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MATERIAL SCIENCE AND NANOTECHNOLOGY

# Fusion reaction study of halo system by quantum mechanical-based model for $^6$He+$^{64}$Zn, $^8$B+$^{58}$Ni and $^8$He+$^{197}$Au systems

**Maryam H. ABD MADHI and Fouad A. MAJEED**[*]

Department of Physics, College of Education for Pure Sciences, University of Babylon, Babylon, Iraq

**Abstract.** In the current work the calculations of the reaction cross-section of total fusion $\sigma_{\text{fus}}$, the fusion barrier distribution $D_{\text{fus}}$, and the probability $P_{\text{fus}}$ were achieved for systems $^6$He+$^{64}$Zn, $^8$B+$^{58}$Ni and $^8$He+$^{197}$Au which involve halo nuclei by using a semiclassical approach. The semiclassical and quantum mechanics treatments comprise the approximation of WKB for describing the relative motion among projectile nuclei and target nuclei, and the method of CDCC (Continuum Discretized Coupled Channel) for describing the intrinsic motion for the projectile and target nuclei. Our semiclassical calculations yielded findings that were compared to obtainable experimental data as well as quantum mechanics calculations. For fusion cross-sections $\sigma_{\text{fus}}$ below and above the Coulomb barrier $V_b$, the quantum mechanics coupled channels are very similar, according to the experimental results.

**Key words:** semiclassical treatment; fusion barrier distribution; halo nuclei; fusion cross-section; continuum discretized coupled channels.

## 1. INTRODUCTION

Nuclear fusion is one of the most promising and forward-looking approaches to finding alternative energy sources [1]. The Coulomb barrier, which is formed using the long-range repulsive Coulomb and the attractive, short-range nuclear force, must be overcome by the relative movement of the colliding nuclei [2]. The tunneling phenomenon can cause fusion reactions at the energies below the current barrier, which are called classical forbidden areas [2,3]. To calculate the tunneling probability, the Schrodinger equation should be approximated in this forbidden field, which is done using the WKB process [2,4]. Two kinds of fusion processes can be distinguished in collisions of weak bound nuclei, ICF (incomplete fusion) when some projectile fragments drift away from the interaction area, and CF (complete fusion) where the nucleons of each projectile-target nucleon combine to form the compound nucleus. The total fusion cross-section represented the sum of cross-sections of both ICF and CF, which was calculated in most experiments [5–9]. The coupling to the continuum was used for investigating the impact of the breakup channel on the system with weak bound projectile [3]. According to CDCC, the continuum was evaluated using a finite set of cases [10–12]. F. A. Majeed et al. [13–23] recently demonstrated that the semi-classical method is very effective for the measurement of fusion reactions when the coupled channel to the continuum is considered by using it on several selected systems including the halo nuclei. This work focuses on using the semi-classical method utilising the adoption of Alder and Winther theory originally employed for treating the Coulomb exciting the nuclei, known as the route of CDCC in which semiclassical and quantum methods were applied for calculating total fusion cross-sections $\sigma_{\text{fus}}$ (mb) and distributions of fusion barrier $D_{fus}$ (mb/MeV) to systems including light halo nuclei $^6$He+$^{64}$Zn, $^8$B+$^{58}$Ni and $^8$He+$^{197}$Au. The results for quantum and semiclassical calculations are compared using the calculations of the single and coupled channel with the available experimental data.

## 2. THEORETICAL BACKGROUND

### 2.1. The coupling channel formalism

Nuclei in the collision can experience internal excitations, and various processes of the particle transfer, all of which impact their total fusion reaction cross-sections $\sigma_{\text{fus}}$. However, the current reaction process includes the effective participation of many freedom degrees to describe it. Hence, the fusion method requires the explicit inclusion of the couplings among the various freedom degrees. Also, this was achieved by incorporating many components into the system wave function equal to the numbers of inherent quantum mechanical cases included [24,25].

Consider the reaction described using the function of total wave $\Psi(r,\tau)$, where $r$ stands for the separation vector of target nuclei and projectile nuclei and $\tau$ for the set of the intrinsic coordinates of target and projectile nuclei. Dynamics of the current reaction was done using Hamiltonian equation:

$$\mathrm{H} = \mathrm{H}_0 + \mathrm{T} + \mathrm{U}, \tag{1}$$

where $\mathrm{H}_0 \equiv \mathrm{H}_0(\tau, \mathrm{p}_\tau)$ is the intrinsic Hamiltonian, $\mathrm{U} \equiv \mathrm{U}(\mathrm{r},\tau)$ is the interaction potential and $\mathrm{T} \equiv -\hbar^2\nabla^2/2\mu$ for the relative

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[*] e-mail: fmajeed@uobabylon.edu.iq

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motion, the kinetic energy operator among target and projectile nuclei, the intrinsic Hamiltonian eigenstates, $|\eta\rangle$, satisfy the Schrödinger equation [4]:

$$(e_\eta - H_0)\,|\eta\rangle . \tag{2}$$

The orthonormality is:

$$\langle\eta\rangle = \int d\tau\, \varphi^*_{\eta'}(\tau)\varphi_\eta(\tau) = \delta_{\eta\eta'}\,, \tag{3}$$

where $\varphi_\eta(\tau)\,\left(\varphi_{\eta'}(\tau)\right)$ is the wave function that corresponds to a state $|\eta\rangle\,(|\eta\rangle)$ in the representation of $\tau$. The interaction potential is split as below:

$$U = U' + U''\,, \tag{4}$$

where $U'$ is diagonal in the space of channel:

$$U' = \sum_\eta |\eta\rangle\, U'_\eta \langle\eta|, \tag{5}$$

$$U'' = \sum_\eta |\eta\rangle U''_{\eta\eta'} \langle\eta'|, \tag{6}$$

where

$$U'_\eta(r) = \int d\tau\, \left|\varphi_\eta(\tau)\right|^2 U'(r,\tau), \tag{7}$$

$$U''_{\eta\eta'}(r) = \int d\tau\, \varphi^*_{\eta'}(\tau)\, U''(r,\tau)\, \varphi_\eta(\tau)\,. \tag{8}$$

The potential $U'$ was arbitrary, except diagonal in the space of the channel. Nevertheless, once it was selected, $U''$ was calculated using this relation $U'' = U - U'$. It was frequently convenient for choosing $U'$ such that $U''$ was purely off-diagonal. But in some states, components of $U''$ were done as below [5]:

$$U''_{\eta\eta'}(r) = \int d\tau\, \varphi^*_{\eta'}(\tau)\, U''(r,\tau)\, \varphi_\eta(\tau) - \delta_{\eta\eta'}\, U'_\eta(r). \tag{9}$$

From the Schrödinger equation, the equations of the coupled channel were derived:

$$(E - H)\left|\Psi_\eta(\eta_0\, k_0)\right\rangle = 0\,, \tag{10}$$

and the channel expansion:

$$\left|\Psi_\eta(\eta_0\, k_0)\right\rangle = \sum_\eta \left|\psi_\eta(\eta_0\, k_0)\right\rangle |\eta\rangle\,. \tag{11}$$

The notation $|\Psi(\eta_0\, k_0)\rangle$ indicated that the collision was started in the channel $\eta_0$, with the wave vector $k_0$, and the scale of energy was selected like that $e_{\eta_0} = 0$. Owing to the reaction off-diagonal part. The solution of the Schrödinger equation has components $|\Psi_\eta(\eta_0 k_0)\rangle$ for both $\eta = \eta_0$ and $\eta \neq \eta_0$. The infinite expansion of equation (11) was cut to include just more suitable channels or closed coupling approximation. For accounting the losing flux throughout the neglected channels, only one maybe involve the imaginary part in potentials of the channel $U'_\eta(r)$. The Hamiltonian must write as below, to calculate the function of wave [5]:

$$H = H_0 + H' + U''\,, \tag{12}$$

where

$$H' = K + U'\,, \tag{13}$$

when equations (11) and (12) were substituted into equation (10), and taken product of the scalar with all intrinsic states $|\eta|$, then gotten equations of the coupled channel:

$$\left(E_\eta - H'_\eta\right)|\psi_\eta(\eta_0\, k_0)\rangle = \sum_{\eta'} U''_{\eta\eta'}(r)\left|\psi_{\eta'}(\eta_0\, k_0)\right\rangle\,, \tag{14}$$

or

$$\left[E_\eta + \frac{\hbar^2}{2\mu}\Delta - U'_\eta(r)\right]\psi_\eta(r) = \sum_{\eta'} U''_{\eta\eta'}(r)\psi_{\eta'}(r)\,, \tag{15}$$

where

$$E_\eta = E - e_\eta\,, \tag{16}$$

$E_\eta$ was the total energy for the relative motion in the channel $\eta$ and

$$H'_\eta = T + U'_\eta\,. \tag{17}$$

Equation (15) turned to the more compact notation $|\psi_\eta(\eta_0\, k_0)\rangle \rightarrow \psi_\eta(r)$, and the channel potentials have been put as:

$$U'_\eta = V_\eta + i\, W_\eta\,, \tag{18}$$

where the flux in channel $\eta$ accounted by the imaginary part $W_\eta$ lost to others not involved in equations of the coupled channel. The non-Hermitian nature consequence of H was that the continuity equation broke down. In general states where the channel coupling interaction $U''_\eta$ was hermitian, the continuity equation was written as below [26]:

$$\nabla \cdot \sum_\eta j_\eta = \frac{2}{\hbar}\sum_\eta W_\eta(r)\left|\psi_\eta(r)\right|^2 \neq 0, \tag{19}$$

where $j_\eta$ is the probability current density in channel $\eta$. Usage of the concept of the absorption cross-section $\sigma_\eta$, integrate the above equation within the broad sphere with a radius greater than the range of interaction [27–30]:

$$\sigma_\eta = \frac{k}{E}\sum_\eta \langle\psi_\eta\rangle\,. \tag{20}$$

If the case of the absorptive potential, the relation is as below:

$$W_\eta = W^D_\eta + W^F_\eta\,, \tag{21}$$

with $W^D_\eta$ is to calculate losing the flux to other direct reaction channels and $W^F_\eta$ to calculate the fusion absorption, according to [14, 16], the fusion reaction cross-section becomes:

$$\sigma_F = \frac{k}{E}\sum_\eta \langle\psi_\eta\rangle\,. \tag{22}$$

In fusion reactions, couplings between multiple channels have significant effects.

### 2.2. Fusion barrier distribution

The influence of coupling of various channels on fusion reactions was well understood for around a quarter-century. Its more dramatic consequence was enhancing the total fusion reaction cross-section $\sigma_{fus}$ at Coulomb sub-barrier energies $V_b$, in several states using many orders of magnitude. The effect of coupling channels can be defined as the division of the fusion barrier into many sections, with the known distribution of fusion barrier $D_{\text{fus}}$ and written by [5, 31]:

$$D_{\text{fus}}(E) = \frac{d^2G(E)}{dE^2}, \tag{23}$$

when $G(E)$ is associated with the total fusion reaction cross-section during:

$$G(E) = E\,\sigma_{\text{fus}}(E). \tag{24}$$

The measured data on the distribution of fusion reaction barrier led to obtaining important information in the understanding of the nature of the fusion reaction, due to the contribution of the coupling channel during a collision. Nevertheless, from equation (23) it was noticed because it must be obtained from values of the reaction cross-section of total fusion. It was susceptible to both experimental and computational uncertainties. The general method is to evaluate the 2nd derivative shown in equation (23) throughout the 3-point difference approach [3, 32]:

$$D_f(E) \approx \frac{G(E+\Delta E)+G(E-\Delta E)-2G(E)}{\Delta E^2}, \tag{25}$$

where $\Delta E$ is the energy interval among the total fusion reaction cross-section measurements. The statistical error related to the distribution of fusion reaction barrier is roughly recorded using [2], according to equation (25):

$$\delta D_f^{\text{stat}}(E) \approx \frac{\sqrt{[\delta G(E+\Delta E)]^2+[\delta G(E-\Delta E)]^2+4[\delta G(E)]^2}}{(\Delta E)^2}, \tag{26}$$

where $\delta G(E)$ is mean the uncertainty in the energy product measurements by the reaction cross-section of total fusion at the known value of energies of collision. When uncertainties were evaluated as below [18]:

$$\delta D_f^{\text{stat}}(E) \approx \frac{\sqrt{6}\,\delta G(E)}{(\Delta E)^2}. \tag{27}$$

## 3. RESULTS AND DISCUSSION

The theoretical results in this section obtained for the cross-section of total fusion $\sigma_{\text{fus}}$, the distribution of the barrier of fusion $D_{\text{fus}}$ and probability of the fusion $P_{\text{fus}}$ by the quantum mechanical route for systems $^6$He+$^{64}$Zn, $^8$B+$^{58}$Ni, and $^8$He+$^{197}$Au. The semiclassical calculations conducted by code SCF and quantum mechanical calculations performed by code CC, the $\sigma_{\text{fus}}$, $D_{\text{fus}}$ and $P_{\text{fus}}$ are compared with measured data. The Akyüz-Winther parameters of the used potential to perform the calculations were listed in Table 1.

**Table 1**
The parameter of the potential of Akyüz-Winther type along with $V_b$

| Proj. + Target | V$_0$ (MeV) | a$_0$ (fm) | r$_0$ (fm) | V$_b$ (MeV) |
|---|---|---|---|---|
| $^6$He+$^{64}$Zn | –43.0 | 0.80 | 1.10 | 8.40 |
| $^8$B+$^{58}$Ni | –96.9 | 0.60 | 1.20 | 20.05 |
| $^8$He+$^{197}$Au | –83.1 | 0.84 | 0.98 | 19.50 |

### 3.1. The reaction $^6$He+$^{64}$Zn

The system $^6$He+$^{64}$Zn includes two neutron light halo nucleus of $^6$He as a projectile nucleus. The calculations for the reaction cross-section of total fusion $\sigma_{\text{fus}}$, the distribution of fusion reaction barrier $D_{\text{fus}}$ and the probability of the fusion $P_{\text{fus}}$ are shown in A, B, and C, respectively, for the system $^6$He+$^{64}$Zn as shown in Fig. 1. The calculations were performed using a CC code.

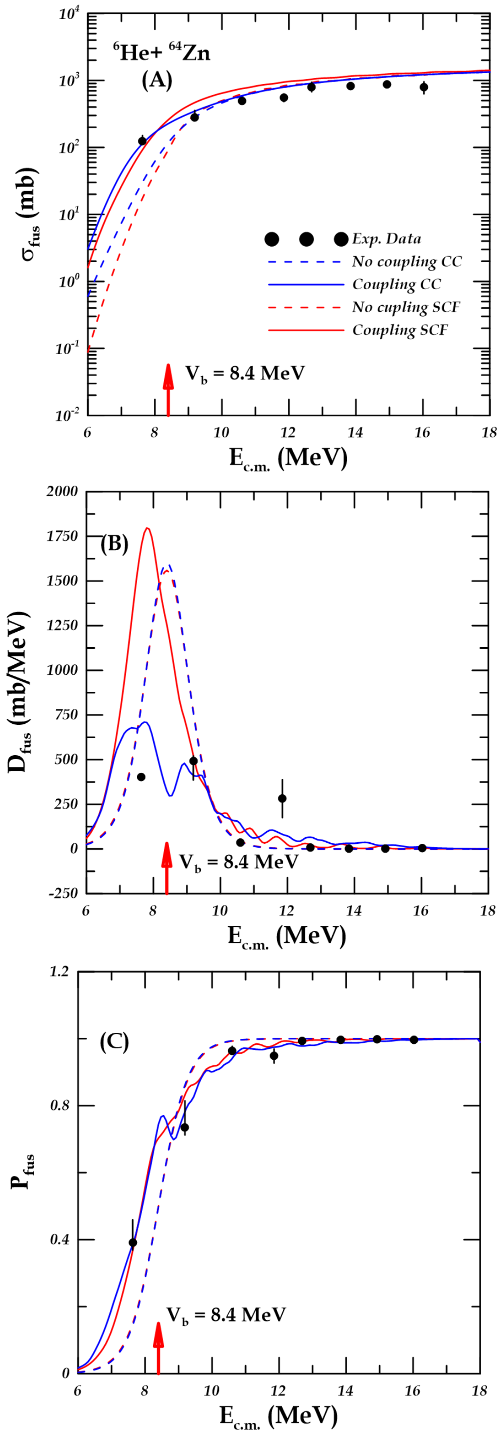

**Fig. 1.** The comparison of the theoretical calculations with measured data [32] for $^6$He+$^{64}$Zn reaction. (A) for cross-section of the total fusion $\sigma_{\text{fus}}$ (mb), and (B) for the distribution of the fusion barrier $D_{\text{fus}}$ (mb/Me V), and (C) the probability $P_{\text{fus}}$

The potential of the Akyüz-Winther type employed in the current study is listed in Table 1. The result of the coupling channel for the $^{6}$He+$^{64}$Zn system agrees with a few numbers of measured data above and below the barrier of Coulomb, as shown in Fig. 1, the data for this system were taken from [33]. Since the studied systems involve halo nuclei (projectile), the calculations below the Coulomb barrier $V_b$ are not in agreement with the measured data, therefore channel coupling is vital in calculations and this fact also must be included in all studied systems in the present work.

### 3.2. The reaction $^{8}$B+$^{58}$Ni

This system includes one proton halo nucleus of projectile $^{8}$B. Figure 1 panel (A), (B) and (C) for $\sigma_{\text{fus}}$ and $D_{\text{fus}}$ and $P_{\text{fus}}$ re spectively, using semiclassical and quantum mechanical treatments for $^{8}$B+$^{58}$Ni fusion reaction were performed by the parameters of Woods-Saxon which are listed in Table 1. The experimental data as shown in Fig. 2 for the current system were collected from [33]. The quantum mechanics theoretical calculations with and without coupling channel show a good match with the measured data.

### 3.3. The reaction $^{8}$He+$^{197}$Au

The cross-section of total fusion $\sigma_{\text{fus}}$, probability $P_{\text{fus}}$ and distribution of fusion barrier $D_{\text{fus}}$ are calculated by using SCF and CC codes, where the projectile $^{8}$He includes four neutrons halo nucleus. The quantum mechanical and semi-classical calculations for this system are shown in Fig. 3A, B and C are for $\sigma_{\text{fus}}$, $D_{\text{fus}}$ and $P_{\text{fus}}$ respectively. The experimental data as shown in Table 1 for this system are obtained from [33]. Theoretical

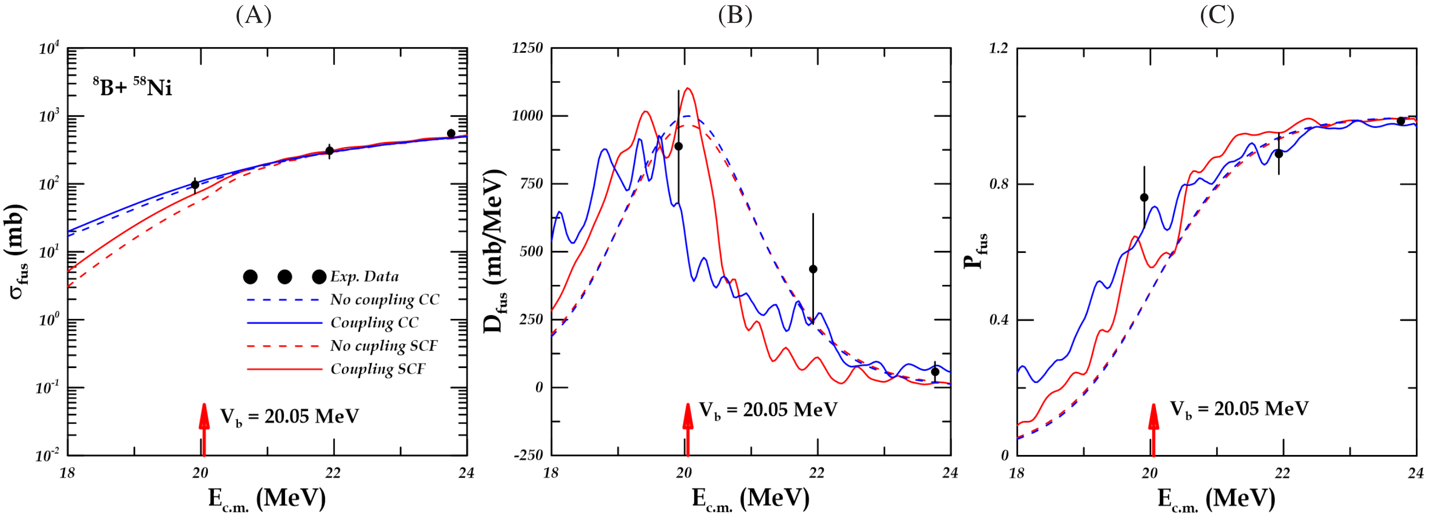

**Fig. 2.** The comparison of theoretical calculations with measured data [33] for $^{8}$B+$^{58}$Ni reaction. (A) for the cross-section of the total fusion $\sigma_{\text{fus}}$ (mb), and (B) for the distribution of fusion barrier $D_{\text{fus}}$ (mb/Me V), and (C) the probability $P_{\text{fus}}$

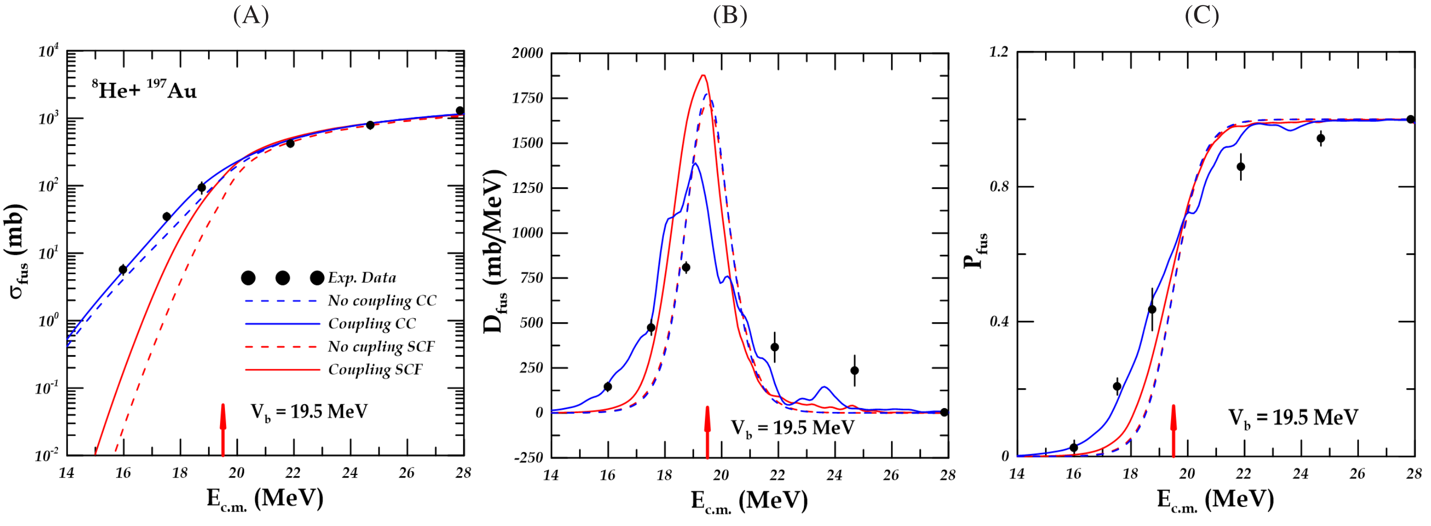

**Fig. 3.** The comparison of theoretical calculations with measured data [33] for $^{8}$He+$^{197}$Au reaction. (A) for cross-section of the total fusion $\sigma_{\text{fus}}$ (mb), and (B) for the distribution of the fusion barrier $D_{\text{fus}}$ (mb/Me V), and (C) the probability $P_{\text{fus}}$

results were compared with the measured data and channel coupling was in good agreement with m below and above the Coulomb barrier.

## 4. CONCLUSION

The conducted study shows clearly the coupling is very important to be considered in the calculations by utilizing the two approaches based on quantum mechanics and semiclassical mechanics for $\sigma_{\text{fus}}$, $D_{\text{fus}}$ and the probability Pfus for the reactions: $^{6}$He+$^{64}$Zn, $^{8}$B+$^{58}$Ni and $^{8}$He+$^{197}$Au. The importance of considering channel coupling arises from the fact that the projectile of the studied systems is loosely bound nuclei. The quantum mechanical results with coupled channels agree reasonably well with the measured data for all reactions under study.

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