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FRACTAL LAWS OF SOUND BACKSCATTERING BY SEA SURFACE AND BOTTOM

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It is demonstrated that the known empirical frequency-angle dependence that characterizes the intensity of high-frequency backscattering of sound by a rough sea surface and is obtained on the basis of generalization of results of many experiments conducted by various researchers under different conditions and in different sea regions has a fractal character. High-frequency backscattering of sound by the sea bottom is also characterized by fractal laws in a broad range of frequency and spatial scales.

INTRODUCTION

Dynamic chaos and fractals occupy an important place among the outstanding scientific discoveries of the 20th century. They are closely associated. Dynamic chaos is chaotic oscillations similar to random in nonlinear determinate systems. Chaotic oscillations have a new dynamic property that is called a fractal structure. In other words, chaotic oscillations (phenomena) arise according to regular laws and originate not from a formless chaos but from the chaos with a concealed order, i.e. fractal structures.

Fractals are self-similar in a certain sense objects with a fractional dimension. They have the property of scale invariance or scaling. The structural (correlation) functions and their spectra characterizing fractal structures are described by power laws with fractional exponents. The concept of fractals and dynamic chaos and the fractal geometry often provide an opportunity to describe more adequately geometric forms and processes observed in nature [1, 2].

The nature in the broad sense is a huge dissipative nonlinear system. There is nothing surprising about the fact that the surface, water masses, and bottom of the ocean are fractal. Motions of the surface and water masses in the ocean are nonlinear dynamic processes in space and time. The ocean bottom as well as the harmony of order and disorder in natural objects (clouds in the atmosphere and clines, their shapes) is dynamic processes frozen in physical forms and a certain interchange of order and disorder is characteristic to them. In this sense nature is fractal.

We demonstrate below on the basis of the experimental and theoretical data given in literature that sound backscattering by the surface and bottom of the ocean is frequently characterized by fractal laws.

1. FRACTAL LAWS OF SOUND BACKSCATTERING BY OCEANIC SURFACE

An empiric frequency-angle dependence characterizing the high-frequency backscattering of sound by rough sea surface is known. This dependence is obtained on the basis of the analysis and generalization of the results of many experiments [3]:

$$\mathrm{N_s} = 10\log(fh\sin\theta)^{0.99} - 45. \tag{1}$$

Here N_s is the strength of sound scattering by sea surface in decibels, f is the sound frequency, h is the amplitude of sea waves, and θ is the glancing angle.

There was no explanation of physical mechanisms of this dependence. Its fractal nature was revealed recently [4]. Indeed, it is interesting to note that Eq. 1 is described by a power law with a fractional (noninteger) exponent that is typical of wave scattering by fractal structures and surfaces. It is known that the power dependence of the intensity of wave scattering on frequency (wavelength) and scattering angle with a fractional power exponent is characteristic to fractals [2]. It is natural to expect Eq. 1 to be caused by fractal characteristics of sea surface.

There is rather convincing evidence of the fact that the sea surface is characterized by fractal properties. For example, Barenblatt and Leikin [5] note the self-similarity of the high-frequency spectrum of wind waves at sea surface and give the formula describing the frequency spectrum of waves that is characterized by a power law with the exponent taking on fractional values. Experimental observations exist indicating the fact that the spectrum of sea-surface waves is described by a power (fractal) law [5].

Expressions characterizing in the space $E = 3$ the rises and the frequency spectrum of a rough sea surface were obtained on the basis of the modified Weierstrass-Mandelbrot function [6], which is used often for description of fractal surfaces. It was noted that the sea surface in the case of large waves is fractal with the dimension of the surface $D \approx 2.25$ in the scale range of surface waves 0.1—100 m.

Recently Qian [7] made an attempt to connect theoretically the dependence given by Eq. 1 with the fractal properties of the sea surface. Sound scattering by a fractal surface in the space $E = 2$ was considered in the Kirchhoff approximation under the condition that the surface was characterized by a generalized Koch's curve. In fact, sound scattering by the surface approximated by the Koch's prefractal was calculated. The expression for the backscattering strength obtained in [7] has the form

$$\mathrm{N_s} = N_0 + 101\log\left[fh\tan\left(\frac{\psi}{2}\right)\sin\theta\right]^{2D} \tag{2}$$

Here ψ is the angle between the elements of the Koch prefractal.

Equation 2 almost coincides with Eq. 1 if we assume that the condition $\psi \approx \pi/2$ and the fractal dimension of the surface $D \approx 0.5$. Meanwhile let us recall that the dimension of the Koch's prefractal obeys the condition $1 < D < 2$.

According to Qian [7], the necessity of the condition $D \cong 0.5$ in Eq. 2 can be explained by the fact that in the case of sound backscattering quasi-degeneration of a continuous fractal Koch's curve into the Cantor set that is known to have the dimension $0 < D < 1$ [2] occurs.

According to our opinion such explanation may seem quite sensible in the light of optical experiments [8] for example. The fractal properties of the zones of falling waves at the sea surface were studied in [8]. The experimental data were obtained by remote optical sensing of sea surface. A series of large-scale photographic pictures of the sea surface with foamy formations accompanying falling of surface gravity waves was taken. In other words, the obtained experimental material contained the results of light backscattering by sea surface in the process of wave falling. It was established that the distribution of zones of falling is fractal with the fractal dimension $D = 0.5$ characterizing the Cantor fractal set.

Comparing Eqs. 1 and 2 and the experimental results [8] one can arrive at the conclusion that the conditions when sound backscattering arises at single troughs of sea surface waves correspond to the results of the acoustic experiments generalized in [3]. However in this case the state of the sea is such that even if falling of sea waves occurs as it is observed in [8], there is no effective formation of the subsurface layer of air bubbles. Otherwise, if a layer of air bubbles forms near the sea surface, the dependence described by Eq. 1 is violated. However, as it is demonstrated in [9], in this case also sound backscattering obeys fractal laws. Here sound scattering is caused not by the sea surface directly but mainly by the layer of air bubbles. It is assumed that the spectrum of sound velocity fluctuations in the layer of subsurface bubbles obeys the Kolmogorov-Obukhov law, which (as it is well known now) reflects the fractal structure of turbulence. The strength of sound backscattering calculated in [9] agrees very well with the experimental data including those for rather low frequencies (starting from 0.1 kHz) and a wide range of glancing angles.

It was noted in [3] that even earlier many experimenters obtained power laws of change of the intensity of sound backscattering by sea surface. As above (see Eq. 1), the experimental dependences obtained earlier can be represented in the form

$$\mathrm{N_S} = 10\log(fh\sin\theta / K_i)^{\nu_i}, \tag{3}$$

where according to [10] $K_1 = 3.28\cdot10^2$ and $\nu_1 = 2.03$; according to [11] $K_2 = 5.57\cdot10^3$ and $\nu_2 =$ $= 1.52$; according to [12] $K_3 = 1.81\cdot10^3$ and $\nu_3 = 1.43$; and according to [13] $K_4 = 6.88\cdot10^3$ and $\nu_4 = 1.03$. Correspondingly, for Eq. 1 in [1] $K = 3.649\cdot10^4$ and $\nu = 0.99$.

One could consider Eqs. 3 as single and in a certain sense occasional facts. Apparently Shulkin and Shaffer [3] treated them just in this way and generalizing they obtained an averaged dependence described by Eq. 1.

In reality, as we think, the results [10—13] are quite justified taken separately. They reflect the experimental conditions and the state of sea surface, its fractal properties, while the value of the exponent of the power dependence reflects the fractal dimension.

The fact that the fractal properties of a rough sea surface can be characterized depending on ambient conditions by different values of fractal dimension is confirmed also by the results of optical experiments [14] and their comparison with the experimental data [8]. The results of experimental studies of statistical characteristics of wind waves at the oceanic surface are given in [14]. The studies were conducted with the help of surface scanning by a laser beam. It turned out that the set of mirror points at a rough oceanic surface was the Cantor set with the fractal dimension $0 < D = 0.8 < 1$. Let us recall that the optical experiments [8] and [14] do not disagree but, on the contrary, indicate the variety of the fractal properties of a rough sea surface.

Thus we can state that sound backscattering by the sea or oceanic surface in the wide range of variability of surface waves is characterized by fractal laws or has the fractal nature. The frequency-angle dependence of the scattering intensity obeys a power law with the fractional value of the exponent. The value of the power index carries information on the fractal properties of the surface and its fractal dimension and can be a characteristic of waves.

2. SOUND BACKSCATTERING BY OCEANIC BOTTOM

There are data indicating the fact that the surface of the bottom, floor sediments, and the so-called acoustic foundation of the bottom are fractal [15, 19]. Fractal dimension was proposed by Barenblatt *et al.* [15] as a quantitative characteristic of ruggedness of the bottom and its acoustic foundation. A natural decrease of the value of the fractal dimension of the bottom relief of the Atlantic Ocean with the increase of the distance from the axis of the mid-oceanic (Mid-Atlantic) range was revealed. As Barenblatt *et al.* [15] assumed, this is the consequence of bottom smoothing. The latter is caused by bottom currents and the processes of sediment accumulation.

Multiple studies demonstrated that sound scattering by the ocean bottom is connected mainly with either scattering at the irregularities of the interface of ocean – bottom sediments or scattering at volume inhomogeneities of bottom sediments. Thorough studies of sound backscattering by the ocean bottom in various regions are presented in [17, 18]. Comparison of experimental and theoretical results has shown that in the case of a sandy bottom sound scattering occurs at its uneven boundary. It has been established that the spatial spectra of inhomogeneities of a sandy bottom are anisotropic and described by power laws with the fractional exponent. Taking this into account has led to a satisfactory agreement of theoretical and experimental results. In the case of silt bottom the dominating role in sound scattering belongs to volume inhomogeneities of bottom sediments. The results of the studies of sound scattering by bottom sediments are given in [19]. It is assumed there that scattering occurs at volume inhomogeneities that are fluctuations of density and sound velocity in sediments. Being guided by the results of experimental studies of the structure of bottom sediments the author [19] describes fluctuations of density and sound velocity by the correlation functions and their spectra characterized by power laws. It is demonstrated that in the case of a hard silt bottom the dominating role in sound scattering belongs to fluctuations of density and in the case of a soft (liquid) bottom, to fluctuations of sound velocity. In these cases the exponents in the power laws for fluctuations of density and sound velocity differ. In particular, the spatial power spectrum of fluctuations of sound velocity in bottom sediments is described by an expression

$$S(k_x,k_y,k_z) = \frac{\beta l^2 B}{2\pi}\left(l^2 k_x^2 + l^2 k_y^2 + k_z^2\right)^{-(\beta+2)/2}, \qquad (4)$$

where k_i are the components of the spatial wave vector, B and l are the constants characterizing the intensity of fluctuations and the scale of their anisotropy in the horizontal plane respectively, and β is the exponent in the power law describing the one-dimensional spatial spectrum of fluctuations in the z coordinate (in depth). Measurements conducted in ten oceanic regions (five shallow-water and five deep-water regions) demonstrated that it changes within the range $1.45 \le \beta \le 2.3$. Its average value for these regions is $\beta = 1.76$. It was noted that the frequency dependence of the strength of sound scattering by the oceanic bottom is determined by this exponent in the conditions when the dominating role belongs to fluctuations of sound velocity in bottom sediments. As one can see, sound backscattering by the oceanic bottom obeys fractal laws.

3. CONCLUSIONS

Thus we can state that high-frequency backscattering of sound by the surface and bottom of the ocean in a wide range of variability of surface waves, bottom roughness, and volume inhomogeneities of bottom sediments are characterized by fractal laws. The frequency-angle dependence of the scattering intensity obeys a power law with a fractional exponent. The exponent value carries information on the fractal properties of the ocean surface and bottom and the fractal dimension that can be used for their characterization.

If the height of unevennesses of the rough surface of the ocean or the interface "ocean – bottom sediments" and the spatial scale of volume inhomogeneities in bottom sediments are small as against the sound wavelength, the fractal properties of the oceanic surface and bottom do not manifest themselves.

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