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A GEOMETRIC ACCURACY DESIGN METHOD OF MULTI-AXIS NC MACHINE TOOL FOR IMPROVING MACHINING ACCURACY RELIABILITY

METODA PROJEKTOWANIA I POPRAWY NIEZAWODNOŚCI DOKŁADNOŚCI OBRÓBCZEJ WIELOOSIOWEJ OBRABIARKI NC WYKORZYSTUJĄCA POJĘCIE DOKŁADNOŚCI GEOMETRYCZNEJ

The reliability of machining accuracy is of great significance to performance evaluation and optimization design of the machine tools. Different geometric errors have various influences on the machining accuracy of the machine tools. The main emphasis of this paper is to propose a generalized method to distribute geometric accuracy of component for improving machining accuracy reliability under certain design requirements. By applying MBS theory, a comprehensive volumetric model explaining how individual errors in the components of a machine affect its volumetric accuracy (the coupling relationship) was established. In order to reflect the ability to reach the required machining accuracy, the concept of machining accuracy reliability is proposed in this paper. Based on advanced first order and second moment (AFOSM) theory, reliability and sensitivity with single failure modes were obtained and the model of machining accuracy reliability and the model of machining accuracy sensitivity with multiple failure modes were developed. By taking machining accuracy reliability as a measure of the ability of machine tool and taking machining accuracy sensitivity as a reference of optimizing the basic parameters of machine tools to design a machine tool, an accuracy distribution method of machine tools for improving machining accuracy reliability with multiple failure modes was developed and a case study example for a five-axis NC machine tool was used to demonstrate the effectiveness of this method. It is identified that each improvement of the geometric errors leads to a decrease in the maximum values and mean values of possibility of failure, and the gaps among reliability sensitivity of geometric parameter errors improved also decreased. This study suggests that it is possible to obtain the relationships between geometric errors and specify the accuracy grades of main feeding components of mechanical assemblies for improving machining accuracy reliability.

***Keywords**: Machine tool; Geometric error; Accuracy distribution; Machining accuracy reliability; Multi-body system theory.*

Niezawodność w zakresie dokładności obróbki ma wielkie znaczenie dla oceny funkcjonowania oraz projektowania optymalizacyjnego obrabiarek. Różne błędy geometryczne mają różny wpływ na dokładność obrabiarek. Głównym celem niniejszej pracy jest zaproponowanie uogólnionej metody rozkładu dokładności geometrycznej elementów składowych obrabiarki, pozwalającej na poprawę niezawodności w zakresie dokładności obróbczej przy spełnieniu pewnych wymagań projektowych. Dzięki zastosowaniu teorii układów wielomasowych MBS, opracowano kompleksowy model wolumetryczny, który wyjaśnia, w jaki sposób pojedyncze błędy występujące w elementach składowych obrabiarki wpływają na jej dokładność wolumetryczną (relacja sprzężeń). Zaproponowane w prezentowanym artykule pojęcie niezawodności dokładności obróbki odnosi się do możliwości uzyskania przez urządzenie wymaganej dokładności obróbki W oparciu o zaawansowaną teorię estymacji momentów AFOSM (Advanced First Order and Second Moment therory), obliczono niezawodność i czułość dla przypadku wystąpienia pojedynczej przyczyny uszkodzenia oraz opracowano model niezawodności dokładności obróbki oraz model czułości dokładności obróbki dla przypadku wystąpienia wielu przyczyn uszkodzeń. Przyjmując niezawodność dokładności obróbki za miarę poprawnego działania obrabiarki oraz przyjmując czułość dokładności obróbki za punkt odniesienia dla optymalizacji projektowej podstawowych parametrów obrabiarek, opracowano metodę, opartą na rozkładzie dokładności obrabiarki, mającą na celu poprawę niezawodności dokładności obróbki dla przypadku wystąpienia wielu przyczyn uszkodzeń. Skuteczność metody wykazano na przykładzie pięcio-osiowej obrabiarki NC. Stwierdzono, że każda korekta błędu geometrycznego prowadzi do spadku maksymalnych i średnich wartości możliwości wystąpienia uszkodzenia oraz zmniejsza rozstęp między poszczególnymi czułościami niezawodnościowymi skorygowanych błędów parametrów geometrycznych. Przedstawione badania wskazują, że możliwe jest ustalenie związku pomiędzy błędami geometrycznymi oraz określenie stopni dokładności głównych elementów składowych zespołów mechanicznych odpowiedzialnych za ruch posuwowy obrabiarki w celu poprawy niezawodności dokładności obróbki.

***Słowa kluczowe**: Obrabiarka; Błąd geometryczny; Rozkład dokładności; niezawodność dokładności obróbki; Teoria układów wielomasowych.*

1. Introduction

Machining accuracy is critical for the quality and performance of a mechanical product and is an important consideration for any manufacturer. It is influenced by machining errors belonging to several categories, e.g. geometric errors caused by mechanical-geometric imperfections, misalignments, wear of the linkages and elements of the machine tool structure, by the non-uniform thermal expansion of the machine structure, and static/dynamic load induced errors [8]. Geometric errors include pitch errors of the lead screws, straightness errors of the guide ways, angular errors of machine slides, and orthogonal error among machine axis [3]. Because its contribution to 30% of the total error so it is given special consideration through the configuration and allocation of appropriate dimensional errors in the design of machine tools with satisfactory machining accuracy [19]. Machine tools are usually made by several assembling parts, and the dimensional and geometric variations of each part have to be specified by tolerances which guarantee a certain level of quality in terms of satisfying functional requirements [20]. A as a result, the distribution of accuracy of machine tools is a problem of distribution of tolerances of these geometric errors. However, this practice of allocation has not been developed and applied in any systematic manner to the design of machine tools.

To enhance the machining accuracy of CNC machine tools, there are two steps included in the accuracy design. The first is accuracy prediction, which refers to forecasting volumetric errors of machine tools based upon the known accuracy for updated and maintained parts, and then predict the machining accuracy of work piece. The other is accuracy allocation, which is to obtain the accuracy of updated and maintained parts from the preset total accuracy of machine tools, and let the accuracy of parts reach optimal scheme [15]. Before accuracy distribution, a error modeling is crucial to maximize the performance of machine tools, and robust and accurate volumetric error modeling is also the first step to correct and compensate these errors [5], therefore, a model explaining how individual error of the components of a machine affect its volumetric accuracy is crucial to the accuracy distribution approach and it is one aspect of importance of this paper.

Geometric errors for each of the assembled parts and components are random variables; the machining errors caused by such errors are also random [40]. As a result, the dimensional and geometric variations for each error source are random and have to be specified by variance (or standard deviation) and the probabilistic nature of the errors produced on the dimensions of a manufactured part is taken into consideration in this paper. Uncertainty in parameters such as material, loading, geometry and the model exist in the process of structural design and optimization, and it results from data shortage, model simplification and human error [18]. Some of these errors can be reduced by collecting more data, with a better understanding of the problem and by implementing strict quality control; however, others such as deviations due to random events cannot be reduced by the above means [6]. To solve this problem, two main philosophies dealing with the uncertain deviations exist: 'Deterministic Structural Optimization' (DSO) uses safety-factors to accomplish the safety, while 'Reliability-Based Structural Optimization' (RBSO) takes the random character of the variables into consideration [27]. Reliability-based design optimization simulates all the uncertain variables to the random variables or random process and minimizes an objective function under probabilistic constraints. The reliability of machine tool reflects the ability to perform its specified functions under the stated conditions for a given period of time and it is often studied by possibility of failure [26], so accuracy distribution based on the reliability theory can maintain and improve the level of quality in terms of satisfying functional requirements while keeping the maximum tolerances of these geometric errors. As a result, the development of a systematic method to realize accuracy distribution of machine tool based on reliability theory according to failure modes for improving machining accuracy reliability is the second very important aspect of this paper.

The rest of this study is organized as follows: In Section 2, the review of accuracy design of machine tool is given. Section 3 explains the process of modeling machining tool geometric errors. Section 4 presents the proposed method to realize accuracy distribution of machine tool. A case study is accomplished as an example in Section 5. The conclusions are presented in the last Section.

2. Accuracy Design of NC Machine Tool

2.1. Accuracy prediction

Up to present, there are many researches on the error modeling technique to show the difference between the actual response of a machine to a command issued according to the accepted protocol of that machine's operation and the response to that command anticipated by that protocol [9]. The development of modeling methods has been experienced many years and it turns out to be various kinds such as matrix translation method, error matrix method, rigid body kinematic, D-H method, model methods based on the multi-body system kinematics theory and so on. In 1973 Love and Scarr obtained the combined effects of the elemental errors in the machine tool and then developed the volumetric errors of a multi-axis machine by using the trigonometric technique [25]. In 1977 a matrix translation method was reported and a calibration technique using three-dimensional metrology on a coordinate measuring machine (CMM) was presented by Hocken et al. [13]. In 1981 Dufour and Groppetti reported the "error matrix" method to obtain error predictions by interpolation between the stored values [10]. In 1982 Portman used rigid body kinematics to evolve an expression for the geometric error of a mechanism [30]. In 1991, Kiriden developed a general model to understand the effects of component geometric errors on the kinematic chain of a machine and the volumetric errors in the work space [21]. In 1993, they used the D–H convention to develop kinematic models for three types of machine. In 1995 a method based on direct consideration of the shape and joint transformations was put forward by Srivastava et al. [33]. In 2007 Bohez et al. presented a new method to identify and compensate the systematic errors in a multi-axis machine tool [1]. In recent years, multi-body system (MBS) theory is used to generalize and provide a unique systematic approach for its advantages such as stylization, normalization, versatility, and ease for computer modeling [36]. There are many investigators have carried out error modeling research for complicated machinery system using MBS [41, 23, 44, 39], mainly focus on designing and constructing a model to determine geometric error of machine tool and developing the key technique for compensation-identifying geometric error parameters.[43, 17, 4, 7, 22, 2, 11] introduce the methods of geometric error compensation, thermal error modeling, position error compensation, position-independent geometric errors modeling, volumetric error modeling and sensitivity analysis and establishing a product of exponential(POE) model for geometric error integration.

2.2. Accuracy allocation

In earlier years, many researches focused their attention to obtain the tolerance allocation on structural design. In 2005 Prabhaharan et al. introduced a kind of metaheuristic approach as an optimization tool for minimizing the critical dimension deviation and allocating the cost-based optimal tolerances [29]. In 2006 Huang and Shiau obtained the optimized tolerance allocation of a sliding vane rotary compressor's components for the required reliability with the minimum cost and quality loss [16]. In 2007 Huang and Zhong established the sequential linear optimization models based on the process capabilities to release the working tolerances, reduce manufacturing costs [14]. Siva Kumar and Stalin [31] used Lagrange multiplier method to simultaneously allocate both design and manufacturing tolerances based on minimum total manufacturing cost. Isabel González et al. [12] developed a methodology to allow an automatic tolerance allocation capable of minimizing manufacturing costs based on statistical approach. Muthu et al. [28] used metaheuristic method to balance the manufacturing cost and quality loss to achieve near optimal design and process tolerances simultaneously for minimum combined manufacturing cost and quality loss over the life of the product. In 2010 K. Sivakumar, et al proposed a novel multi-objective optimization method to enhance the operations of the non-traditional algorithms and Multi-Objective Particle Swarm Optimization and systematically distribute the tolerances among various the components of mechanical assemblies [32]. From the above literature, the previous researches on tolerance allocation mainly focus on structural design taking manufacturing cost or manufacturing process into consideration, besides, there are many of accuracy allocation in the field of hull construction, robotics, military application, and instruments [35, 24, 37, 42, 34]. However, works on accuracy allocation of multi-axis machine tools are few. Reliability is a specification to measure the ability of machine tool to overcome a certain functions and reliability sensitivity reflects the influence of basic parameters to the possibility of failure, so reliability theory plays an important role in accuracy allocation of NC machine tools by determining the levels for these geometric parameter errors, what's more, multi-axis NC machine tools composed of various parts are complex structures and so they have multiple failure modes. In 1994 Dorndorf U proposed an error allocation approach to optimize allocation of manufacturing and assembly tolerances along with specifying the operating conditions to determine the optimal level for these errors so that the cost is minimized [9], it is regardless of reliability sensitivity and the model is a two-axis machine. In 2013 Yu proposed a geometric error propagation model and reliability approximately model to by response surface method with error samples and improved the functions of machine tools by optimization of the sensitivity [40] with single failure model. As a result, the continuous effort lies on accuracy allocation of NC machine tools taking machining accuracy reliability and sensitivity with multiple failure modes into consideration.

3. Error modeling of NC machine tool

There are two important aspects in this study: the first is the development of a systematic approach to obtain geometric/kinematic errors on the kinematic chain of a machine tool. Another aspect, considered more important, is the proposing of the concept of machining accuracy reliability and the addressing a reliability and sensitivity analysis method in the multiple failure modes to realize the distribution of the standard deviation for the geometric errors.

A method called MBS is used to establish a machine tool geometric/kinematic error model showing the relation between the individual error of the components of a machine, and its volumetric accuracy is crucial to the allocation of standard deviation for the geometric parameter errors. In this paper, a 5-axis machine tool is used to analyze geometric errors and the geometric/kinematic error model is developed. This XKH1600 five-axis machining center is designed for leaf blade machining, configured as three linear axes X, Y, Z axes and two rotary axes A, B axis. The 3-dimension digital structure model of the machine tool is shown in Figure 1.

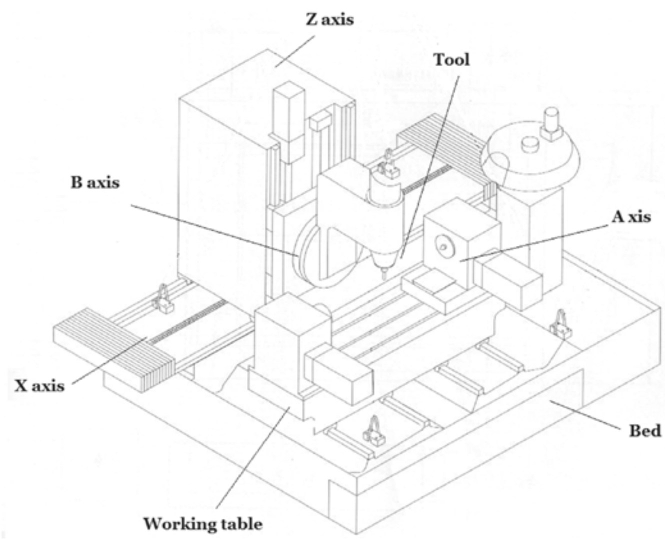

Fig. 1. The 3-dimension digital structure model of the NC machine tool

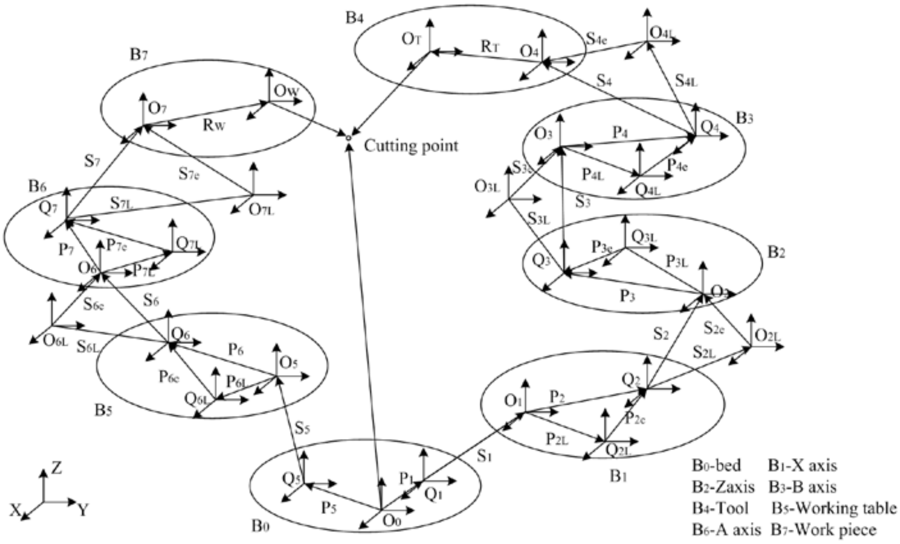

Fig. 2. The coordinate system structure diagram

Table 1 Lower body array of five-axis NC machine tool

Topic body j	1	2	3	4	5	6	7
$L^0(j)$	1	2	3	4	5	6	7
$L^1(j)$	0	1	2	3	0	5	6
$L^2(j)$	0	0	1	2	0	0	0
$L^3(j)$	0	0	0	1	0	0	0
$L^4(j)$	0	0	0	0	0	0	0
$L^5(j)$	0	0	0	0	0	0	0
$L^6(j)$	0	0	0	0	0	0	0

Taking the error factors and coupling relations of the various parts into consideration, based on the MBS theory, the five-axis machine tool can be abstracted into a multi-body system. The coordinate system structure diagram is shown in Figure 2. The error model between B_K and B_j revealed, when the displacement and the displacement error are zero, O_k and Q_k coincide, q_k refers to the initial position vector between O_j (B_j 's origin) and O_k (B_K 's origin), and q_{ke} is the position error vector, including load error and thermal error. S_k is the displacement vector between B_K and B_j , and S_{ke} is the displacement error vector, including geometric error and dynamic error. When there is a displacement in one part or component of a machine tool, this displacement is the position increment. The lower body array is listed in Table 1.

$$\mathbf{P}_t = \begin{bmatrix} P_{tx} & P_{ty} & P_{tz} & 1 \end{bmatrix}^T \tag{1}$$

The work-piece forming point in the work-piece coordinate system coordinate is:

$$\mathbf{P}_w = \begin{bmatrix} P_{wx} & P_{wy} & P_{wz} & 1 \end{bmatrix}^T \tag{2}$$

When the machine tool moves in ideal form, that means the machine tool is without error, the ideal forming function of tool forming point in work-piece coordinate system is:

$$\left(\prod_{k=n,L^n(t)=0}^{k=1} \boldsymbol{T}^P_{L^k(t)L^{k-1}(t)} T^S_{L^k(t)L^{k-1}(t)} \right) \boldsymbol{P}_t = \left(\prod_{u=n,L^n(w)=0}^{u=1} \boldsymbol{T}^P_{L^u(w)L^{u-1}(w)} \boldsymbol{T}^S_{L^u(w)L^{u-1}(w)} \right) \boldsymbol{P}_w \tag{3}$$

In this paper, the ideal forming function of tool forming point of this five-axis machine tool is:

$$\begin{aligned} \boldsymbol{P}_t &= \left(\prod_{k=n,L^n(t)=0}^{k=1} \boldsymbol{T}^P_{L^k(t)L^{k-1}(t)} T^S_{L^k(t)L^{k-1}(t)} \right)^{-1} \left(\prod_{u=n,L^n(w)=0}^{u=1} \boldsymbol{T}^P_{L^u(w)L^{u-1}(w)} \boldsymbol{T}^S_{L^u(w)L^{u-1}(w)} \right) \boldsymbol{P}_w \\ &= \begin{pmatrix} 1 & 0 & 0 & -x_t \\ 0 & 1 & 0 & -y_t \\ 0 & 0 & 1 & -z_t \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -\cos\varphi & \sin\varphi & 0 & 0 \\ -\sin\varphi & -\cos\varphi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\cos A & \sin A & 0 \\ 0 & -\sin A & -\cos A & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\ & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\ & \begin{pmatrix} -\cos B & 0 & -\sin B & 0 \\ 0 & 1 & 0 & 0 \\ \sin B & 0 & -\cos B & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & x_{wd} \\ 0 & 1 & 0 & y_{wd} \\ 0 & 0 & 1 & z_{wd} \\ 0 & 0 & 0 & 1 \end{pmatrix} \boldsymbol{P}_w \end{aligned} \tag{4}$$

Machining accuracy is determined by relative displacement error between the tool forming point of machine and work-piece. During the actual machining process, the actual position of cutting tool point will inevitably deviate from the ideal location, which results in volumetric error. Table 2 lists the characteristic matrices of this 5-axis CNC machining center, including body ideal static, motion characteristic matrix ($\boldsymbol{T}^P_{L^k(t)L^{k-1}(t)}, T^S_{L^k(t)L^{k-1}(t)}$) and body static, kinematic error characteristic matrix ($\Delta\boldsymbol{T}^P_{L^k(t)L^{k-1}(t)}, \Delta T^S_{L^k(t)L^{k-1}(t)}$).As a result, the comprehensive volumetric error caused by the gap between actual point and ideal point in this paper can be written as:

In the above equation, the values and means of the expressions such as T^P_{05} , ΔT^P_{05} , T^S_{05} , ΔT^S_{05} and so on can be obtained from Table 2.

4. Accuracy allocation based on Reliability Theory

In Section 3, the development of a systematic approach to obtain geometric/kinematic errors on the kinematic chain of a machine tool has been finished. As a result, the concept of machining accuracy reliability should be proposed, besides, a reliability and sensitivity analysis method with multiple failure modes for improving machining accuracy reliability to realize the geometric errors allocation should be addressed.

4.1. The concept of Machining Accuracy Reliability and AFOSM Theory

The reliability of structure reflects the ability to perform its specified functions under the stated conditions for a given period of time and it is often studied by possibility of failure. In order to reflect the ability of machine tools to reach the required machining accuracy, machining accuracy reliability is proposed in this paper and can be defined as possibility of fulfilling the specified machining accuracy, expressed by the possibility of failure of machine accuracy. Besides, the probabilistic nature of the errors produced on the dimensions of a manufactured part is taken into consideration in this paper. In order to develop an approach to obtain the machining accuracy reliability and sensitivity of this five-axis machine tool, there is an assumption that the errors produced on a dimension are drawn from a Gaussian distribution. Our task of error allocation is therefore to determine the optimal levels for these geometric parameter errors according to the machining accuracy reliability. In order to overcome this task, Advanced First Order and Second Moment (AFOSM) theory is introduced at first.

Supposing that there is a performance function $Z = g(x_1, x_2, \ldots, x_n)$, with some uncorrelated parameters $x_i \sim N(\mu_{x_i}, \sigma_{x_i})$, which are subject to random variation about their nominal values, let the functional requirements be of the form Z=0 (limit state equation) which divides the parameter space into two regions, one a failure domain can be expressed as $F = \{T : g(x) \le 0\}$ and the other is the safe domain.

Supposing that there is a design point (the most possibility failure point) $\boldsymbol{P}^*(x_1^*, x_2^*, ..., x_n^*)$ in the failure domain, then $g(x_1^*, x_2^*, ..., x_n^*) = 0$ is obtained and $G_i = g(x_1^*, x_2^*, ..., x_n^*) + \sum_{i=1}^{n} (\frac{\partial g}{\partial x_i})_{\mathrm{p}^*} (x_i - x_i^*)$ can be developed to represent the linear part as to $Z = g(x_1, x_2, \ldots, x_n)$ by Taylor, so the performance equation is $\sum_{i=1}^{n} (\frac{\partial g}{\partial x_i})_{\mathrm{p}^*} (x_i - x_i^*)$ =0.

Suppose $\mu_z = \sum_{i=1}^{n} (\mu_{x_i} - x_i^*)(\frac{\partial g}{\partial x_i})_{\mathrm{p}^*}$, $\sigma_z^2 \approx \sum_{i=1}^{n} [(\frac{\partial g}{\partial x_i})_{\mathrm{p}^*} \sigma_{x_i}]^2$,

$\alpha_i = \dfrac{(\frac{\partial g}{\partial x_i})_{\mathrm{p}^*} \sigma_{x_i}}{\{\sum_{i=1}^{n} [(\frac{\partial g}{\partial x_i})_{\mathrm{p}^*} \sigma_{x_i}]^2\}^{1/2}}$, $(i = 1, 2, ..., n)$ which refers to the Sensi-

$$\begin{aligned} E &= \left[\prod_{u=n,L^n(7)=0}^{u=5} T^P_{L^u(7)L^{u-1}(7)} \Delta T^P_{L^u(7)L^{u-1}(7)} T^S_{L^u(7)L^{u-1}(7)} \Delta T^S_{L^u(7)L^{u-1}(7)} \right] P_w - \left[\prod_{u=n,L^n(4)=0}^{u=1} T^P_{L^u(4)L^{u-1}(4)} \Delta T^P_{L^u(4)L^{u-1}(4)} T^S_{L^u(4)L^{u-1}(4)} \Delta T^S_{L^u(4)L^{u-1}(4)} \right] p_t \\ &= T^P_{05} \Delta T^P_{05} T^S_{05} \Delta T^S_{05} T^P_{56} T^S_{56} \Delta T^S_{56} T^P_{67} \Delta T^P_{67} T^S_{67} P_w - T^P_{01} T^S_{01} \Delta T^S_{01} T^P_{12} \Delta T^P_{12} T^S_{12} \Delta T^S_{12} T^S_{23} \Delta T^P_{23} \Delta T^S_{23} T^P_{34} \Delta T^P_{34} p_t \end{aligned} \tag{5}$$

Table 2. Characteristic matrices of the 5-axis NC machining center

adjacent body	Body ideal static, motion characteristic matrix ($T^P_{L^k(t)L^{k-1}(t)}$, $T^S_{L^k(t)L^{k-1}(t)}$)	Body static, kinematic error characteristic matrix ($\Delta T^P_{L^k(t)L^{k-1}(t)}$, $\Delta T^S_{L^k(t)L^{k-1}(t)}$)
0-1 X axis	$T_{01P}=I_{4\times4}$ $T_{01s}=T_{01s(x)}=\begin{pmatrix}1&0&0&x\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$	$\Delta T_{01s}=\begin{pmatrix}1&-\Delta\gamma_x&\Delta\beta_x&\Delta x_x\\\Delta\gamma_x&1&-\Delta\alpha_x&\Delta y_x\\-\Delta\beta_x&\Delta\alpha_x&1&\Delta z_x\\0&0&0&1\end{pmatrix}$
1-2 Z axis	$T_{12P}=I_{4\times4}$ $T_{12s}=T_{12s(z)}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&z\\0&0&0&1\end{pmatrix}$	$\Delta T_{12p}=\begin{pmatrix}1&0&\Delta\beta_{xz}&0\\0&1&-\Delta\alpha_{yz}&0\\-\Delta\beta_{xz}&\Delta\alpha_{yz}&1&0\\0&0&0&1\end{pmatrix}$ $\Delta T_{12s}=\begin{pmatrix}1&-\Delta\gamma_z&\Delta\beta_z&\Delta x_z\\\Delta\gamma_z&1&-\Delta\alpha_z&\Delta y_z\\-\Delta\beta_z&\Delta\alpha_z&1&\Delta z_z\\0&0&0&1\end{pmatrix}$
2-3 B axis	$T_{23P}=I_{4\times4}$ $T_{23s}=\begin{pmatrix}\cos B&0&\sin B&0\\0&1&0&0\\-\sin B&0&\cos B&0\\0&0&0&1\end{pmatrix}$	$\Delta T_{23p}=\begin{pmatrix}1&-\Delta\gamma_{yb}&0&0\\\Delta\gamma_{yb}&1&-\Delta\alpha_{yb}&0\\0&\Delta\alpha_{yb}&1&0\\0&0&0&1\end{pmatrix}$ $\Delta T_{23s}=\begin{pmatrix}1&-\Delta\gamma_B&\Delta\beta_B&\Delta x_B\\\Delta\gamma_B&1&-\Delta\alpha_B&\Delta y_B\\-\Delta\beta_B&\Delta\alpha_B&1&\Delta z_B\\0&0&0&1\end{pmatrix}$
3-4 Tool	$T_{34p}=\begin{pmatrix}1&0&0&x_{td}\\0&1&0&y_{td}\\0&0&1&z_{td}\\0&0&0&1\end{pmatrix}$ $T_{34S}=I_{4\times4}$	$\Delta T_{34p}=\begin{pmatrix}1&-\Delta\gamma_{td}&\Delta\beta_{td}&\Delta x_{td}\\\Delta\gamma_{td}&1&-\Delta\alpha_{td}&\Delta y_{td}\\-\Delta\beta_{td}&\Delta\alpha_{td}&1&\Delta z_{td}\\0&0&0&1\end{pmatrix}$ $T_{34P}=I_{4\times4}$
0-5 Y axis	$T_{05P}=I_{4\times4}$ $T_{05s}=\begin{pmatrix}1&0&0&0\\0&1&0&y\\0&0&1&0\\0&0&0&1\end{pmatrix}$	$\Delta T_{05p}=\begin{pmatrix}1&-\Delta\gamma_{xy}&0&0\\\Delta\gamma_{xy}&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$ $\Delta T_{05s}=\begin{pmatrix}1&-\Delta\gamma_y&\Delta\beta_y&\Delta x_y\\\Delta\gamma_y&1&-\Delta\alpha_y&\Delta y_y\\-\Delta\beta_y&\Delta\alpha_y&1&\Delta z_y\\0&0&0&1\end{pmatrix}$
5-6 A axis	$T_{56P}=I_{4\times4}$ $\mathrm{T}_{56s}=\begin{bmatrix}1&0&0&0\\0&\cos A&-\sin A&0\\0&\sin A&\cos A&0\\0&0&0&1\end{bmatrix}$	$\Delta T_{56\mathrm{p}}=\begin{bmatrix}1&-\Delta\gamma_{ya}&\Delta\beta_{xa}&0\\\Delta\gamma_{ya}&1&0&0\\-\Delta\beta_{xa}&0&1&0\\0&0&0&1\end{bmatrix}$ $\Delta T_{56\mathrm{s}}=\begin{bmatrix}1&-\Delta\gamma_a&\Delta\beta_a&\Delta x_a\\\Delta\gamma_a&1&-\Delta\alpha_a&\Delta y_a\\-\Delta\beta_a&\Delta\alpha_a&1&\Delta z_a\\0&0&0&1\end{bmatrix}$
6-7 Work piece	$T_{67p}=\begin{pmatrix}1&0&0&x_{wd}\\0&1&0&y_{wd}\\0&0&1&z_{wd}\\0&0&0&1\end{pmatrix}$ $T_{67P}=I_{4\times4}$	$\Delta T_{67p}=\begin{pmatrix}1&-\Delta\gamma_{wd}&\Delta\beta_{wd}&\Delta x_{wd}\\\Delta\gamma_{wd}&1&-\Delta\alpha_{wd}&\Delta y_{wd}\\-\Delta\beta_{wd}&\Delta\alpha_{wd}&1&\Delta z_{wd}\\0&0&0&1\end{pmatrix}$

tivity coefficient, reflecting the influence of the random parameter x_i acting on the σ_z, so $\sigma_z = (\frac{\partial g}{\partial x_i})_{p^*} \sigma_{x_i} \alpha_i$, and then the reliability index and the possibility of failure are obtained as follows[26]:

$$\beta = \frac{\mu_z}{\sigma_z} = \frac{\sum_{i=1}^{n} (\mu_{x_i} - x_i^*)(\frac{\partial g}{\partial x_i})_{p^*}}{(\frac{\partial g}{\partial x_i})_{p^*} \sigma_{x_i} \alpha_i} \quad (6)$$

$$P_f = \Phi(-\beta) \quad (7)$$

Compare with machining accuracy reliability as a specification to measure the ability of machine tool to overcome a certain functions, machining accuracy sensitivity reflects the influence of basic parameters to the possibility of failure, which can be used for improving and optimizing the basic parameters of machine tool. As a result, machining accuracy sensitivity plays an important role in determining the levels for these geometric parameter errors. Based on the AFOSM theory, the performance function $Z = g(x_1, x_2, \dots, x_n)$ can be changed:

4.2. An AFOSM Reliability and Sensitivity Analysis Method with Multiple Failure Modes

Failure mode is critical to the reliability of any structure. The number of limit state equations divides failure mode into two parts: single failure mode and multiple failure modes. Single failure mode which means that there is only one limit state equation in the whole system or structure, in contract, multiple failure modes refers to multiple limit state equations in the system and its complexes lies in the logical relationship, correlation coefficient and joint probability density of a system or a structure with multiple failure modes[38]. According to the logical relationship with the failure modes of a machine tool, this machine tool has multiple failure modes and so it is a serial system. As a result, this paper aims to the reliability and sensitivity analysis of such serial system.

4.2.1. The Narrow Bounds Method

The narrow bounds is the main method used for the reliability and sensitivity analysis of a serial system though it fails to obtain the certain valves of reliability and sensitivity, as a result, it is used for verification of the method introduced in this paper.

Supposing that there are "m" failure modes, the reliability P {F} and the reliability sensitivity $\frac{\partial P\{F\}}{\partial \sigma_{x_i}}$ based on AFOSM Theory are expressed as follows [9]:

$$P\{F\} \subseteq [P\{F_1\} + \sum_{2}^{m} \max[P\{F_i\} - \sum_{j=1}^{i-1} P\{F_i \cap F_j\}; 0], \sum_{i=1}^{m} P\{F_i\} - \sum_{i=2}^{m} \max(j < i) P\{F_i \cap F_j\}] \quad (9)$$

$$\frac{\partial P\{F\}}{\partial \sigma_{x_i}} = \sum_{j=1}^{n} [f(\beta_j) \frac{\partial(\beta_j)}{\partial \sigma_{x_i}}(-1)] + \sum_{j=2}^{n} \sum_{k=1}^{j-1} [{}^{(3)}\tau_{jk} [{}^{(1)}\tau_{jk} ({}^{(4)}\tau_{jk} \cdot {}^{(4)}\zeta_{\sigma_{x_i}} + {}^{(5)}\tau_{jk} \cdot {}^{(5)}\zeta_{\sigma_{x_i}}) + {}^{(2)}\tau_{jk} \cdot {}^{(2)}\zeta_{\sigma_{x_i}}]] \quad (10)$$

into the following equation at the design point $\boldsymbol{P}^*(x_1^*, x_2^*, \dots, x_n^*)$,

$$g \approx g(x_1^*, x_2^*, \dots, x_n^*) + \sum_{i=1}^{n} (\frac{\partial g}{\partial x_i})_{p^*} (x_i - x_i^*).$$

Suppose $c_0 = g(x_1^*, x_2^*, \dots, x_n^*) - \sum_{i=1}^{n} (\frac{\partial g}{\partial x_i})_{p^*} x_i^*$, $c_i = (\frac{\partial g}{\partial x_i})_{p^*}$,

Then,

$$g(x) \approx G(x) = c_0 + \sum_{i=1}^{n} c_i x_i$$

In this way, $\beta = \frac{\mu_G}{\sigma_G} = \frac{c_0 + \sum_{i=1}^{n} c_i \mu_{x_i}}{\sqrt{\sum_{i=1}^{n} c_i^2 \sigma_{x_i}^2}}$ and $P_f = \Phi(-\beta)$ are obtained.

From the following equations, the reliability sensitivity can be obtained as follows:

$$\frac{\partial P_f}{\partial \sigma_{x_i}} = \frac{\partial P_f}{\partial \beta} \frac{\partial \beta}{\partial \sigma_{x_i}} = -\frac{c_i^2 \sigma_{x_i}^2 \mu_G}{\sqrt{2\pi} \sigma_G^3} \exp[-\frac{1}{2}(\frac{\mu_G}{\sigma_G})^2] \quad (8)$$

In the above expressions, F_i and F_j are the events of failing with respect to the Z_i and Z_j condition respectively.

$P(F_i) = \Phi(-\beta_i)\Phi(-\frac{\beta_j - \rho_{ij}\beta_i}{\sqrt{1-\rho_{ij}^2}})$, $P(F_j) = \Phi(-\beta_j)\Phi(-\frac{\beta_i - \rho_{ij}\beta_j}{\sqrt{1-\rho_{ij}^2}})$,

$\rho_{ij} = \frac{q_i q_j Cov(g_i, g_j)}{\sqrt{Var[g_i]}\sqrt{Var[g_j]}}$, β_i and β_j are the reliability indices with respect to the ith and jth failure conditions respectively, and ρ_{ij} is the correlation between the two failure conditions, q_i is positive when Z>0 and is negative when Z≤0.

$${}^{(2)}\zeta_{\sigma_{x_i}} = {}^{(4)}\zeta_{\sigma_{x_i}} + {}^{(5)}\zeta_{\sigma_{x_i}},$$

$${}^{(4)}\zeta_{\sigma_{x_i}} = \phi(-\beta_j)\frac{\partial}{\partial \sigma_{x_i}}(-\beta_j)\Phi(-u_{jk}) + \phi(-u_{jk})\frac{\partial}{\partial \sigma_{x_i}}(-u_{jk})\Phi(-\beta_j)$$

$${}^{(5)}\zeta_{\sigma_{x_i}} = \phi(-\beta_k)\frac{\partial}{\partial \sigma_{x_i}}(-\beta_k)\Phi(-v_{jk}) + \phi(-v_{jk})\frac{\partial}{\partial \sigma_{x_i}}(-v_{jk})\Phi(-\beta_k),$$

$$\phi(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2},$$

$$\frac{\partial \beta_j}{\partial \sigma_{x_i}} = \frac{\left(c_0 + \sum_{i=1}^{n} c_i \mu_{x_i}\right) c_i^2 \sigma_{x_i}}{[\sum_{i=1}^{n} c_i^2 \sigma_{x_i}^2]^{3/2}}, \; u_{ij} = \frac{\beta_j - \beta_i \rho_{ij}}{\sqrt{1-\rho_{ij}^2}}, \; v_{ij} = \frac{\beta_i - \beta_j \rho_{ij}}{\sqrt{1-\rho_{ij}^2}},$$

$${}^{(1)}\tau_{jk} = \begin{cases} 1 & \tau_{jk} \geq 0 \\ 0 & \tau_{jk} < 0 \end{cases},$$

$${}^{(2)}\tau_{jk} = \begin{cases} 0 & \tau_{jk} \geq 0 \\ 1 & \tau_{jk} < 0 \end{cases},$$

$${}^{(1)}\tau_{jk} + {}^{(2)}\tau_{jk} = 1$$

$${}^{(3)}\tau_{jk} = \begin{cases} 1 & if\ P_{fjk} = \max(k<j) P_{fjk} \\ 0 & otherwise \end{cases}$$

$${}^{(4)}\tau_{jk} = \begin{cases} 1 & if\ \Phi(-\beta_j)\Phi(-u_{jk}) \geq \Phi(-\beta_k)\Phi(-v_{jk}) \\ 0 & otherwise \end{cases}$$

$${}^{(5)}\tau_{jk} = \begin{cases} 1 & if\ \ \Phi(-\beta_k)\Phi(-v_{jk}) > \Phi(-\beta_j)\Phi(-u_{jk}) \\ 0 & otherwise \end{cases}$$

$${}^{(4)}\tau_{jk} + {}^{(5)}\tau_{jk} = 1$$

4.2.2. A Method for Reliability and Sensitivity Analysis with Multiple Failure Modes

For the failure to obtain the certain valves of the reliability and sensitivity by the narrow bounds, a method used for reliability and sensitivity analysis with multiple failure modes was introduced in this paper and the solution procedure roughly follows the approach developed by[38].

Given that there are "m" multiple failure modes in a system, the calculation of its possibility of failure is to obtain the integration of a multidimensional normal joint distribution. As a result, the possibility of failure of a serial structure with multiple failure modes is given by:

$$P\{F\}=P(\bigcup_{i=1}^{m} F_i < 0)=1\text{-}\Phi(\beta;\rho)=1\text{-}\int_{-\infty}^{\beta_1}\int_{-\infty}^{\beta_2}...\int_{-\infty}^{\beta_m} \varphi_m(m;\rho) dm_1 dm_2 ... dm_m \tag{11}$$

As to a general serial system, the possibility of failure can be expressed as follows:

$$P\{F\} = P\{(F_1 \leq 0) \bigcup (F_2 \leq 0) \bigcup ... \bigcup (F_m \leq 0)\} \tag{12}$$

In order to calculate the equation (12), the correlation coefficient is proposed to show the correlation with multiple failure modes. If there are two failure modes, the possibility of failure can be expressed as follows:

$$P\{F_{12}\} = (F_1 \bigcup F_2) = P(F_1) + P(F_2) - P(F_1 \bigcap F_2) = P(F_1) + P(F_2) - P(F_1 F_2) \tag{13}$$

Supposing $P(F_1 F_2) = \alpha_{12} P(F_2)$, α_{12} is the correlation coefficient of these two failure modes. As a result, $P\{F_{12}\} = P(F_1) + P(F_2) - \alpha_{12} P(F_2) = P(F_1) + (1-\alpha_{12}) P(F_2)$ and the possibility of failure with three failure modes can be given by:

$$\begin{aligned} P(F_{123}) &= P(F_1 \bigcup F_2 \bigcup F_3) = P(F_{12} \bigcup F_3) \\ &= P(F_{12}) + P(F_3) - \alpha_{123} P(F_3) = P(F_{12}) + (1-\alpha_{123}) P(F_3) \end{aligned} \tag{14}$$

In a similar way, the possibility of failure with "m" multiple failure modes in a serial system can be obtained by:

$$\begin{aligned} P(F) &= P(F_{12...m-1}) + (1-\alpha_{12...m}) P(F_m) \\ &= P(F_1) + (1-\alpha_{12}) P(F_2) + (1-\alpha_{123}) P(F_3) + ... + (1-\alpha_{12...m}) P(F_m) \end{aligned} \tag{15}$$

In the equation (15), $\alpha_{ij} \approx \Phi(\frac{-\beta_i + \rho_{ij}\Upsilon_j}{\sqrt{1-\rho_{ij}^2 \Upsilon_j(-\beta_j + \Upsilon_j)}})$,

$\Upsilon_j = \frac{\varphi(-\beta_j)}{\Phi(-\beta_j)}$.

Up to now, the possibility of failure with "m" multiple failure modes is obtained and the next work is to focus attention on the calculation of reliability sensitivity. For lack of space, a description of the complete theoretical analysis is avoided and only a conceptual description and important results are given.

$$\begin{aligned} \frac{\partial P(F)}{\partial \sigma_{x_i}} &= \frac{\partial P(F_1)}{\partial \sigma_{x_i}} + (1-\alpha_{12})\frac{\partial P(F_2)}{\partial \sigma_{x_i}} + (-P(F_2))\frac{\partial \alpha_{12}}{\partial \sigma_{x_i}} + (1-\alpha_{123})\frac{\partial P(F_3)}{\partial \sigma_{x_i}} \\ &+(-P(F_3))\frac{\partial \alpha_{123}}{\partial \sigma_{x_i}} + ... + (1-\alpha_{12...m})\frac{\partial P(F_m)}{\partial \sigma_{x_i}} + (-P(F_m))\frac{\partial \alpha_{12...m}}{\partial \sigma_{x_i}} \end{aligned} \tag{16}$$

In the equation (16), $\frac{\partial P(F_i)}{\partial \sigma_{x_i}}$ can be obtain by equation(8) and $\frac{\partial \alpha_{ij}}{\partial \sigma_{x_i}} = \phi(\beta_{ij})\frac{\partial \beta_{ij}}{\partial \sigma_{x_i}}$, $\Xi_j = \Upsilon_j(-\beta_j + \Upsilon_j)$,

$$\frac{\partial \beta_{ij}}{\partial \sigma_{x_i}} = \frac{1}{\sqrt{1-\rho_{ij}^2 \Xi_j}}(\frac{\partial \beta_i}{\partial \sigma_{x_i}} + \rho_{ij}\frac{\partial \Upsilon_j}{\partial \sigma_{x_i}}) + \frac{-\beta_i + \rho_{ij}\Upsilon_j}{2(1-\rho_{ij}^2 \Xi_j)^{\frac{3}{2}}} \rho_{ij}^2 \frac{\partial \Xi_j}{\partial \sigma_{x_i}},$$

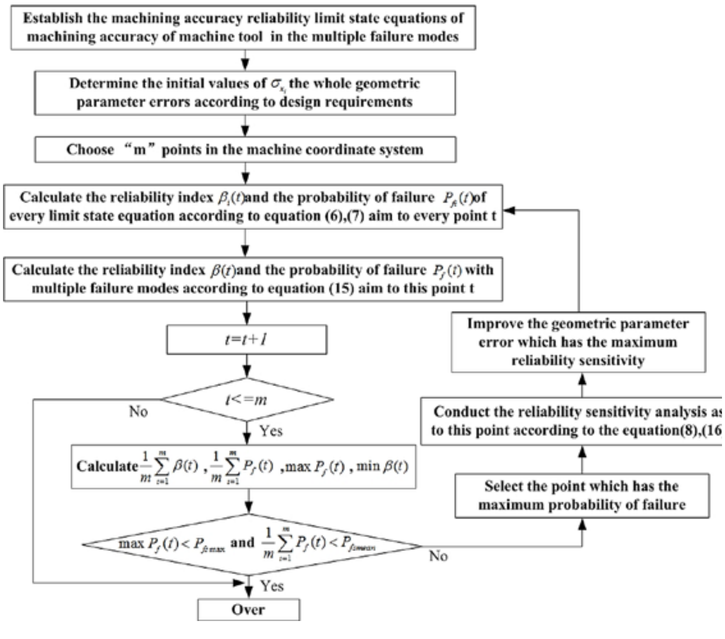

Fig. 3. The flowchart of method for geometric accuracy allocation of multi-axis machine tool for improving machining accuracy reliability

$$\frac{\partial \Upsilon_j}{\partial \sigma_{x_i}} = \frac{-\Phi(-\beta_j)\beta_j\phi(-\beta_j)+\phi^2(-\beta_j)}{\Phi^2(-\beta_j)}(-\frac{\partial \beta_j}{\partial \sigma_{x_i}}),$$

$$\frac{\partial \Xi_j}{\partial \sigma_{x_i}} = (2\Upsilon_j - \beta_j)\frac{\partial \Upsilon_j}{\partial \sigma_{x_i}} - \Upsilon_j \frac{\partial \beta_2}{\partial \sigma_{x_i}}$$

In order to develop an approach to distribute this five-axis machine tool on machining accuracy, the method used for the machining accuracy reliability and sensitivity with multiple failure modes is introduced in this paper and the calculation of the reliability and reliability sensitivity is developed in MatLab software. Besides, the process of this accuracy distribution method in this paper can be shown in Figure 3.

5. Case Study

In the preceding section, a method of geometric accuracy design of multi-axis machine tool for improving machining accuracy reliability was introduced and a machining accuracy reliability model and a sensitivity model with multiple failure modes based on AFOSM theory were established. Previous to it, a method for relating the volumetric accuracy of a machine to the errors on individual links and joints in its kinematic chain was described and the error model can be expressed by the form:

$$\boldsymbol{E} = \left[E_X, E_Y, E_Z\right]^T \tag{17}$$

In this paper, the machining accuracy reliability limit state equations of the machine tool are subject to the expression as follows:

$$\begin{aligned} l_{11} \le E_x \le l_{12} \\ l_{21} \le E_Y \le l_{22} \\ l_{31} \le E_Z \le l_{32} \end{aligned} \tag{18}$$

From the equation (18), it is not difficult to notice that this machine tool has 26 failure modes in total, including 6 single failure modes, 12 double failure modes and 8 triple failure modes.

One of important aspect of the design requirements of such machine tool is that the probability of failure of position error less than 0.03mm should be no more than 5%. According to “Test code for machine tools- Part 1: Geometric accuracy of machines operating under no-load or finishing conditions” and “Test code for machine tools-Part 2: Determination of accuracy and repeatability of positioning numerically controlled axes”, the values of geometric parameter errors of five-axis NC machine tool were set initially and were shown in Table 3.

Table 3. Initial values of geometric parameter errors of five-axis NC machine tool (mm)

Number i	1	2	3	4	5	6	7	8
Parameter	Δx_x	Δy_x	Δz_x	$\Delta\alpha_x$	$\Delta\beta_x$	$\Delta\gamma_x$	Δx_y	Δy_y
Value	0.0065	0.0065	0.0065	0.0037/1000	0.0037/1000	0.0037/1000	0.007	0.007
Number i	9	10	11	12	13	14	15	16
Parameter	Δz_y	$\Delta\alpha_y$	$\Delta\beta_y$	$\Delta\gamma_y$	Δx_z	Δy_z	Δx_z	Δx_z
Value	0.007	0.0028/1000	0.0028/1000	0.0028/1000	0.007	0.007	0.007	0.0028/1000
Number i	17	18	19	20	21	22	23	24
Parameter	$\Delta\beta_z$	$\Delta\gamma_z$	Δx_A	Δy_A	Δz_A	$\Delta\alpha_A$	$\Delta\beta_A$	$\Delta\gamma_A$
Value	0.0028/1000	0.0028/1000	0.0058	0.0058	0.0058	0.0061/1000	0.0061/1000	0.0061/1000
Number i	25	26	27	28	29	30	31	32
Parameter	Δx_B	Δy_B	Δz_B	$\Delta\alpha_B$	$\Delta\beta_B$	$\Delta\gamma_B$	$\Delta\gamma_{xy}$	$\Delta\beta_{xz}$
Value	0.0068	0.0068	0.0068	0.0049/1000	0.0049/1000	0.0049/1000	0.0037/500	0.0037/500
Number i	33	34	35	36	37			
Parameter	$\Delta\alpha_{yz}$	$\Delta\gamma_{xa}$	$\Delta\beta_{xa}$	$\Delta\gamma_{yb}$	$\Delta\alpha_{yb}$			
Value	0.0037/500	0.011/300	0.011/300	0.011/300	0.011/300			

In order to improve the machining accuracy reliability of this five-axis NC machine tool, the models based on AFOSM theory addressed in section 4 was used. The process was divided into two steps: one was to determine some points by orthogonal sampling method in the X-, Y- plane of this machine tool to obtain the reliability, and the other was to realize accuracy allocation of all the geometric parameter errors of this machine tool. Five points 50, 225, 275, 325, 500 were chosen in X-axis and −225, −50, 0, 50, 225 were selected in Y-axis, as a result, there were 25 points in X-,Y- plane. The machining accuracy reliability model for accuracy allocation of machine tool addressed in this paper used to optimize errors was given by the expression (15), and its process of calculation was operated in the MatLab environment. According to the constraint conditions, if $\max P_f(t) > 5\%$ or $\frac{1}{m}\sum_{t=1}^{m} P_f(t) > 3\%$, $\max P_f(t)$ was obtained and chosen to calculate $\frac{\partial \max P_f(t)}{\partial \sigma_{x_i}}$ to determine the geometric parameter error to be improved. This process was over until $\max P_f(t) < P_{fs\max}$ and $\frac{1}{m}\sum_{t=1}^{m} P_f(t) < P_{fsmean}$. In this expression, $l_{11}, l_{12}, l_{13}, l_{21}, l_{22}, l_{23}$ were obtained though putting the vector of each point into the equation (5) and σ_{x_i} should be no more than the initial valves shown in Table 3. In order to verify the effectiveness of the method introduced in this paper, the possibility of failure of this machine tool based on the narrow bounds method was also calculated. The values of the possibility of failure of machining accuracy before improvement were acquired and shown in Table 4.

$$
\begin{aligned}
M \quad &: \max P_f(t), \max \frac{\partial \max P_f(t)}{\partial \sigma_{x_i}} \\
\text{S. t.} \quad &: l_{11} \le E_x \le l_{12} \\
&\quad l_{21} \le E_Y \le l_{22} \\
&\quad l_{31} \le E_Z \le l_{32} \\
&\quad 0 < \sigma_{x_i} \le initial\ values \\
&\quad \frac{1}{m}\sum_{t=1}^{m} P_f(t) \le 3\% \\
&\quad \max P_f(t) \le 5\%
\end{aligned}
$$

It is not difficult to notice that there are two important points in Table 4: one is that the method developed in this paper is verified because the values of possibility of failure calculated by such method are in the intervals obtained according to the narrow bounds method, and the other is that the initial values of the reliability of this machine tool failed to satisfy the design requirement (no more than 5% and 3% respectively). As a result, it is necessary to reallocate the initial values of geometric parameter errors in Table 3 according to the sensitivities of such errors shown in Figure 4. In order to satisfy the design requirement five times improvements have been developed. From the Figure 4 to Figure 8 we can learn that geometric parameter errors $\Delta\beta_{xz}$, $\Delta\alpha_{yz}$, $\Delta\alpha_y$, $\Delta\gamma_x$, $\Delta\beta_x$ have the largest values of reliability sensitivity and have been improved as the changing objects in turn as a result, besides, the machine accuracy reliability (expressed by possibility of failure) with these five improvements have been calculated as shown in Table 5. It suggests that the whole process of accuracy distribution have been finished after the forth improvements.

Figure 9 illustrates the relation between the probability of failure and each improvement, it is clear that each improvement leads to a decrease in the maximum values and mean values of possibility of failure and such decrease becomes very tight after a certain improvement. Figure 10 illustrates the relation between the reliability sensitivity of geometric parameter errors improved and each improve-

Table 4. Values of the possibility of failure of machining accuracy before improvement

Vector of point i	Narrow bounds method	The method in this paper	
		Reliability index	Possibility of failure
(50, 225)	[0.0961,0.1132]	1.236	0.1079
(50,50)	[0.0689,0.0821]	1.427	0.0766
(50.0)	[0.0531,0.0598]	1.578	0.0572
(50,-50)	[0.0689,0.0821]	1.427	0.0766
(50, -225)	[0.0961,0.1132]	1.236	0.1079
(225, 225)	[0.0997,0.1124]	1.234	0.1083
(225,50)	[0.0689,0.0821]	1.426	0.0766
(225,0)	[0.0531,0.0598]	1.578	0.0572
(225,-50)	[0.0689,0.0821]	1.426	0.0766
(225, -225)	[0.0997,0.1124]	1.234	0.1083
(275, 225)	[0.0997,0.1124]	1.233	0.1083
(275,50)	[0.0689,0.0821]	1.426	0.0766
(275,0)	[0.0556,0.0606]	1.576	0.0575
(275,-50)	[0.0689,0.0821]	1.426	0.0766
(275, -225)	[0.0997,0.1124]	1.233	0.1083
(325, 225)	[0.1042,0.1134]	1.231	0.1088
(325,50)	[0.0689,0.0821]	1.421	0.0772
(325,0)	[0.0556,0.0606]	1.576	0.0575
(325,-50)	[0.0689,0.0821]	1.421	0.0772
(325, -225)	[0.1042,0.1134]	1.231	0.1088
(500, 225)	[0.1042,0.1134]	1.231	0.1088
(500,50)	[0.0668,0.0792]	1.421	0.0772
(500,0)	[0.0567,0.0605]	1.564	0.0584
(500,-50)	[0.0668,0.0792]	1.421	0.0772
(500, -225)	[0.1042,0.1134]	1.231	0.1088
Mean value		1.378	0.0856

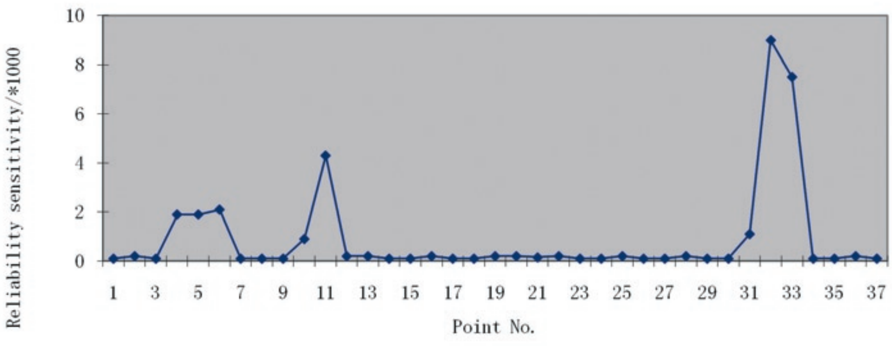

Fig. 4. Initial values of the reliability sensitivity of geometric parameter errors

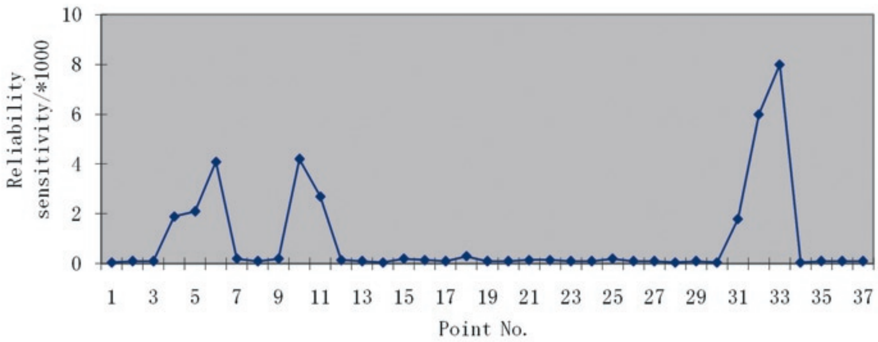

Fig. 5. Reliability sensitivity of geometric parameter errors after the first improvement

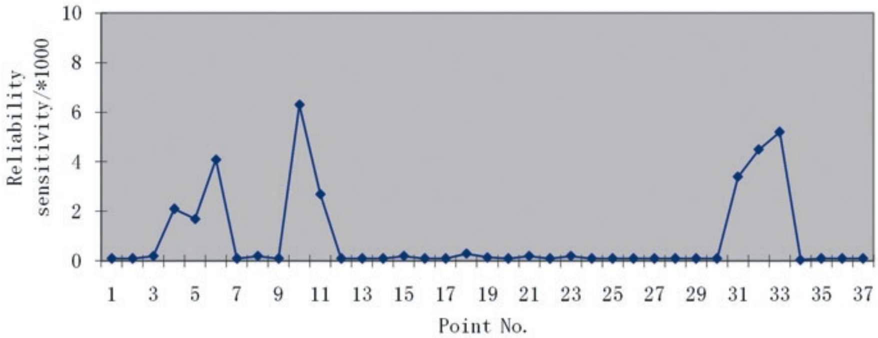

Fig. 6. Reliability sensitivity of geometric parameter errors after the second improvement

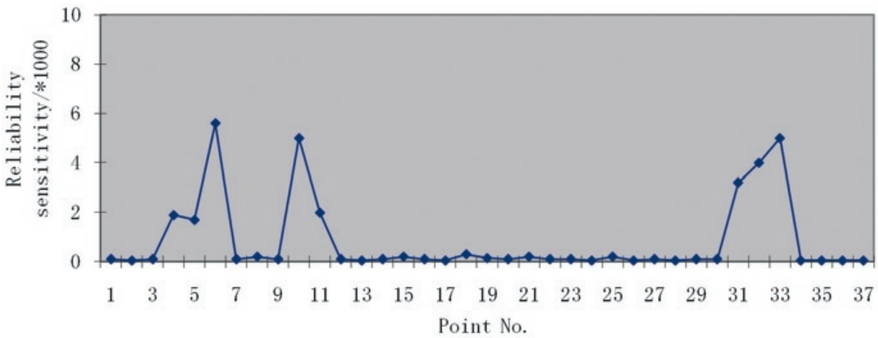

Fig. 7. Reliability sensitivity of geometric parameter errors after the third improvement

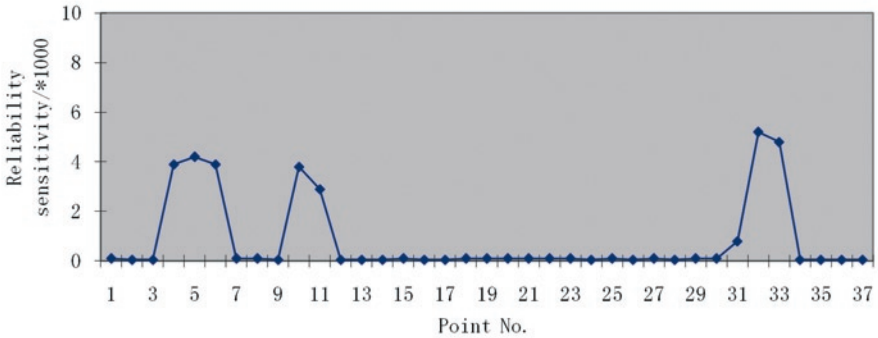

Fig. 8. Reliability sensitivity of geometric parameter errors after the fourth improvement

ment. As expected, with each improvement, the gaps among reliability sensitivity of geometric parameter errors improved decreased because the aim of each improvement is to reduce or eliminate the largest reliability sensitivity of geometric parameter errors. Figure 11 illustrates that the values of reliability sensitivity of geometric parameter errors unimproved change in a small scope, which means that these errors have smaller sensitivity to the reliability of machine tool in this machining case.

Conclusions

Different geometric errors have varying influence on accuracy of the machine tools. The main emphasis of this research is to develop a generalized method to obtain an optimal σ_i of each geometric error under certain machine accuracy design requirements. For machine accuracy reliability can be as a specification to measure the ability of machine tools to overcome a certain functions and reliability sensitivity can reflect the influence of basic parameters to the possibility of failure, which can be used for improving and optimizing the basic parameters of machine tools, the accuracy distribution of machine tools for improving the machine accuracy reliability was a process of optimizing or reallocating σ_i of each geometric error by taking reliability as a measure of the ability of machine tool and taking reliability sensitivity as a reference of optimizing the basic parameters of machine tools to design a machine tool.

In order to realize the distribution of σ_i of each geometric error, a generalized machine accuracy reliability model and a sensitivity model for accuracy distribution of machine tools with multiple failure modes are developed here. Based on such models, the optimal σ_i of each geometric error can be obtained for improving accuracy of machine tools. This study contains:

(1) A comprehensive volumetric model explains how individual errors in the components of a machine affect its volumetric accuracy (the coupling relationship). This was established by MBS theory, which shows the geometric structure, the change of comparative position and orientation, and the geometric errors.

(2) The definition of machine accuracy reliability of machine tool was given .Because it can reflect the ability to perform its specified functions and machine accuracy sensitivity can be used for optimizing the basic parameters of machine tools, accuracy distribution based on the reliability theory can improve the level of machining accuracy in terms of satisfying design requirements such as machining accuracy reliability while keeping the maximum tolerances of these geometric errors. As a result, the development of a systematic method to realize accuracy distribution of machine tool based on reliability theory according to failure modes is the second very important aspect of this paper.

(3) In order to use the systematic method to realize accuracy distribution of machine tool, a reliability theory called advanced first order and second moment (AFOSM) theory was introduced to obtain the reli-

Table 5. Improved values of errors and possibility of failure in different improvements

		The first improvement	The second improvement	The third improvement	The forth improvement	The fifth improvement
Improved object		$\Delta\beta_{xz}$	$\Delta\alpha_{yz}$	$\Delta\alpha_y$	$\Delta\gamma_x$	$\Delta\beta_x$
Parameter errors	Initial value	0.0037/500	0.0037/500	0.0037/1000	0.0028/1000	0.0037/1000
	Improved value	0.003/500	0.003/500	0.003/1000	0.002/1000	0.003/1000
Possibility of failure	maximum value	8.96%	7.32%	5.89%	4.57%	3.95%
	Mean value	7.36%	5.53%	4.25%	2.79%	2.43%

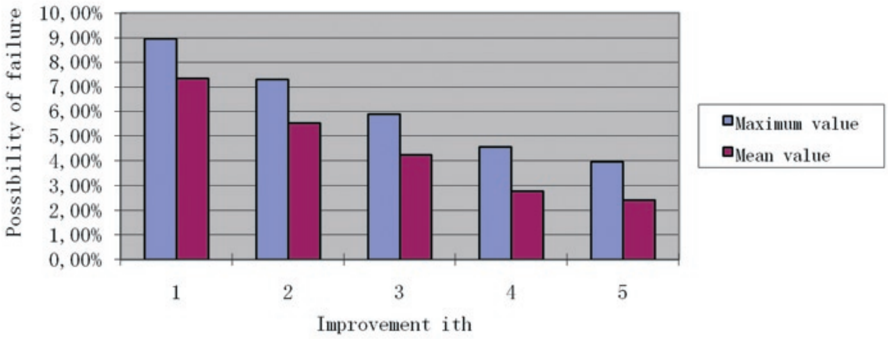

Fig. 9. Tendency of the maximum values and mean values of possibility of failure with different improvements

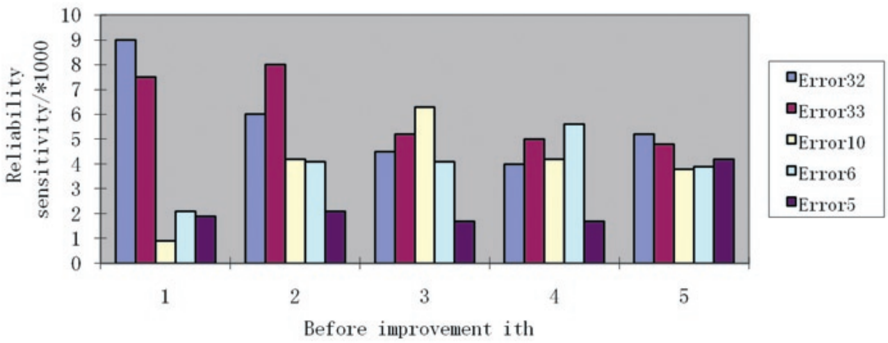

Fig. 10. Tendency of the reliability sensitivity of geometric parameter errors improved

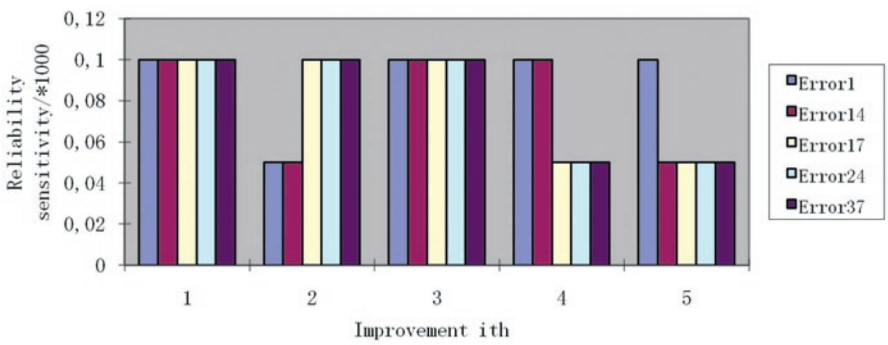

Fig. 11. Tendency of the reliability sensitivity of geometric parameter errors unimproved

ability and sensitivity with single failure mode. Besides, the multiple failure modes based on AFOSM theory were studied and the model of reliability and the model of sensitivity of a serial system with multiple failure modes were established.

(4) Based on the previous theoretical research, a case study example for a five-axis machine tool was used to demonstrate the effectiveness of this method. Each geometric error under certain accuracy design requirements was optimized. Accuracy grades of main feeding components of mechanical assemblies were also specified.

Despite the progress, a number of issues need to be further investigated to improve the method proposed here: the proposed method is developed on the premise that the volumetric machining accuracy is influenced only geometric errors, however, thermal errors and dynamic errors as other main errors can arise as a result of joint interface deformation between machine tool structural components and contribute to volumetric machining accuracy. As a result, the development of a method of accuracy allocation of machine tool for improving machining accuracy reliability while considering such errors would be a focus of future research.

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Appendix Nomenclature

Δx_x = Positioning error
Δy_x = Y direction of straightness error
Δz_x = Z direction of straightness error
$\Delta\alpha_x$ = Rolling error
$\Delta\beta_x$ = Britain swing error
$\Delta\gamma_x$ = Yaw error
Δx_y = X direction of straightness error
Δy_y = Positioning error
Δz_y = Z direction of straightness error
$\Delta\alpha_y$ = Rolling error
$\Delta\beta_y$ = Britain swing error
$\Delta\gamma_y$ = Yaw error
Δx_z = X direction of straightness error
Δy_z = Y direction of straightness error
Δz_z = Positioning error
$\Delta\alpha_z$ = Rolling error
$\Delta\beta_z$ = Britain swing error
$\Delta\gamma_z$ = Yaw error
Δx_B = X direction run-out error
Δy_B = Y direction run-out error
Δz_B = Z direction run-out error
$\Delta\alpha_B$ = Around the X axis turning error
$\Delta\beta_B$ = Turning error
$\Delta\gamma_B$ = Around the Z axis turning error
Δx_A = X direction run-out error
Δy_A = Y direction run-out error
Δz_A = Z direction run-out error
$\Delta\alpha_A$ = Turning error
$\Delta\beta_A$ = Around the Y axis turning error
$\Delta\gamma_A$ = Around the Z axis turning error
$\Delta\gamma_{xy}$ = X, Y -axis perpendicularity error
$\Delta\beta_{xz}$ = X, Z -axis perpendicularity error
$\Delta\alpha_{yz}$ = Y, Z -axis perpendicularity error
$\Delta\gamma_{yb}$ = B, X- axis parallelism error in XY plane
$\Delta\alpha_{yb}$ = B, Z- axis parallelism error in ZY plane
$\Delta\gamma_{xa}$ = A, Y-axis perpendicularity error
$\Delta\beta_{xa}$ = A, Z-axis perpendicularity error

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