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Time between inspections optimization for technical object with time delay

Keywords

delay-time modelling, single-unit system

Abstract

In the presented paper, authors are concerned with a single-element system liable to costly failure. Failure is taken here to mean a breakdown or catastrophic event, after which the system is unusable until repaired or replaced. The mathematical modelling of maintenance decisions for such a system is provided with the use of delay-time and delay-time analysis. The directions for further research work are defined.

1. Introduction

Papers published over recent fifty years which contribute to maintenance theory cover the full range of potential models for maintenance management problems. Especially, the maintenance issues for one-component system have been much discussed in the literature (see e.g. [20], [37], [41]).Of particular interest is, of course, time between inspections optimization problem. Moreover, *the one-component unit is an important case for study, since it is the building block for the main cases of practical interest which entail systems of components* [9].
Taking one step further, usually in the real-life systems, their components do not fail suddenly. Thus, before a component breaks down, there will be some sings of reduced performance or abnormalities. The time between the first identification of abnormalities (called initial point - *u*) and the actual failure time (failure point) will vary depending on the deterioration rate of the component. This time period is called a delay time *h* to carry out maintenance or an inspection [34], and the modelling concept is called a Delay-time modelling approach. During the period of *h* there is an opportunity to identify and prevent failure (*Figure 1*). This concept, which provide useful means of modelling the effect of periodic inspections on the failure rate of repairable technical systems, was developed by Christer et al., see e.g. [9], [10], [13]-[15],[18], [38].

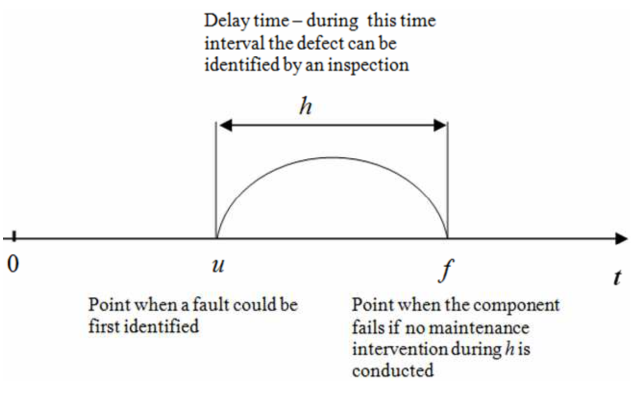

Figure 1. Time delay modelling concept [30]

Following this, the focus of this study is to develop a mathematical model for maintenance of single-element technical system with time delay.
The inspection models for new elements are known in the literature. However, when assuming that the working element is not defected at the point of inspection maintenance time, the optimization model for time between next inspections performance is not investigated.
Thus, in the next Section, there is discussed the literature overview in the area of delay time

modelling for one-component systems. Later, there is presented the investigated maintenance model. In the next Section, authors provide the sensitivity analysis of the investigated model. The work ends up with summary and directions for further research.

In conclusion, this article is to be the starting point of consideration about mathematical modelling of technical objects with time delay maintenance. Moreover, it is also a continuation of the considerations about future research, connected with delay time modelling for complex and multi-unit systems in given reliability structure, given in [24] - [27], [43]. In work [25] authors investigate the effectiveness of a system when defined maintenance strategy is one of two kinds. First, the typical group maintenance policy is considered. Second one regards to delay time model. Authors assume that the system is a three-component with a k-out-of-n reliability structure. To make a comparison, the basic assumptions are defined to be the same for both the investigated models. Next, in [26] authors focus on investigation of Block-Inspection Policy performance level with economical and availability point of view. Later, in [27] authors analyse the problem connected with model parameters estimation process and its influence on Block Inspection Policy results. The imperfect inspection problem for the defined technical system is investigated in [24].

In the last work [43] author investigates the possibilities of basic DT model implementation in the area of logistic system of sixteen forklifts performance analysis.

2. Delay-time models for single-unit systems – literature review

A literature review, in which delay-time models are investigated along with other PM models are given in [19], [21], [22], [29], [33], [36], [37].The state of art works, dedicated strictly to DT modelling are given in [1], [3], [6] - [8], [11], [12], [28], [30], [35], [38].

One of the first works dedicated to delay time modelling of single-component system is [13]. Authors in this work investigate a model of equipment-replacement decision process based upon a survey of companies, designed to investigate how replacement decisions are made in practice. Later, in [4], there is analysed the repairable machine that may fail or suffer breakdown many times during the course of its service lifetime, and is inspected for visible faults at intervals. The authors mostly focus on the problem of model parameter estimation with the use of maximum likelihood method and Akaike information criterion, providing also a model for imperfect inspections performance.

Delay time models of single-unit systems reliability are developed in [2], [5], [9], [23], and [45]. In [9], author presents simple reliability model of a single component subject to one type of inspect able defect, when inspections are assumed to be perfect. This problem is later investigated in [5], where author simplifies the developed model and provides some numerical examples. The same model, when delay time and time to failure densities are exponentially distributed, is provided in [2]. Later, in [23], authors develop a delay time model with periodic testing process. Authors analyse different variants of the models, like no inspection actions performance during maintenance processes, or zero delay times occurrence. In the next work, [45], authors investigate the model to evaluate the reliability and optimise the inspection schedule for a multi-defect component. There is also considered the situation of non-constant inspection intervals.

Inspection models for single-component system under condition based maintenance are presented in [16], and [31]-[32]. In [16], authors analyse an inspection modelling problem for a production plant under condition monitoring. Later, in works [31]-[32], authors develop a method for determining the discrete time points of inspection for a deteriorating single-unit system under condition-based maintenance. The delay-time model is here utilized to describe the transition of the system’s states. Moreover, two types of probabilities with respect to inspections are considered – a failed-dangerous probability of type I error, and a failed-safe probability of type II error.

Moreover, there can be found some models dedicated to single-component systems’ risk analysis of maintenance activities performance (see e.g. [40]), semi-Markov processes implementation (see e.g. [17]), or safety inspection processes optimization (see e.g. [42]). The example of delay time model for a single unit case is given in [44], where authors propose a method to find cost-optimized maintenance of an elevator.

3. Delay-time model for single-unit systems

In this Section, there is considered a basic delay-time model for single-unit systems in the comparison to the new approach.

In accordance with the literature concept of the delay time, known models assume that an inspected element is newly replaced at the beginning of the inspection period, e.g. the previous inspection exposed a defect of the element and caused a maintenance action. However, the optimization of a constant time between inspections seems to be impossible without considering the element operation process performance longer than its first cycle. That is why the expected maintenance cost of an element should be developed to the form that includes long term costs in the series form.

3.1. Basic delay-time maintenance model

Assuming, that p.d.f and c.d.f. of the initiation time, u are given as $g(u)$ and $G(u)$, respectively. Likewise, the delay time, h, has p.d.f and c.d.f. $f_h(h)$ and $F_h(h)$, respectively, independent of u. There is a possibility to define the c.d.f. of time to failure, $F(x)$, as the convolution of u and h such that $u + h \leq x$ [35] (*Figure 1*):

$$F(x) = \int_{u=0}^{x} g(u) F_h(x-u) du \quad (1)$$

And the reliability, $R(x) = 1 - F(x)$.

Taking into account the following additional assumptions and notation [35], [39]:

- the system is a three state system where, over its service life, it can be either operating or down for necessary repair or planned maintenance,
- the system is renewed at either a failure repair or at a repair done at an inspection if a defect is identified,
- after either a failure renewal or inspection renewal the inspection process restarts,
- maintenance actions restores maintained components to as good as new condition,
- failures of the system are identified immediately and repairs or replacements are made as soon as possible,
- perfect inspections case, i.e. every defect is assumed to be identified during inspection action performance,
- system incurs costs of: failure replacement c_f, planned defect replacement c_r, and inspection performance c_i,
- failure replacement, inspection replacement and inspection replacement require d_f, d_r and d_i time units respectively,

the expected cost over each cycle, $C(T)$, may be defined by the following equation [35]:

$$C(T) = c_f F(T) + (c_r + c_i) \int_{u=0}^{T} g(u)(1 - F_h(T-u)) du + c_i (1 - G(T)) \quad (2)$$

Thus,

$$C(T) = (c_f - c_r - c_i) F(T) + c_r G(T) + c_i \quad (3)$$

The expected downtime $E_d(T)$ of an inspection cycle of length T is given as [8]:

$$E_d(T) = (d_f - d_r - d_i) F(T) + d_r G(T) + d_i \quad (4)$$

Assuming instantaneous inspection and replacement times, the expected cycle length, $T_M(T)$, is given by [8], [35]:

$$T_M(T) = \int_{x=0}^{T} x F'(x) dx + TR(T) \quad (5)$$

The examples of such the model implementation are given e.g. in [35], [39].

3.2. Extended delay-time maintenance model

The cost and availability models presented in the previous Section (Eq. (2)-(5)) describes the first inspection cycle results, i.e. when a new element is run to work at the beginning of the cycle. On this base, one can choose the length of the cycle T, and thus – the moment of the first inspection, which yields the minimum expected costs per the cycle. However, if an element does not expose any symptoms of a defect while is inspected first time, the next problem arises: when should it be tested again? The presented models do not lay any foundation to define the best further maintenance cycles length. One of approaches which may be used in this case is to define the length of the first inspection cycle individually and then, dependently on inspection results, to plan moments of inspection action performance in the future. The next inspection cycles lengths may vary when the decision about the nearest cycle is taken during an inspection. Independently on a taken strategy of a period between inspections determination, the cost model should be developed to the form, which includes information about inspection effects:

$$C(T_i) = C(t_{i-1}, t_i) = \frac{c_f \int_{t_{i-1}}^{t_i} g(u) F_h(t_i - u) du + (c_r + c_i) \int_{t_{i-1}}^{t_i} g(u)(1 - F_h(t_i - u)) du + c_i (1 - G(t_i))}{1 - G(t_{i-1})} \quad (6)$$

where:
$C(t_{i-1}, t_i)$ - the expected maintenance costs in the period between i-1-th and i-th moments of inspection.

The equation (6) presents the expected cost of failure, inspection and planned defect replacements calculated on the base of the conditional probability that the defect is not discovered during inspection at the beginning of a cycle. The long term expected costs may be expressed in the form of the series:

$$C(T) = C(T_1) + \sum_{i=2}^{\infty} C(T_i) \quad (7)$$

where:
$C(T_1)$ - expected cost over the first inspection cycle, calculated in (2)
T_i – the length of the i-th inspection cycle.

The consideration of the lowest-cost inspection cycle cannot be done without determining the expected length of element lifetime. If we assume that an element may operate longer than to the first inspection, the expected value may be expressed as follows:

$$T_p(T) = T_M(T_1) + \sum_{i=2}^{\infty} T_M(T_i) \quad (8)$$

$$T_M(T_i) = \int_{t_{i-1}}^{t_i} \frac{(x - t_{i-1}) F'(x)}{1 - G(t_{i-1})} dx + \left(\frac{1 - F(t_i)}{1 - G(t_{i-1})} \right) T_i \quad (9)$$

where:
$T_M(T_1)$ – the expected length of the first inspection cycle, given in equation (5)
$T_M(T_i)$ – the expected length of the i-th inspection cycle

Hence, the long-term expected costs per unit time $c(T)$ is given by:

$$c(T) = \frac{C(T)}{T_p(T)} \quad (10)$$

4. Sensitivity analysis

The cost model developed in the previous Section has been analyzed and the chosen results are discussed. The objective of the analysis was to find out whether it is reasonable to develop the model to the form presented in Eq. (6) - (10) or it is enough to find the most profitable solution for the first inspection cycle and to reiterate it in the future. Authors have assumed that if the cost per unit of element lifetime changes dependently on the number of cycle, it is rational to determine the length of an every inspection cycle on the base of the proposed cost model.
Figures 2 - 5 present chosen effects of the carried out research. The cost model was tested for variables u and h described by Weibull probability distributions with different shape parameter, thus when "exponential distribution" is presented, the shape parameter of the both variable are equal to 1 and "Weibull distribution" means the both parameters are greater ($\alpha = 3,5$). The scale parameters of the variables u and h are 65 and 35, respectively. The expected cost of a failure and summary cost of single inspection cycle are presented in the relation to the number of inspection cycle when the element still works without defects (NC symbol) and the cycle length (marked as CL).

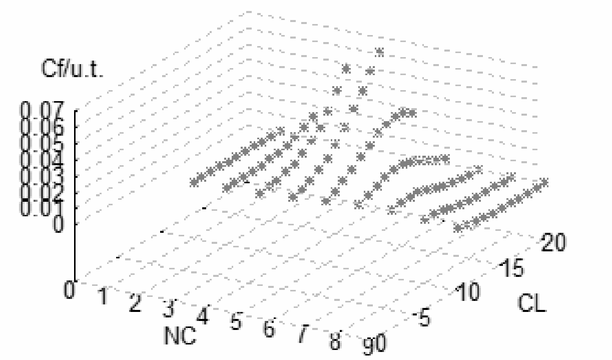

Figure 2. The expected cost of failure per unit time for Weibull distributions of times u and h.

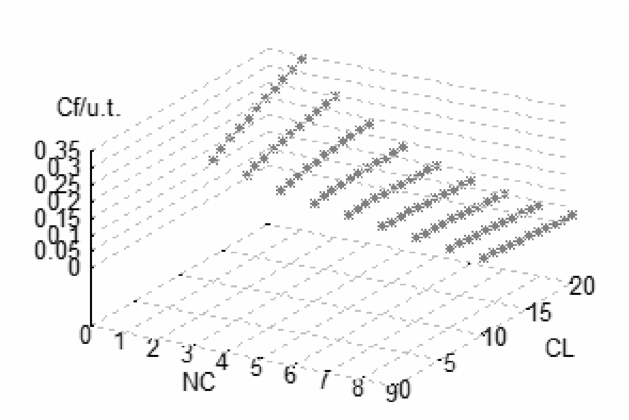

Figure 3. The expected cost of failure per unit time for exponentially distributed times *u* and *h*.

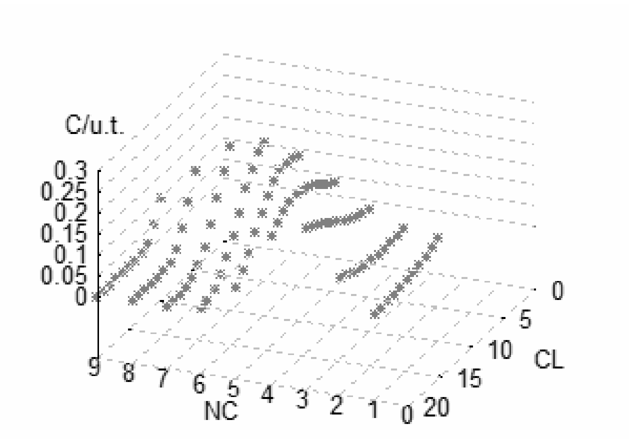

Figure 4. Cost per unit lifetime of element for Weibull distributions of times *u* and *h*.

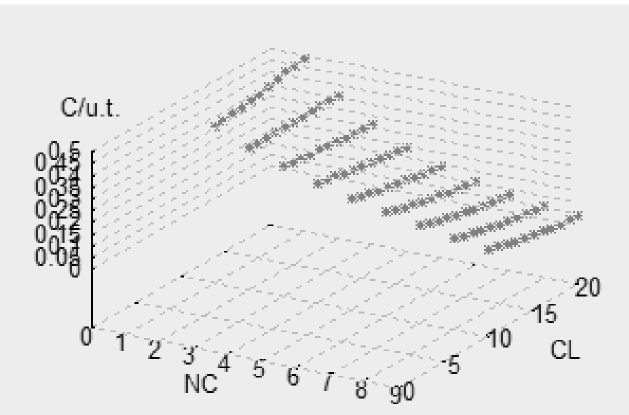

Figure 5. Cost per unit lifetime of element for exponentially distributed times *u* and *h*.

The results of the research provide the arguments for the use of the developed cost model while determining the least expensive cycle length of inspection. While the "exponential case", what is obvious, does not expose any need to carry out the preventive inspections, the "Weibull case" reveals the explicit dependency between the cost, the cycle length and the number of period for which inspection is executed. One can see that when an element is still operating during third and fourth inspection cycle of 20 units of time length (*Figure 3*), failure costs per unit time increases and the cycles might be reduced in order to lower the expected failure cost. Similar dependency may be seen in *Figure 4*. Independently on the length of the single inspection cycle, first two periods generates low maintenance costs while in the neighbourhood of the expected lifetime of an element, the costs per unit of lifetime increase. It may be the effect of growing possibilities of preventive replacement and failure as well as the lower expected lifetime in a cycle.

5. Conclusions

The optimal time between inspections for technical object with time delay may be defined in the two ways. First, there is a possibility to define a constant time between inspections *T* with the use of basic delay time model. The second approach gives the possibility to define optimal time period *T* in accordance to the following inspection actions performance obtained results. Thus, the optimal time between inspections *T* will not be a constant one, what should provide better dependability of technical objects achievement.
Following this, in the next step, authors plan to carry out the simulation analysis for the case of multi-element system with delay time. This work, as well as [24] – [27] provide the research analysis results to define some rules how to choose a PM policy from an engineering point of view.

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