}{\partial t}\right|^{x_2=l_2}\right)^2 \tag{2}$$

$$\begin{aligned}V=&\frac{1}{2}\sum_{i=1}^{2}EJ_i\int_0^{l_i}\left(\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2}\right)^2dx_i+\frac{1}{2}C\left(\left.\frac{\partial W_1(x_1,t)}{\partial x_1}\right|^{x_1=l_1}-\left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|_{x_2=0}\right)^2+\\&-P\frac{1}{2}\sum_{i=1}^{2}\int_0^{l_i}\left(\frac{\partial W_i(x_i,t)}{\partial x_i}\right)^2dx_i+\frac{1}{2}P\frac{1}{l_c}W_2(l_2,t)^2\end{aligned} \tag{3}$$

On (1) the integration and variation operations are performed and finally inter alia the differential equations of motion in transversal direction (4) are found:

$$EJ_iW_i^{''''}(x_i,t)+PW_i^{''}(x_i,t)+\rho A_i\ddot{W}_i(x_i,t)=0 \quad i=1,2 \tag{4}$$

As well as the natural boundary conditions. The complete set of natural and geometrical boundary conditions is presented by 4(a-h):

$$\begin{gathered}W_1(0,t)=W_1^I(0,t)=0 \qquad W_1(l_1,t)=W_2(0,t) \qquad W_2^{II}(l_2,t)=0\\EJ_1W_1^{III}(l_1,t)+PW_1^I(l_1,t)-EJ_2W_2^{III}(0,t)+PW_2^I(0,t)=0\\-EJ_2W_2^{II}(0,t)+C\left[W_2^I(0,t)-W_1^I(l_1,t)\right]=0\\EJ_1W_1^{II}(l_1,t)-C\left[W_2^I(0,t)-W_1^I(l_1,t)\right]=0\\EJ_2W_2^{III}(l_2,t)+P\left[W_2^I(l_2,t)-\frac{1}{l_c}W_2(l_2,t)\right]-m\ddot{W}_2(l_2,t)=0\end{gathered} \tag{4a-h}$$

On the basis of solution of formulated boundary problem connected with free vibrations characteristic curves on the plane: external load – natural vibration frequency for given parameters such as crack size/location or position of pole can be calculated.

3. Results of numerical simulations

The results of numerical simulations are shown in the non-dimensional form:

$$p=\frac{Pl^2}{EJ_1},c=\frac{Cl}{EJ_1+EJ_2},d=\frac{l_1}{l},\mu=\frac{EJ_2}{EJ_1},m_b=m\rho A_1l,l_{CB}=\frac{l_c}{l} \tag{5(a-f)}$$

The use of non-dimensional parameters does not require to specify the material properties or cross section area.

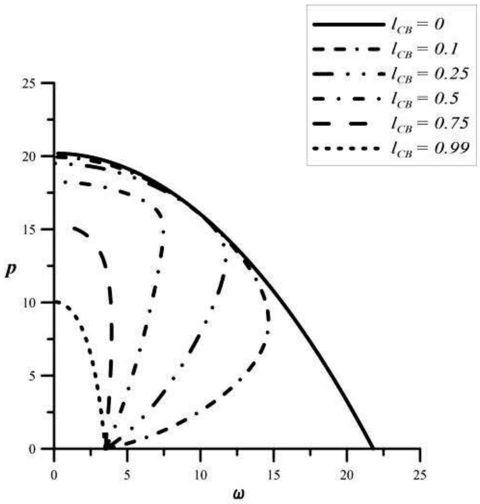


Figure 2. Characteristic curves of reference structure at different transom length; $d = 0.5$, $\mu = 1$, $m_b = 0.15$, $c = \infty$

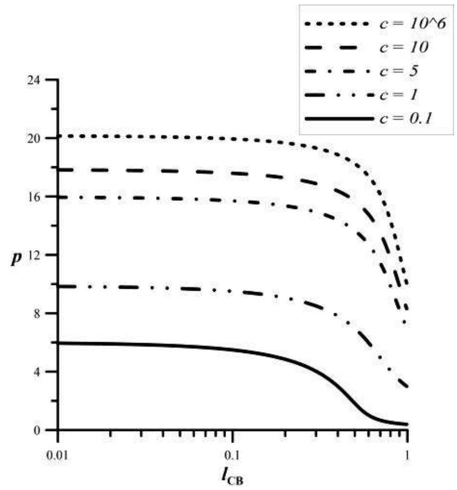


Figure 3. An influence of transom length on loading capacity at different crack size; $d = 0.5$, $\mu = 1$, $m_b = 0.15$

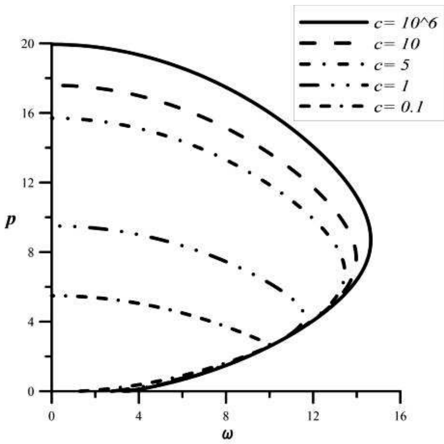


Figure 5. Characteristic curves of at different crack size; $d = 0.5$, $\mu = 1$, $m_b = 0.15$, $l_{CB} = 0.1$

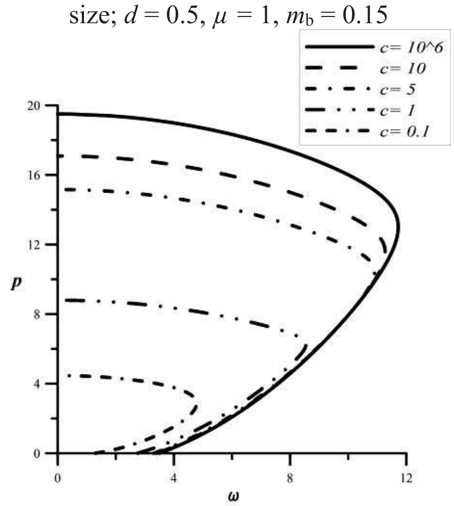


Figure 6. Characteristic curves of at different crack size; $d = 0.5$, $\mu = 1$, $m_b = 0.15$, $l_{CB} = 0.25$

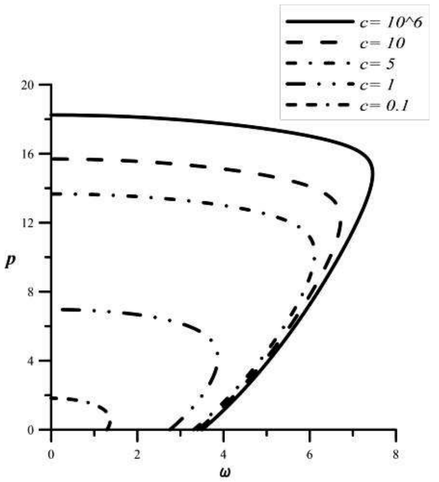


Figure 6. Characteristic curves of at different crack size; $d = 0.5$, $\mu = 1$, $m_b = 0.15$, $l_{CB} = 0.5$

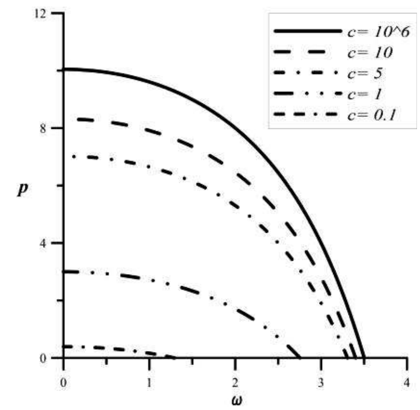
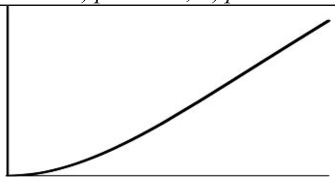
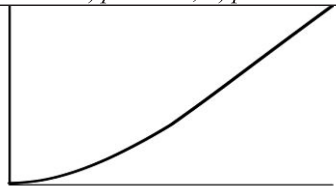
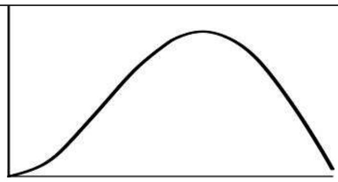
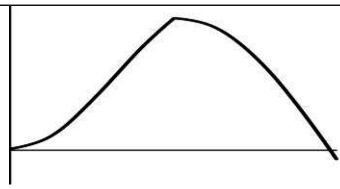
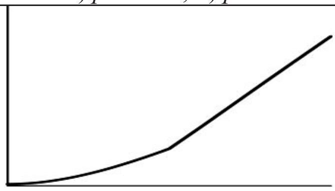
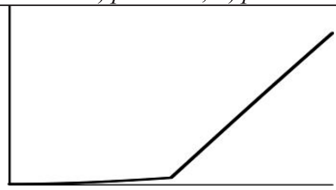
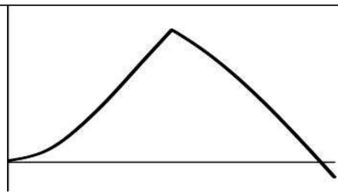
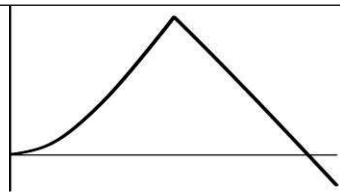


Figure 7. Characteristic curves of at different crack size; $d = 0.5$, $\mu = 1$, $m_b = 0.15$, $l_{CB} = 0.99$

In the figures 2, 3 an influence of the transom length on the natural vibrations and loading capacity are presented. An analysis of the figure 2 allows one to conclude that if no transom is used the column behaves like a fixed - pinned system. The use of short transom results in change of the dynamic behaviour in relation to the reference ($l_{CB} = 0$) column. Furthermore the characteristic curve of the column with short transom has initially positive slope what results in increase of the natural vibration frequency along with increase of the external load magnitude. After reaching the highest natural vibration frequency magnitude point the frequency decreases while the external load is getting greater. This type of characteristic curve relates to the structures with divergence – pseudo flutter instability type. The vibration modes are being changed along the characteristic curves – see table 1 at $c = \infty$ a) $p = 0.97$, b) $p = 14.6$. As shown in the figure 2 an installation of the transom leads to the reduction of vibration frequency by reduction of transversal displacements of the free end of the column. An increase of its length finally leads to the very rapid decrease of the loading capacity in the structure without any defects and causes the change of instability into divergence one (refer to figure 2 and 3). There exists such transom length above which the change of instability can be observed. If in the column the crack appears its loading capacity is reduced. The size and nature of this reduction highly depends on transom length. The greater the crack and the longer transom the more smooth loading capacity reduction can be observed (see figure 3). When the characteristic curves of the cracked system are taken into account (figures 5 – 8) it allows one to state that at short transom (figure 5) an appearance of the crack does not affect the initial shape of the investigated curves – the curves are overlapping each other. Along with external load increase the curves are being shifted in relation to the reference curve (continues one) and the loading capacity drop is observed. The size of this reduction highly depends on crack size and transom length. At longer transom (figures 6, 7) the smaller crack causes the shift of the characteristic curve in relation to the reference one. When the transom length is equal to the one of the column's (figure 8) the system has divergence instability type. The crack appearance shifts the characteristic curves even at small size of the defect. It can be stated that the crack does not affect the type of instability. The first and second vibration modes are plotted in the table 1. The plots are done at different crack sizes of the divergence – pseudo flutter system. Due to large number of the results the table 1 corresponds only to one configuration of the investigated system. An analysis of the vibration modes can easily lead to detection of the structure defect but it must cover at least first and second modes.

Table 1. Vibration modes of the divergence – pseudo flutter system: $d = 0.5$, $\mu = 1$, $m_b = 0.15$, $l_{CB} = 0.1$

$c = \infty$ a) $p = 0.97$, b) $p = 14.6$	$c = 5$ a) $p = 0.97$, b) $p = 14.6$
a) 	a) 
b) 	b) 
$c = 1$ a) $p = 0.97$, b) $p = 8.75$	$c = 0.1$ a) $p = 0.97$, b) $p = 4.85$
a) 	a) 
b) 	b) 

Taking into account the results from table 1 it can be concluded that the loading structure by means of which the load by a force directed towards a positive pole is created has an influence on the vibration modes. The size of an influence depends on crack size as well as on the parameters of the loading structure which are present in the boundary condition (4h).

4. Conclusions

In this paper the studies on natural vibration frequency and loading capacity of a column subjected to a force directed towards a positive pole are done. Additionally an influence of the defect in the form of crack is taken into account. On the basis of the results of numerical simulations it can be concluded that:

- in the reference structure an installation of transom of greater length causes a decrease of maximum loading capacity in relation to the shorter elements,
- in relation to transom length the divergence or divergence – pseudo flutter characteristic curves can be obtained,
- presence of a crack causes a reduction of maximum loading capacity. This change highly depends on transom length – the shorter transom the more rapid loading capacity decrease can be observed,
- the crack affects the shape of characteristic curves but doesn't change the type of instability (divergence or divergence – pseudo flutter),
- at short transom the appearance of the crack doesn't change the initial shape of characteristic curve of divergence – pseudo flutter system,
- the crack presence and location can be found on the basis of the analysis of the vibration modes. An analysis of the higher modes allows on to indicate the crack even if it is unseen the lower modes.

The obtained results can be used in the structure health monitoring in order to find a defect which can lead to the destruction of the column. The presented studies should be expanded by an analysis of the crack location on the instability and natural vibrations of the slender system as well as by the experimental verification of the proposed mathematical model. Additionally the investigations of the parameters of the loading structure at which the column is the least sensitive to the crack presence can be performed.

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