

SELECTED ISSUES IN MODELLING OF TRAFFIC FLOWS IN CONGESTED URBAN NETWORKS

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Abstract: *Making rational decisions about the planning and designing the traffic management in the city requires a proper description of traffic flows following through the various elements of the transportation network. This issue is the subject of many studies, resulting in a wide variety of models used in this field. Generally they can be divided into two main groups: models describing the distribution of traffic flows in the transportation network and models describing the transition of traffic flow by individual elements of the transportation network. This article reviews the models used to describe the traffic shaping in such an arrangement. Then the way of describing traffic flows, which may be used in the construction and calibration of dynamic traffic models has been formalized. The article also includes a calculation example with application of the proposed description of the components of traffic flows on the link of urban network.*

Key words: *transportation system, congestion, traffic flows, urban networks*

1. Introduction

Traffic modelling in urban transportation networks is particularly complex. Urban area is the place of the strong interaction of different socio-economic, technical, organizational and economic issues. This affects the functioning of the different transportation subsystems, which on the one hand, are characterized by distinct features, on the other - are interdependent. This interaction also relates to traffic flows passing through the transportation network.

In urban transportation system diverse needs of users are competing at the same time for a limited capacity of road network elements. This often leads to congestion that results in the formation of significant queues, and even blocking large part of the city to traffic. These phenomena, in turn, could contribute to a reduction in safety, performance and efficiency of movement. They may also adversely affect on the environment and health of residents [10]. Such situations can be a potential cause of disturbances in traffic flow as well as of changes in decision-making of users of the transportation system related to the way they travel [68]. Structural imbalance of the proper operation of the transportation system in urban areas causes thus interference in the functioning of the entire settlement unit [60].

Multidimensionality of the issue requires the introduction of the notion that is wider than transportation system. Therefore, for the purposes of modelling of traffic flow in congested networks, the notion of the urban system has been applied. It covers not only real objects associated with transportation infrastructure, but also the relationships of an economic, legal, technological, and organizational nature. Such system also includes the social characteristics of users and spatial development of the study area. Its structure is presented in the form of urban network [9, 31, 65, 66]. The transportation network is thus a part of the urban network, which also contains, inter alia, zone centroids of the individual areas that represent aggregated locations of activity system of the area. The activity system one should be treated as a set of individual, social, and economic behaviors and interactions that give rise to travel demand [6]. The way of linking the activities with the transportation system, defined by a certain level of accessibility, affects the formation of traffic flows in the urban area under study.

Congestion may be defined in different ways. The very notion of congestion should be understood as a situation where the larger number of purchasers is applying for some of the good that cannot be supplied in the form of separate units

[20, 58]. The essence of transportation congestion is the interaction between users resulting in adverse exploitative and economic effects [11]. From the operational point of view, congestion is the difference in the costs of resources between road network functioning under real conditions and the network operated under ideal conditions where there is no delays and traffic flow is moving at maximum safe speed [52, 60]. Transportation congestion is also defined as the mutual obstruction in moving by vehicles, which is a consequence of the relationship between the speed of the traffic and its intensity, under conditions where the level of service—understood as a degree of capacity utilization—of the transportation system reaches a value close to its limit [13, 56].

Participants of the OECD countries Conference on Managing Urban Traffic Congestion found that there is no universal definition of congestion occurring on the road. Therefore it should be treated primarily as a relative phenomenon, which is determined by the difference between the performance expected by the users of the road system and the way in which this system operates [49]. From this perspective congestion may be considered in the context of qualitative gap between expected and realized quality. User expectations regarding the performance of the road system are thus essential to understand the perception of congestion. The same levels of congestion may be seen both as a burdensome (unacceptable) traffic conditions and the satisfactory ones depending on the road users-related attributes, such as age, driving experience, temperament, and others. Further factors may vary depending on the destination and the knowledge of the network by travellers.

Assessment of traffic conditions by the users of the transportation system and their perception of the level of congestion determine the choice of means of transport and the way of passing through the transportation network. Hence, modelling of traffic flows in the urban network is a particularly difficult issue. The first attempts to mathematical description of traffic flows in the transportation network date back to the 20th of the last century. It was then that Frank Knight [36] described the equilibrium of traffic in network, which was formalized by J. G. Wardrop

in the form of first and second principles of equilibrium only in 1952 [25, 61]. He also introduced the alternative behaviour postulate of the minimization of the total travel costs. The first mathematical model of network equilibrium was formulated by M. J. Beckmann, C. B. McGuire and C. B. Winston in 1956 [1]. However, despite the significant progress of technology and possibilities in efficient use of IT tools, it has yet set any universal theory that could describe the real traffic conditions in a comprehensive way.

Description of the traffic flow is complex and is of a non-linear nature, which is strongly determined by the interactions between individual units of traffic flow. Due to the individual reactions of users of transportation network, the analogies referring to the rules in force in the mechanical sciences are not sufficient for mapping the structure of traffic and its dynamics. In the description of traffic flow one should also reach for models of human behaviour applied in sociology, demography and psychology. This is particularly important in the case of high traffic density where the user decision-making processes with respect to the way of travelling play an important role.

In general, models describing the traffic flow may be divided into two groups:

- models describing the distribution of traffic flows in the transportation network,
- models describing the transition of traffic flow by individual elements of the transportation network.

2. Models describing the distribution of traffic flows in the transportation network

Models belonging to the first group are used in the stage of the traffic flow planning and take into account the relationships between the demand side and the supply one of the transportation system in the city [6, 50, 51]. Travel demand, as an aggregation of individual trips, is formed based on the spatial distribution of the activities subsystem elements (locations of households and economic activity objects) causing transport needs and as a result - trips of persons and transport of goods. Household members make long-term mobility choices (e.g. holding a driving license, owning a car, etc) and short-term travel choices (e.g. trip frequency, time, its destination,

modes of transport used and the path of travel). They use transportation system so that they could undertake different activities (work, study, shopping) in various places. The choices made by users result in travel demand flows understood as the trips made by people between the different zones of the city, for different purposes, in different periods of the day and by means of the different transportation modes available for them. Similarly, economic activities require transportation of goods that are consumed by other activities or by households. Finally, the aggregated traffic flows—consisting of vehicles carrying people and freight—rise in urban network. Summing up, travel demand model may be defined as a mathematical relationship between travel demand trips and their characteristics on the one hand, and given activity and transportation supply systems and their characteristics on the other one. Table 1 shows the basic demand modelling methods [6].

Table 1. Classification of travel demand models

TYPE OF MODEL	MAIN ASSUMPTIONS
<i>trip-based travel-demand models</i>	choices relating to each origin-destination trip are made independently of the choices for other trips within the same and other journeys
<i>trip-chaining travel-demand models</i>	choices concerning the entire journey influence each other
<i>activity-based demand models</i>	predict travel demand as the outcome of the need to participate in different activities in different locations and at different times

Source: own research based on [6].

Travel demand models used in the planning of traffic flows in the cities consist of several submodels. In the classical approach, based on the foundations of microeconomic theory (e.g. [29, 44]), it includes four parts, which are also a mathematical representation of the subsequent stages of the formation of traffic flows in urban network. Successively, they are following partial models [6, 33, 37, 51]:

- trip generation model,
- trip distribution model,
- modal split model,
- traffic assignment model.

The classic approach assumes that, despite some feedbacks, individual submodels are implemented in a sequential manner. This means that, at each stage the results achieved in the previous model are applied. In the literature one can meet a wide variety of modifications of such a system, involving the aggregation of certain stages or changing the sequence of their implementation (e.g. [6, 16, 51]).

Four-step demand model is based on a behavioural approach which assumes that users behave rationally when making decisions about the need, time, place and manner of travel in the urban network. In this approach, the objective function—defined from the point of view of the user—is the difference between the benefits resulting from the joining the two activities conducted in separate geographic locations, and the costs of travelling (e.g. the cost of fuel, travel time, etc.) [54]. It is important to differentiate this function from the perspective of different groups of users. This means that each of them selects a specific destination, mean of transport and path of travel in such a way that the difference between the subjective benefits and costs was maximal [54]. However, in terms of transport it is difficult to assess the benefits of the travel for individual users. Therefore, the benefits are often ignored in the analyses and the objective function is expressed as a generalized cost of travel. This means that each user tends to minimize his or her general costs related to the sequential passing individual elements of urban network [67].

One of the main disadvantages of the classical travel demand models is the loss of many detailed information about how the individual trip is made due to the aggregation to the level of the trip. Accordingly, activity-based demand models, which take account of the relationship between individual chains of trip made by individuals or households during a day, are increasingly applied in modelling of traffic flows [6, 12, 38, 43]. These models are mainly based on the assumptions of the Swedish theory of time geography developed by T. Hägerstrand [23] in the 70s of the last century. The theory emphasizes the importance of

temporal and spatial constraints in accessibility to transport. On this base so called. potential path space is constructed [46]. The big advantage of these behavioural models is the ability to description and to modelling multipurpose trips made by the users of the transportation system with the distinction of their social-economic features such as gender, age or ethnic group. However, the description of traffic flows based on the time geography is difficult due to the problems with obtaining accurate data on the schedule of the daily activities of urban network users. These models are also very complex in the case when the assumption of different speeds of traffic flows is made.

In turn, authors of the study [3] applied nonlinear model of traffic flow (ang. PFE - Path Flow Estimator) to determine the origin-destination (O-D) matrix. This model is based on the assumption of stochastic user equilibrium (SUE), which gives a clearly identified volume of traffic flow on the individual links of the network and does not require knowledge of volume of traffic flow on the other links. This assumption is treated as an advantage of PFE model because it allows also to choose several paths with different travel times for the fixed O-D pair. The main component of PFE model is logit path choice submodel, which—after taking account the cost function on the sections of network—gives the traffic assignment of the stochastic nature.

Also noteworthy is the application of agent models in modelling of traffic flows. In these models, individual households are represented by so called. agents who can make independent decisions relating to their behaviour. Such an approach can be used for different time horizons, ranging from long-term decisions relating to place of residence, through medium-decisions related to planning activities during the day, and ending on short-term decisions made during the movement in the urban network [45].

3. Models describing the transition of traffic flow by individual elements of the transportation network

Models describing the passing of traffic flow through a single element of transportation network are also called traffic flow propagation models [42]. Sections of the transportation network in an urban area are loaded with traffic

flows, which may be decomposed into individual traffic streams. Analysis of traffic stream passing through the individual elements of the urban network mainly involves determining the empirical relationship between its basic characteristics such as intensity, density and velocity [21, 22, 34, 35, 42, 59]. The relations between these values for uniform traffic flow are presented in the form of so called. fundamental traffic diagram. Functional form describing the dependence studied is similar for all types of roads, but actual road and traffic conditions—that are specific for every road section—affect the exact shape of the curves [15, 21, 22, 47, 48]. Among the most important factors influencing the fundamental form of empirical relationships one should mention: the structure of the traffic flow, behaviour of drivers, the size of the intervals between successive vehicles, the geometrical parameters and the view of the road, weather conditions, road environment and factors associated with the measurement technique [19]. Table 2 shows the classification of such models. The level of aggregation has been assumed as a criterion for division. A detailed description of each of these types of models can be found, inter alia, in [19, 30, 42].

Selection of the parameters describing the traffic flows varies depending on the research needs. For example, in planning the quantitative characteristics of traffic stream, such as travel time of traffic stream, its uniformity and structure may be relevant [31]. In turn, from the point of view of traffic management an important are such features of traffic stream as intensity, the way of organizing (e.g. traffic completely organized, self-organizing and borderland situations), the way of control (traffic full or partly controlled and uncontrolled traffic) or the flexibility in the traffic organization [57]. Traffic conditions may also be described in a qualitative way by means of so called. level of service, which are the measure that takes into account the feelings of drivers and of other road users [26].

J. Woch [62] applied the theory of smoothness to description of disruptions in traffic flows. Smoothness of traffic flow is an important measure of the efficiency of the transportation network nodes and corresponds to the expected number of units in the traffic stream that will not

be disturbed in the fixed node [53]. According to J. Woch, critical nodes of transportation network where capacity reserves are stuck in smooth streams, accompanying of the flows disturbed, are sites of interferences. Appropriate changes in the structure of traffic flows in these locations may lead to improved capacity and reduce the total delays in the network. Therefore, it is important to precise identification of disruption at the microscopic level. Taking advantage of features of hierarchical networks, some nodes can be decomposed into smaller parts, called elementary nodes [62]. Only the analysis at the level of elementary nodes gives a proper assessment of the degree of disruption of the traffic stream in the transportation network.

The relationship between smoothness and the intensity of traffic flow has been developed by J. Woch [62] based on the work of F. Haight [24] and D. Heidemann [27]. The queue by F. Height is a condition in which there is a close

relationship between the vehicles on the road. It consists in the fact that the delays for the specific vehicle are caused by the previous vehicle, and their size depends on the difference in speed between them. This leads to interdependence of headways between vehicles. In turn, smooth traffic flow is defined by F. Haight as the queue of vehicles forming a sequence of independent headways. In such a situation, the queue length is described geometric distribution.

Similar assumptions adopted D. Drew, who developed mathematical relationships also based on the geometric distribution [14]. The applying of queuing models to describe the maximum smoothness of traffic flow was also proposed in [27, 28].

The effect of moving bottlenecks on flow of traffic is an important factor in estimating travel times and paths following by commuters. In the paper [39].

Table 2. Classification of traffic flow propagation models

TYPE OF MODEL	MAIN ASSUMPTION	EXAMPLES
macroscopic	highest level of aggregation, lowest level of detail, based on continuum mechanics, typically entailing fluid-dynamic models	– hydrodynamic Lighthill- Whitham model, – Newell model, – Richards model, – Payne model.
mesoscopic	high level of aggregation, low level of detail, typically based on a gas-kinetic analogy in which driver behaviour is explicitly considered	– cluster Ben-Akiva model, – headway distribution Branstom model, – kinetic Prigogine-Herman model.
microscopic	description of the interaction between vehicles in the traffic stream low level of aggregation, high level of detail, typically based on models that describe the detailed interactions between vehicles in a traffic stream.	– Chandler model, – Kometani-Sasaki model, – Gazis model, – Treiber-Helbing model, – Wiedemann model, – cellular automata Nagela-Schrackenberg model, – queue Heidemann model.
submicroscopic	lowest level of aggregation, highest level of detail, like microscopic models but extended with detailed descriptions of a vehicles' inner workings	– van Arem model, – Minderhoud model, – Ludmann model.

Source: own research based on [42].

Authors proposed a fully coupled, multi-scale mathematical model in which the microscopic position of a single car is taken together with the macroscopic car density on the road. They used micro-macro model with dynamics of a moving bottleneck caused by a slow-moving cars on a street.

In one of the first models describing traffic flows in the transportation network the graph theory has been applied [17]. Currently, among the models used in the mapping the traffic flows one should mention game theory, Petri networks or artificial intelligence methods. J. Žak presented an interesting approach in modelling of transportation processes in [63]. She used the original method in which the queuing theory has been joined to graph theory. Such models can also be used in describing the traffic flows in terms of congestion.

4. Representation of traffic flow in congested urban network

Following are the notations—in alphabetical order—used to express the short outline of the model described in this section.

- A** - set of numbers of vertices representing zone centroids, which produce travel demand flow,
- a** - number of vertex representing zone centroid, which produce travel demand flow (origin node),
- (a, b)** - O-D pair,
- B** - set of numbers of vertices representing zone centroids, in which travel demand flow disappears,
- b** - number of vertex representing zone centroid, in which travel demand flow disappears (destination node),
- E** - set of O-D pairs,
- G** - graph, which is representation of the urban network structure,
- L** - set of the arcs of graph **G**,
- L^p** - set of arcs belonging to the given path $p \in \mathbf{P}$,
- P** - set of numbers of all paths in urban network,
- P^(a,b)** - set of path numbers for given of O-D pair $(a, b) \in \mathbf{E}$,
- p** - path number,
- PK** - set of numbers of level of congestion,

- pk** - number of level of congestion,
 - Pr^{p,pk}(t)** - probability of choice the path with number $p \in \mathbf{P}$ at the beginning of trip in time interval $t \in \mathbf{T}$ for level of congestion $pk \in \mathbf{PK}$,
 - T** - set of numbers of sequential time intervals,
 - T^{p,pk}(t)** - set covering all numbers of sequential time intervals, in which whole flow $x^{p,pk}(t)$ is passing through the urban network,
 - t** - number of time interval,
 - t_{0(w,w')}** - average travel time for the link represented by arc $(w, w') \in \mathbf{L}$ in uncongested network,
 - tz_(w,w')^{pk}** - additional time (in relation to the time of trip in uncongested urban network) in the link represented by arc $(w, w') \in \mathbf{L}$ determined for level of congestion $pk \in \mathbf{PK}$,
 - V** - set of numbers of vertices representing nodes of transportation network,
 - v** - number of vertex representing node of transportation network,
 - W** - set of the numbers of vertices of graph **G**,
 - w** - number of vertex of graph **G**,
 - (w, w')** - arcs of graph **G**,
 - x^(a,b)(t)** - O-D flows for $(a, b) \in \mathbf{E}$ that begins the trip in time interval $t \in \mathbf{T}$,
 - x^{p,pk}(t)** - average number of users, who in time interval $t \in \mathbf{T}$ begin to follow the path with number $p \in \mathbf{P}$ for level of congestion $pk \in \mathbf{PK}$,
 - x_(w,w')^{pk}(t)** - traffic volumes (link flow) on link $(w, w') \in \mathbf{L}$ in time interval $t \in \mathbf{T}$ for level of congestion $pk \in \mathbf{PK}$,
 - δ_(w,w')^{p,pk}(j, t)** - rate of flow $x^{p,pk}(t)$ (expressed in range $\langle 0, 1 \rangle$) that is found in time interval $t \in \mathbf{T}$ in the link represented by arc $(w, w') \in \mathbf{L}$,
 - ϑ(p)** - path with number $p \in \mathbf{P}$.
- In static models, often used in the planning of traffic flows in cities, it is assumed that the whole process of traffic flow passing through the urban network from the starting to the ending location is held in a sufficiently long period of time. Analysis of disruptions caused by congestions

and assessment of their impacts, however, requires a dynamic approach. Therefore, for modelling purposes it is necessary to introduce a suitable model of time. Thus it was assumed that the entire reference period is divided into a predetermined number of intervals of equal length, which is dependent on the desired level of detail. A set T includes the numbers of sequential time intervals, i.e.:

$$T = \{1, \dots, t, \dots, \bar{T}\} \quad (1)$$

where \bar{T} denotes number of all time intervals (the size of the set T).

In general, trip undertaken in a given urban area may start and end at a large number of points. To model the system, it is necessary to divide the study area (and possibly the part of its environment) into a number of discrete geographic units called traffic analysis zones. For simplicity, it is assumed that all socioeconomic variables related to the activity system and the decision makers are concentrated at a single fictitious point in space, which is called zone centroid and is located in the center of gravity of each zone. All transportation needs are generated in this place. The structure of the urban network can be represented using graph theory [8, 31, 32, 40, 65, 66] as:

$$G = \langle W, L \rangle \quad (2)$$

where set W of the numbers of vertices of graph G is representation of all nodes in urban network, i.e.:

$$W = \{1, \dots, w, w', \dots, \bar{W}\} \quad (3)$$

where \bar{W} denotes the size of the set W (number of all nodes of graph G), and set L of the arcs of graph G is representation of all links in urban network, i.e.:

$$L = \{(w, w') : (w, w') \in W \times W, w \neq w'\} \quad (4)$$

Set W can be decomposed according to the role they play in the traffic flow into three subsets [31, 32]:

A - set of numbers of vertices representing zone centroids, which produce travel demand flow (origin nodes, start points of trip), i.e. $A = \{a : a \in W\}$,

B - set of numbers of vertices representing zone centroids, in which travel demand flow disappears (destination node, end points of trip), i.e. $B = \{b : b \in W\}$,

V - set of numbers of vertices representing nodes of transportation network, i.e.

$$V = \{v : v \in W\},$$

These sets are pairwise disjoint, i.e.

$$A \cap V = \emptyset, \quad B \cap V = \emptyset, \quad A \cap B = \emptyset \quad (5)$$

and the following condition is satisfied:

$$W = A \cup B \cup V \quad (6)$$

The single O-D pair (a, b) describes the relationship between the individual node which is the origin of traffic flow and the individual node which is the destination of traffic flow. The set of all O-D pairs may be describe as [31, 32]:

$$E = \{(a, b) : (a, b) \in A \times B\} \quad (7)$$

Furthermore it is assumed that for every O-D pair there is at least one path with number p that connects given vertices, understood as a sequence of not repeated arcs, in which sequentially traffic flow is passing through the network. This path is described as:

$$\vartheta(p) = \langle (a, v), (v, \dots), \dots, (\dots, v') \rangle, \quad (8)$$

where $a \in A, v, \dots, v' \in V, b \in B$.

Set P of numbers of all paths in urban network may be described as:

$$P = \{1, \dots, p, \dots, \bar{P}\} \quad (9)$$

where \bar{P} denotes the size of the set P (number of all paths in urban network).

Set P can be decomposed into the subsets $P^{(a,b)}$, that cover path numbers for given of O-D pair $(a, b) \in E$, i.e.:

$$P^{(a,b)} = \{p : p \in P\}, \quad (a, b) \in E \quad (10)$$

where

$$P = \bigcup_{(a,b) \in E} P^{(a,b)} \quad (11)$$

Thus, the set L^p of arcs belonging to the given path $p \in P$ is specified as:

$$L^p = \left\{ (w, w') : (w, w') \in L \wedge (w, w') \subset \vartheta(p) \right\}, \quad p \in P \quad (12)$$

Traffic flow passing through the individual elements of the urban network is a representation of the trips undertaken by persons and (or) transport of goods in the transportation network. The size of the traffic flow is determined by the number of units of traffic flow per unit time. As the unit of traffic flow one can take both the transport unit (e.g. vehicle) and transported goods (e.g. person, freight). When conducting the analyses related to the supply side of the transportation system it is more convenient to operate on the concept of the so-called. passenger

car unit (PCU) with fixed technical parameters [19]. The task of the transportation system is not only a realization of displacement, but also the fulfilment of the specific conditions that can be broadly defined as [41]:

- providing the displacement at a particular period of time,
- providing arrival or delivery into exactly specified destination,
- providing the displacement exactly by the means of transport that best suit the transportation susceptibility of the goods or has the least burdensomely travel conditions from the point of view of a particular criterion.

Every disruption caused by congestions in the urban network can induce a violation of these conditions, and thus the decrease in the quality of travelling. Thus, the concept level of congestion has been adopted for the purposes of representation of traffic flow in congested urban network. It corresponds to fixed level of traffic quality losses beyond which disruptions effects are experienced by users of the urban network. This level is largely determined by specific characteristics of the area under study. Acceptance of certain traffic conditions and related quality of travelling is subjective issue and depends on the user-related attributes, time of day, type of travel, as well as on the function and location of the elements of the transportation network. The set of numbers of congestion levels is determined as:

$$\mathbf{PK} = \{0, 1, \dots, pk, \dots, \overline{PK}\} \quad (13)$$

where \overline{PK} denotes the size of the set \mathbf{PK} (number of all levels of congestion accepted for analysis). Flow $x^{p,pk}(j)$ in the path with number $p \in \mathbf{P}$ beginning travelling in time interval $j \in \mathbf{T}$ for level of congestion $pk \in \mathbf{PK}$ may be determined as:

$$x^{p,pk}(j) = x^{(a,b)}(j) \cdot \text{Pr}^{p,pk}(j), \quad j \in \mathbf{T} \quad (14)$$

The probability $\text{Pr}^{p,pk}(j)$ of choice the path with number $p \in \mathbf{P}$ at the beginning of the trip in time interval $j \in \mathbf{T}$ for level of congestion $pk \in \mathbf{PK}$ depends on the traffic conditions and the level of congestion in the transportation network of the city. The models used to estimate such probability are presented, among others, in [2, 4, 5, 18].

Rate $\delta_{(w,w')}^{p,pk}(j, t)$ (expressed in range $\langle 0, 1 \rangle$) may be also understood as the probability that flow $x^{p,pk}(j)$ is found in time interval $t \in \mathbf{T}$ in the link represented by arcs $(w, w') \in \mathbf{L}$. It is therefore strongly dependent on the traffic conditions in the transportation network. It is also known that the flow $x^{p,pk}(j)$ in each time interval with the number $t \in \mathbf{T}^{p,pk}(j)$ charged particular links of urban network in different rates, and the condition must be met:

$$\sum_{(w,w') \in L^p} \delta_{(w,w')}^{p,pk}(j, t) = 1, \quad \forall t \in \mathbf{T}^{p,pk}(j), \quad p \in \mathbf{P}, \quad j \in \mathbf{T}, \quad pk \in \mathbf{PK} \quad (15)$$

Time of following through the individual elements of the urban network is the measure that is very sensitive to variations in traffic conditions. For the purpose of modelling the additional time $tz_{(w,w')}^{pk}$ (separately for each link) resulting from traffic conditions has been fixed for each of the levels of congestion. The assumption that the traffic conditions described by certain level of congestion are homogenous throughout the length of the link is made.

In dynamic approach the traffic flow $x_{(w,w')}^{pk}(t)$ passing through an individual link $(w, w') \in \mathbf{L}$ in the time interval $t \in \mathbf{T}$ for the level of congestion $pk \in \mathbf{PK}$ consists of the average numbers of users $x^{p,pk}(j)$, who in time intervals $j \in \mathbf{T}$, $j \leq t$ begin to follow the path with number $p \in \mathbf{P}$ for level of congestion $pk \in \mathbf{PK}$. That can be determined as:

$$x_{(w,w')}^{pk}(t) = \sum_{(a,b) \in E} \sum_{p \in \mathbf{P}^{(a,b)}} \sum_{j=1}^t \delta_{(w,w')}^{p,pk}(j, t) \cdot x^{p,pk}(j), \quad \forall t \in \mathbf{T}, \quad j \leq t, \quad \forall (w, w') \in \mathbf{L}, \quad pk \in \mathbf{PK} \quad (16)$$

Equations (16) for the individual time intervals numbered $t \in \mathbf{T}$ and links represented by arcs $(w, w') \in \mathbf{L}$ describe traffic shaping in the urban network for a certain level of congestion $pk \in \mathbf{PK}$. In practice, the traffic flow $x_{(w,w')}^{pk}(t)$ corresponds to the number of traffic flow units measured at the specified place of the link $(w, w') \in \mathbf{L}$ in the time interval $t \in \mathbf{T}$. Consequently one should remember that the value obtained in this way is only an estimation of the real one.

5. Calculation example

A simple example network shown in Figure 1 has been developed as an exemplification presented approach.

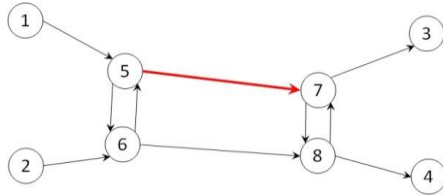


Fig. 1. Scheme of simple urban network
Data needed to determine traffic flows in urban network shown in Fig.1 is following:

– set of numbers of vertices of graph G , which are the representation of all nodes in urban network:

$$W = \{1,2,3,4,5,6,7,8\},$$

where:

$$A = \{1,2\}, \quad B = \{3,4\}, \quad V = \{5,6,7,8\},$$

– set of arcs of graph G , which are the representation of all links in urban network:

$$L = \left\{ (1,5), (5,7), (7,3), (2,6), (6,8), (8,4), (5,6), (6,5), (7,8), (8,7) \right\}$$

– set of O-D pairs:

$$E = \{(1,3), (1,4), (2,3), (2,4)\},$$

– set of numbers of all paths in urban network:

$$P = \{1,2,3,4,5,6,7,8\}$$

where

$$P = \{P^{(1,3)}, P^{(1,4)}, P^{(2,3)}, P^{(2,4)}\} \text{ and}$$

$P^{(1,3)} = \{1,2\}$, where:

$$\vartheta(1) = \langle (1,5), (5,7), (7,3) \rangle,$$

$$\vartheta(2) = \langle (1,5), (5,6), (6,8), (8,7), (7,3) \rangle,$$

$P^{(1,4)} = \{3,4\}$, where:

$$\vartheta(3) = \langle (1,5), (5,7), (7,8), (8,4) \rangle,$$

$$\vartheta(4) = \langle (1,5), (5,6), (6,8), (8,4) \rangle,$$

$P^{(2,3)} = \{5,6\}$, where:

$$\vartheta(5) = \langle (2,6), (6,8), (8,7), (7,3) \rangle,$$

$$\vartheta(6) = \langle (2,6), (6,5), (5,7), (7,3) \rangle,$$

$P^{(2,4)} = \{7,8\}$, where:

$$\vartheta(7) = \langle (2,6), (6,8), (8,4) \rangle,$$

$$\vartheta(8) = \langle (2,6), (6,5), (5,7), (7,8), (8,4) \rangle,$$

– set of numbers of time intervals:

$$T = \{1,2,3,4,5\}$$

– travel demand flows $x^{(a,b)}(j)$ for O-D pair $(a,b) \in E$ which begin to follow through the urban network in time interval $j \in T$:

$$[x^{(a,b)}(j)] = \begin{bmatrix} 30 & 50 & 20 & 10 \\ 50 & 60 & 30 & 30 \\ 70 & 70 & 50 & 60 \\ 50 & 50 & 40 & 40 \\ 40 & 60 & 30 & 50 \end{bmatrix}$$

– values of average travel time for the link represented by arc $(w,w') \in L$ in uncongested network:

$$t0_{(w,w')} = [4 \ 8 \ 5 \ 5 \ 7 \ 6 \ 2 \ 2 \ 2 \ 2]$$

– set of numbers of level of congestion:

$$PK = \{0,1,2,3\}$$

– values of additional time in the link represented by arc $(w,w') \in L$ determined for level of congestion $pk \in PK$:

$$[tz_{(w,w')}^{pk}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ 2 & 4 & 2 & 2 & 4 & 2 & 2 & 2 & 2 & 2 \\ 3 & 6 & 3 & 3 & 6 & 3 & 3 & 3 & 3 & 3 \end{bmatrix}$$

– values of probabilities $Pr^{p,pk}(j)$ of choice the path with number $p \in P$ at the beginning of the trip in time interval $j \in T$ for level of congestion $pk \in PK$,

(for ease of understanding the example the simplifying assumption that the choice of the road does not depend on the level of congestion has been adopted):

$$[Pr^{p,pk}(j)] = \begin{bmatrix} 0,7 & 0,3 & 0,5 & 0,5 & 0,5 & 0,5 & 0,5 & 0,5 & 0,5 & 0,7 & 0,3 \\ 0,7 & 0,3 & 0,5 & 0,5 & 0,5 & 0,5 & 0,5 & 0,5 & 0,5 & 0,7 & 0,3 \\ 0,7 & 0,3 & 0,5 & 0,5 & 0,5 & 0,5 & 0,5 & 0,5 & 0,5 & 0,7 & 0,3 \\ 0,7 & 0,3 & 0,5 & 0,5 & 0,5 & 0,5 & 0,5 & 0,5 & 0,5 & 0,7 & 0,3 \end{bmatrix} \forall pk \in PK$$

According to formula (14) the average numbers of users $x^{p,pk}(j)$, who in time interval $j \in T$ begin to follow the path with number $p \in P$ for level of congestion $pk \in PK$ have been calculated, i.e.:

$$[x^{p,pk}(j)] = \begin{bmatrix} 21 & 9 & 25 & 25 & 10 & 10 & 7 & 3 \\ 35 & 15 & 30 & 30 & 15 & 15 & 21 & 9 \\ 49 & 21 & 35 & 35 & 25 & 25 & 42 & 18 \\ 35 & 15 & 25 & 25 & 20 & 20 & 18 & 12 \\ 28 & 12 & 30 & 30 & 15 & 15 & 35 & 15 \end{bmatrix}$$

In the example the equations of traffic flows for the selected link of urban network $(5,7) \in L$ according to the formula (16) have been formulated for the following four cases:

- case I – uncongested urban network (level of congestion $pk = 0$),

$$\begin{aligned} x_{(5,7)}^0(t) = & \\ & 0,4 \cdot x^{1,0}(t-1) + 0,4 \cdot x^{3,0}(t-1) + \\ & + 0,6 \cdot x^{1,0}(t-2) + 0,6 \cdot x^{3,0}(t-2) + \\ & + 0,8 \cdot x^{6,0}(t-2) + 0,8 \cdot x^{8,0}(t-2) + \\ & + 0,2 \cdot x^{6,0}(t-3) + 0,2 \cdot x^{8,0}(t-3) \end{aligned}$$

- case II – urban network congested at the level $pk = 1$,

$$\begin{aligned} x_{(5,7)}^1(t) = & \\ & x^{1,1}(t-2) + x^{3,1}(t-2) + \\ & + 0,2 \cdot x^{6,1}(t-2) + 0,2 \cdot x^{8,1}(t-2) + \\ & + 0,8 \cdot x^{6,1}(t-3) + 0,8 \cdot x^{8,1}(t-3) \end{aligned}$$

- case III – urban network congested at the level $pk = 2$,

$$\begin{aligned} x_{(5,7)}^2(t) = & \\ & 0,6 \cdot x^{1,2}(t-2) + 0,6 \cdot x^{3,2}(t-2) + \\ & + 0,4 \cdot x^{3,2}(t-3) + 0,4 \cdot x^{6,2}(t-3) + \\ & + 0,6 \cdot x^{6,2}(t-3) + 0,6 \cdot x^{8,2}(t-3) + \\ & + 0,4 \cdot x^{6,2}(t-4) + 0,4 \cdot x^{8,2}(t-4) \end{aligned}$$

- case III – urban network congested at the level $pk = 3$,

$$\begin{aligned} x_{(5,7)}^3(t) = & \\ & 0,2 \cdot x^{1,3}(t-2) + 0,2 \cdot x^{3,3}(t-2) + \\ & + 0,8 \cdot x^{1,3}(t-3) + 0,8 \cdot x^{3,3}(t-3) + \\ & + x^{6,3}(t-4) + x^{8,3}(t-4) \end{aligned}$$

The results for all four cases for the time interval $t = 5$ are given in Table 3.

Table 3. Traffic flow for the selected link of urban network $(5,7) \in L$.

Case I	Case II	Case III	Case IV
$x_{(5,7)}^0(5)$	$x_{(5,7)}^1(5)$	$x_{(5,7)}^2(5)$	$x_{(5,7)}^3(5)$
113,6	111,8	96	81,8

It is therefore apparent that the proper segregation of trip matrix and appropriate assumptions on the terms of traffic condition are essential in receiving the values of traffic flow characteristics based on traffic flow equations. The estimated values could be compared with the actual ones—delivered from traffic measurements—and then be used to build and calibrate a dynamic model of traffic. Such analyses for networks with more complex topologies require the application of specialized tools.

The example assumes a uniform distribution of demand in the reference period of time. However, this model can also be used for other distributions assuming adequate relationships between $\delta_{(w,w')}^{p,pk}(j,t)$ and $tz_{(w,w')}^{pk}$.

6. Conclusions

The main objective of modelling of traffic flow for the analysis of disruptions caused by congestion is proper assessment of their level of impact. Disturbances in the urban network are carried by the traffic flows. Thus a suitable method for representation the stream of traffic in the urban network allows you to take into account a number of factors that influence to obtain reliable results. This is particularly important in the traffic management, where the transfer of current, reliable and properly formulated information to users of the urban network can improve the functioning of the transportation system of the city.

Every user of the transportation system undertake his or her trip in a specified time interval. From the point of view of modelling it determining the origin node and the moment when user will be in certain points the urban network are very important. The certain points include both places where the traffic conditions are extremely burdensome and decision points where the user has to choose the way in which he or her will continue the trip. Unfortunately, it is difficult to get such detailed information for each user of the transportation system, although it would be very useful for a precise description of the traffic flows in the urban network. In the presented approach, the reference period is divided into elementary time intervals of equal length. This allows adequate segregation of trip matrix and take into account the variability of trip demand in different periods of the day. Moments of beginning of the trip for a specified number of users are assigned to the appropriate period of time, the length of which depends on the desired level of detail determined by purpose of modelling. Furthermore, the selection of paths depends on the level of congestion in a certain period of time. This allows the determination of different paths in different time intervals for the same O-D pair. The approach presented in the article may be used both to identify disruptions in the transportation

networks (and to determine their level) and the construction and calibration of the dynamic traffic models [7, 55, 64], that are applied in traffic management. The issue requires further study, particularly in terms of the selection of measures, which would examine the level of impact of disruptions on the various elements of the transportation system and its environment in a comprehensive way. It is also worth to extend the model by taking into account the different characteristics for various groups of users and transport subsystems in representation the traffic flows in the urban network.

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