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The simulation analysis of the bridge rectifier continuous operation in AC circuit

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The paper deals with a nonlinear model of the full–wave bridge rectifier in AC circuit with RC load for non–rigid supply system. The mathematical model of the AC circuit for this load was worked out using dimensionless variables. The model of the circuit was created and analyzed with MATLAB/Simulink. Analysis of harmonic distortions of the currents and voltages in considered circuit was carried out. Indices of power quality are described. The current–voltage characteristics for different values of capacitive load are presented. The balances active and reactive powers in the circuit were performed.

KEYWORDS: nonlinear load, bridge rectifier, modelling, active and reactive power

1. Introduction

Harmonics are one of the main problems of the widely understood power quality. Whereas nonlinear load are the main source of their propagation in the power system. The energy is supplied by the first harmonic from source to the load. In nonlinear load this energy is convert to higher harmonics energy and returned to the supply system. Number of nonlinear loads constantly is growing and according to [1] estimated that over the next 10 years, more than 60 % of all loads will constitute the nonlinear load. Therefore, assessment of interaction of nonlinear load and power system is important issue. The interaction may be considered using computer simulation and credible mathematical models that describe the phenomena occurring in the process of energy transfer from the producer to the consumer.

For the assessment of interaction of nonlinear loads and power system i.e. the propagation of harmonics and reactive power a mathematical model of such circuit was created.

The analyzed circuit diagram is presented in Figure 1. The circuit contains the full–wave bridge rectifier loaded by the capacitor C and the resistance of the load R_O. The rectifier is supplied by sinusoidal voltage with the angular frequency ω, through the series connected inductance L and the resistance R_S

which model the series equivalent impedance of the power supply system and other elements.

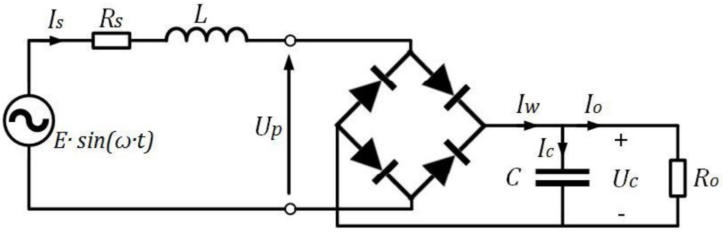

Fig. 1. The analyzed AC circuit with bridge rectifier

The inductance L is the sum of the inductance of the power supply system and additional series inductance by AC side of rectifier. This diagram differs from the ones known from the literature [2, 3] in which the additional inductance L is considered, but on the DC side of the rectifier. Both continuous and discontinuous modes operations of the bridge rectifier are possible for proposed circuit model.

2. Mathematical model of the circuit

The model of the AC circuit with the bridge rectifier in Figure 1 can be described with following equations:

$$L\frac{dI_s}{dt} = E\cdot\sin(\omega t) - I_s\cdot(R_s + 2R_d) - (2U_d + U_c)\cdot sign(I_s) \tag{1}$$

$$C\frac{dU_c}{dt} = |I_s| - \frac{1}{R_o}\cdot U_c \tag{2}$$

where: I_S – represents the instantaneous value of the current flowing from the source to the rectifier circuit, L, R_S – are respectively the inductance and the resistance of the supply system, R_d – is the series resistance of the diode in conduction state, U_d – is the voltage drop across of the diode in conduction state, U_C – the output voltage of the bridge rectifier, E – the supply voltage, R_O – resistance of the load.

The mathematical model of the rectifier (the bridge rectifier) was created using the signum function of the current I_S for which a proportion coefficient is the sum of the voltage on the capacity C and voltage drops on two conductive diodes. With the bridge rectifier put out the absolute value of the input current I_S of the rectifier filtered on parallel connected: the capacitor C and the resistance of the load R_o.

The number of parameters (R_d, R_S, R_O, L, C, ω, t, E, U_d) of such model is significant and in effect the analysis of interactions is complicated. For the purpose of the simplification of the analysis, scaling of the time and the reference variables were used:

$$\tau = \omega t;\ X = \omega L;\ I_m = \frac{E}{X};\ Y = \omega C \tag{3}$$

Using the dimensionless variables:

$$i_s = \frac{I_s}{I_m};\ u_d = \frac{U_d}{E};\ u_c = \frac{U_c}{E} \tag{4}$$

$$r_z = \frac{R_s + 2R_d}{\omega L};\ r_o = \frac{R_o}{\omega L};\ c = XY = \omega^2 LC; \tag{5}$$

equations (1 and 2) can be written as:

$$\frac{di_s}{d\tau} = \sin(\tau) - i_s \cdot r_z - (2u_d + u_c) \cdot sign(i_s) \tag{6}$$

$$\frac{du_c}{d\tau} = \frac{1}{c \cdot r_o}(|\, i_s \,| \cdot r_o - u_c) \tag{7}$$

Use of transformation caused significantly reduction the number of input parameters. Their initial number was 9, and after transformation the model consists of 5 parameters (r_o, c, r_z, u_d, τ).

3. The model of the circuit in Simulink

The operational diagram of considered circuit with Figure 1 was modelled in Simulink by the use of dimensionless equations 6 and 7. This model presents the Figure 2.

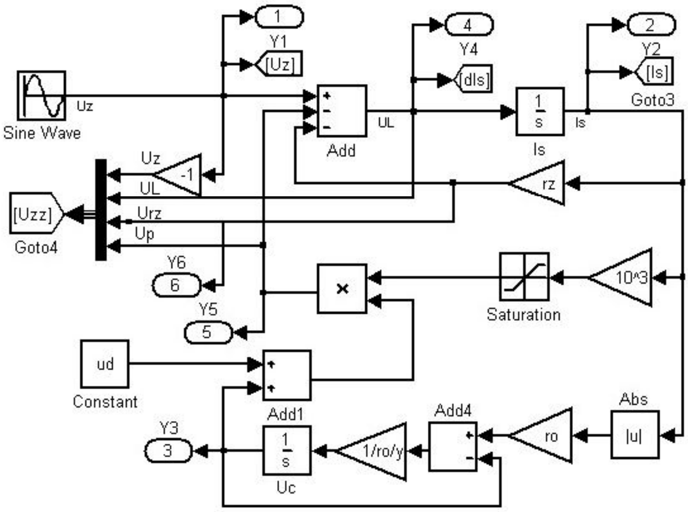

Fig. 2. The operational diagram in Simulink

In the diagram adders, gain blocks and integrators may be distinguish. The instantaneous values voltages and current were led out to MATLAB workspace using output ports. On the basis of these values the harmonic analysis of curves voltages and current was carried out and balances active and reactive power were performed.

The simulation of the model and the analysis of results was carried out with input parameters r_o, c, u_d and r_z. The Figure 3 shows voltages and current waveforms for the sinusoidal function $sin(\tau)$ supply and input parameters $r_o = 1$, $c = 6$, $u_d = 0.06$ and $r_z = 0.05$.

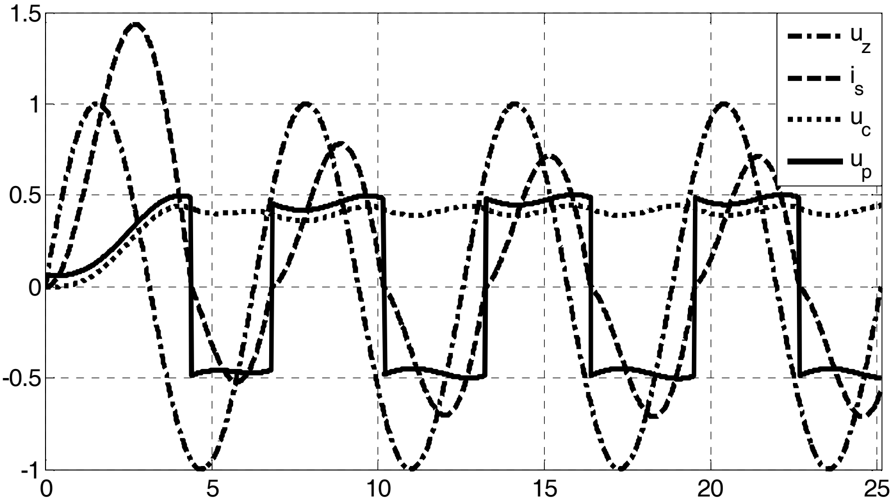

Fig. 3. The courses of the voltages and current in circuit from Figure 1.
Where: u_z – the supply voltage, u_c – the output voltage of bridge rectifier, u_p – the voltage on bridge rectifier by AC side, i_s – the current flowing in the power supply circuit

In the figure the dimensionless instantaneous waveforms of supply voltage u_z, current i_s flowing in supply circuit, voltage u_p on the rectifier on the AC side and the output voltage u_c are presented. The current i_s has negative phase shiftment in relation to the supply voltage u_z. The variables r_o and c express respectively dimensionless value of resistive and capacitive load of the rectifier bridge in reference to reactance ωL.

For constant input parameters $r_o = 1$, $u_d = 0.06$, $r_z = 0.05$ and different values of capacitive load c the current – voltage $u_p(i_s)$ characteristics were computed. The characteristics are presented in Figure 4. The characteristics show the dependence of the voltage u_p on the rectifier by AC side in function of the current i_s. On these characteristics fluctuations of the output voltage u_c in function of dimensionless capacity c are observed.

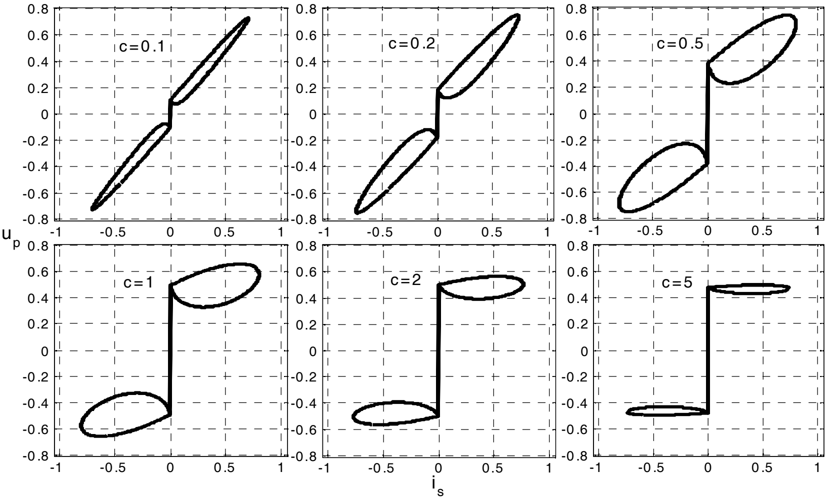

Fig. 4. The current – voltage $u_p(i_s)$ characteristics for $r_o = 1, r_z = 0.05, u_d = 0.06$ and different values c

The lowest level of these fluctuations is observed for $c = 5$. When for such case we will consider different values of the resistance r_o then characteristics $u_p(i_s)$ are such as in Figure 5. The increase of the voltage on the rectifier u_p and the decreasing value of the current i_s in supply circuit are observed for growing values of parameters r_o.

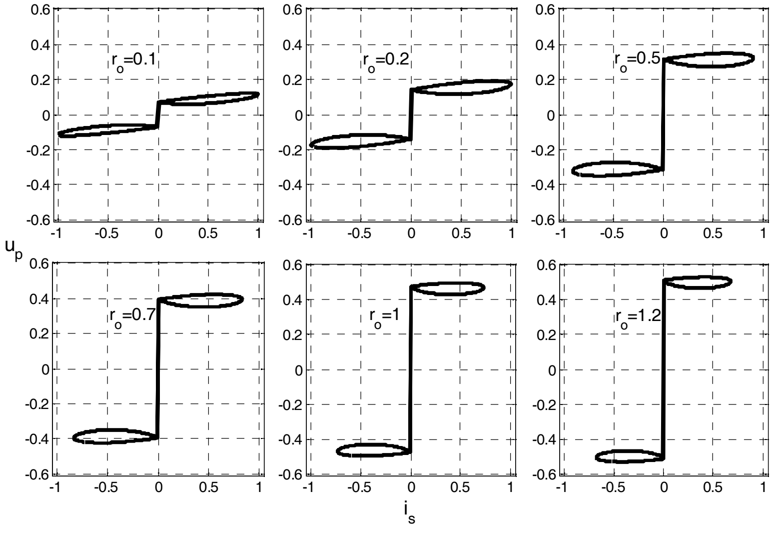

Fig. 5. The current – voltage $u_p(i_s)$ characteristics for $c = 5, r_z = 0.05, u_d = 0.06$ and different values r_o

One of the fundamental output quantities of the rectifier circuit is the mean value of the output voltage u_s. Assuming constant values of $u_d = 0.06$ and $r_z = 0.05$ the mean values of the dimensionless output voltage u_s was obtained in function r_o and c. The mean value of the output voltage u_s is presented respectively in Figures 6 and 7.

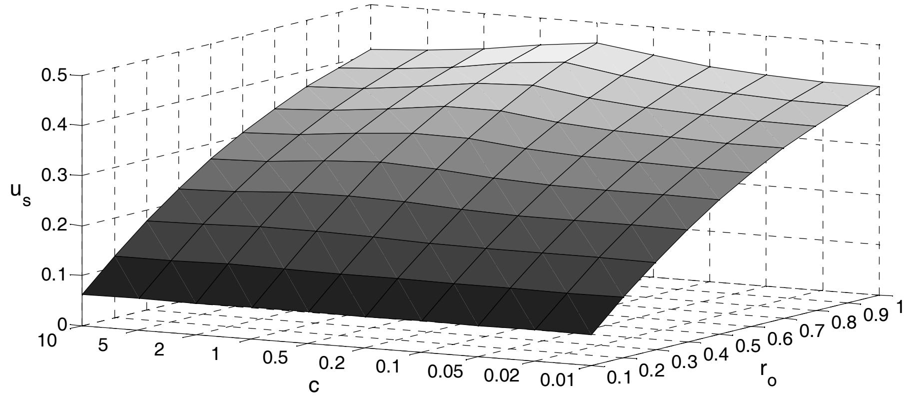

Fig. 6. The surface plot of the output voltage mean value u_s for varying load r_o and c

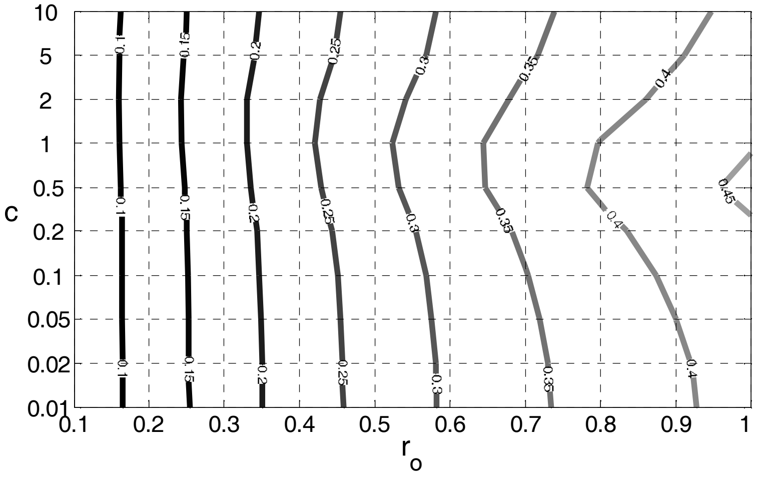

Fig. 7. The contour plot of the output voltage mean value u_s for varying load r_o and c

It can be noted that voltage u_s for $r_o > 0.3$ has maximum, which value increase together with more and more r_o. Using characteristics from Figure 6 and 7 it can be determined for which value of capacity c the voltage u_s has the maximal values. For that purpose helpful is the graphs in Figure 8, where the dependence of the voltage u_s in function of the load resistance r_o for selected values c is presented.

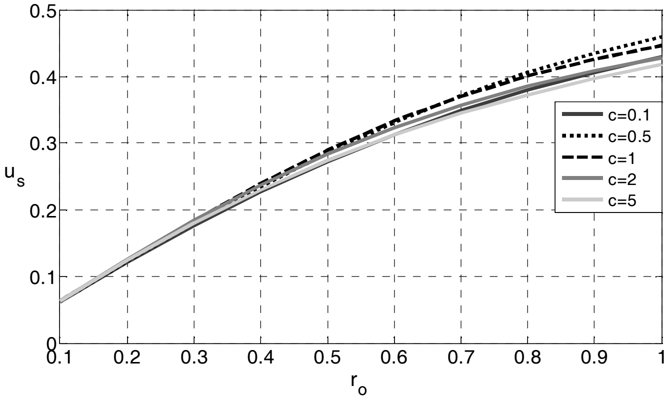

Fig. 8. The maximum values of the mean output voltage u_S in function of the resistance load r_O, for different values of capacity c

Fluctuations of the output voltage u_c for different values r_o and c are shown in Fig. 9 and 10. It can be seen that the voltage u_t is getting lower for increasing values of c and establishes on the constant level in the wide range of load variations r_o.

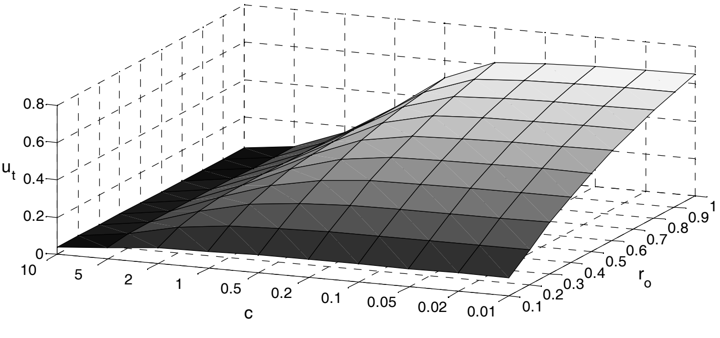

Fig. 9. The surface plot of the output voltage fluctuations u_t in the function of r_o and c

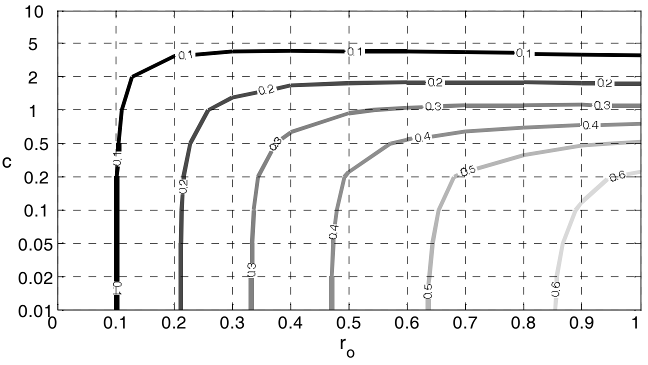

Fig. 10. The contour plot of the output voltage fluctuations u_t in the function of r_o and c

4. Harmonics and indicators of power quality

For current and voltages from figure 3 the harmonic analysis was performed. For periodical waveforms the harmonic analysis was carried out using Fourier series. For this purpose the circuit model in Figure 2 created in Simulink was controlled by the MATLAB script. The instantaneous values of currents and voltages were collected, and then their discrete Fourier transformation was carried out.

The Figure 11 shows that the voltage on the rectifier u_p has the highest harmonics contents. The values of these harmonics are characteristic for such nonlinear load. The amplitudes of the individual harmonics decrease with an increase their orders.

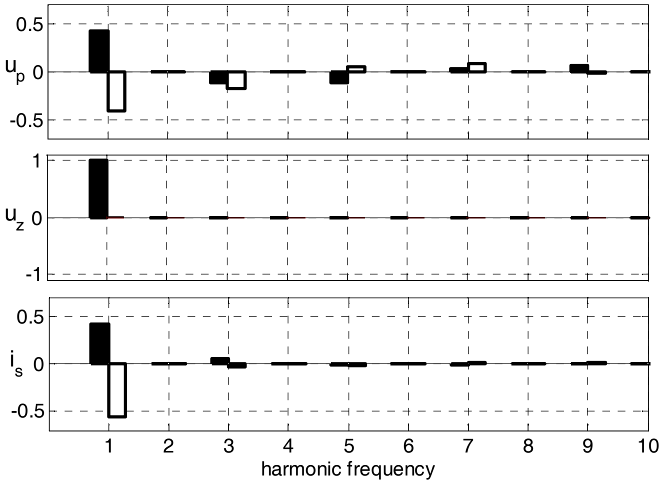

Fig. 11. The amplitudes of the real part (black) and imaginary (white) of harmonics of voltages and current in Figure 3

Harmonics contents in the current is not large. There is observed significant value fundamental harmonic, small value of the third harmonic and minimal values of the remaining harmonics. For the assessment of the shape of the voltage and current waveforms is used the distortion factor harmonic, which is defined according to the IEEE 519 formula [4]:

$$THDi = \frac{\sqrt{i_2^2 + i_3^2 + i_4^2 + \cdots}}{\sqrt{i_1^2 + i_2^2 + i_3^2 + \cdots}} \tag{8}$$

where: i_n – RMS values successive harmonics.

Total harmonic distortion values for voltages and current i_s of the circuit are shown in Table 1.

Table 1. The values of THD coefficient for voltages and current i_s in circuit from Figure 1

$THDu_z$	$THDu_p$	$THDi_s$	$THDu_c$
0.000	0.41	0.11	0.09

THD coefficient values in table 1 significantly depend on the changing parameters r_o and c. The influence changes of load capacitance and resistance of the rectifier on the THD coefficient of the current i_s is presented in Figure 12 and 13.

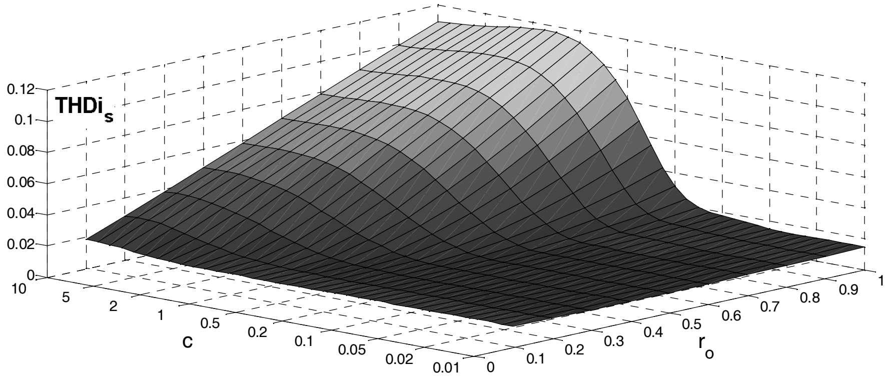

Fig. 12. The surface plot of the current i_s THD coefficient in function of r_o and c

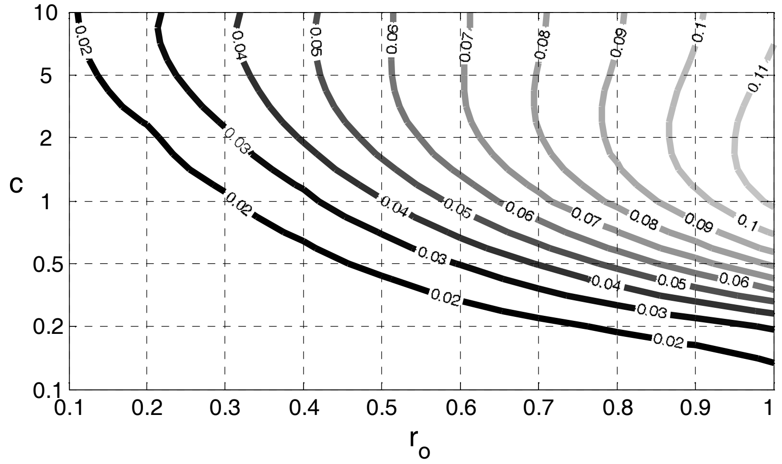

Fig. 13. The contour plot of the current i_s THD coefficient in function of r_o and c

The shape of the graph demonstrates that values THD of the current i_s depend both on parameters r_o and c. For considered values of parameters r_o and c the maximum value THD is 11 % and appears when $r_o > 0.95$.

The value of the power factor for considered circuit in the function of r_o and c, for constant values $u_d = 0.06$ and $r_z = 0.01$ is presented in Figures 14 and 15. For $r_o > 0.98$ and $c = 0.2$ the power factor achieves the maximum value which is equal 0.8. In such case the circuit has the reactive power least value.

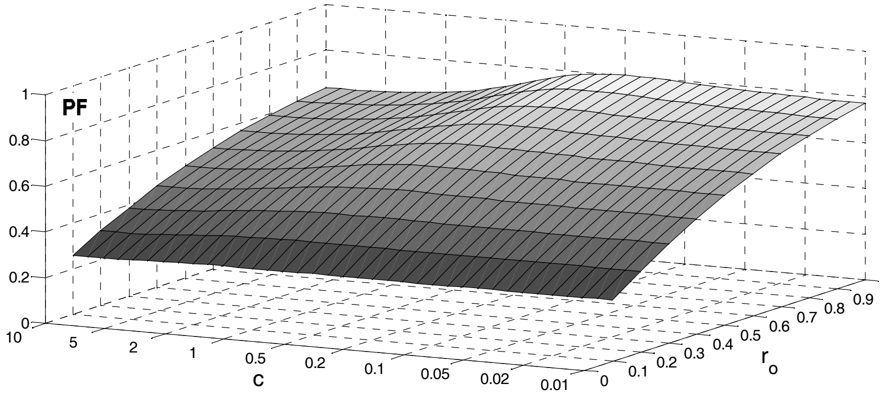

Fig. 14. The surface plot of the power factor PF in fucntion of r_o and c

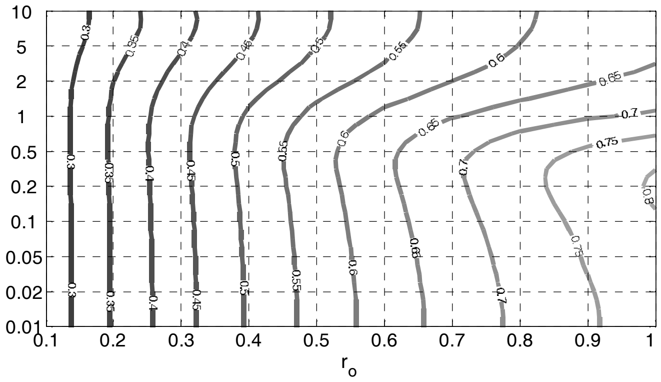

Fig. 15. The contour plot of the power factor PF in fucntion of r_o and c

5. Power balance

Using the Simulink model of the measuring system proposed in [5] the instantaneous values of the active and reactive power on individual elements of the circuit were designated. The operational diagram of the model is presented in Figure 16.

The instantaneous values of the active and reactive power are designated as products of the multiplication of the vector of voltages drops on the circuit elements by the current i_s and current derivative $di_s / d\tau$. The respectively instantaneous values vectors of the active and reactive power are averaged on digital filters MeanV1 and MeanV2 over the period. The results are shown on displays and led out to ports P and Q.

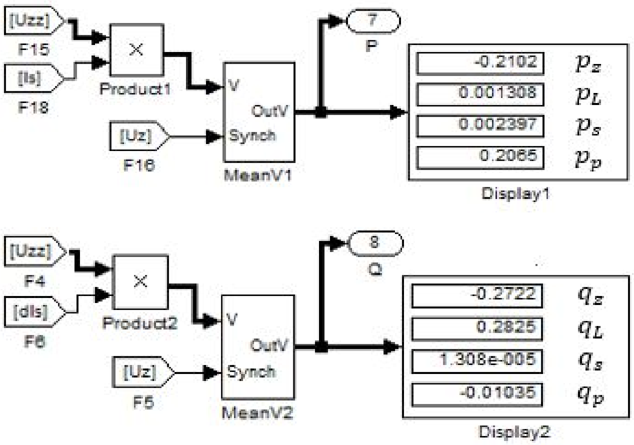

Fig. 16. The diagram of the active and reactive power measuring system. Parameters of simulation: $r_o = 1$, $c = 5$, $u_d = 0.06$ and $r_z = 0.01$; p_Z, q_Z; p_L, q_L; p_S, q_S; p_P, q_P – denote respectively active and reactive power supply source, inductance, resistance r_z and rectifier

The graphs of active and reactive power for constant values $r_o = 1$, $u_d = 0.06$, $r_z = 0.01$ and different c are placed in the Figures 17 and 18. Balances of the power in the circuit are fulfilled. Interesting phenomenon can be observed for reactive power. The rectifier has a capacitive reactive power. The value of this power is much smaller than the inductive reactive power.

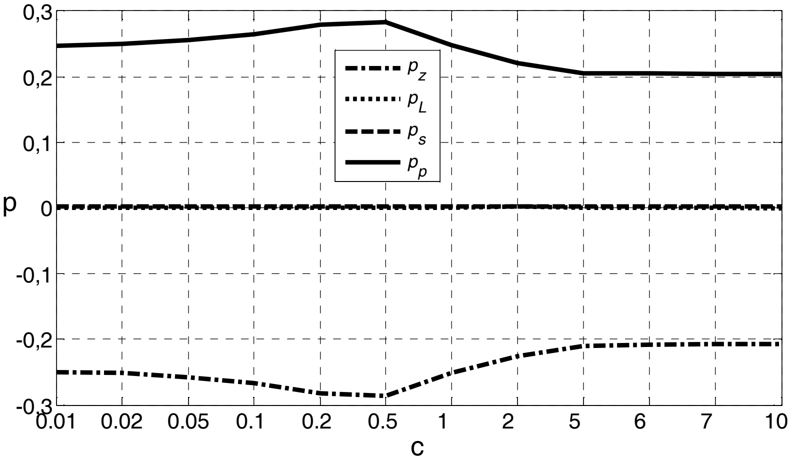

Fig. 17. Active power of the elements of the circuit for different value of c

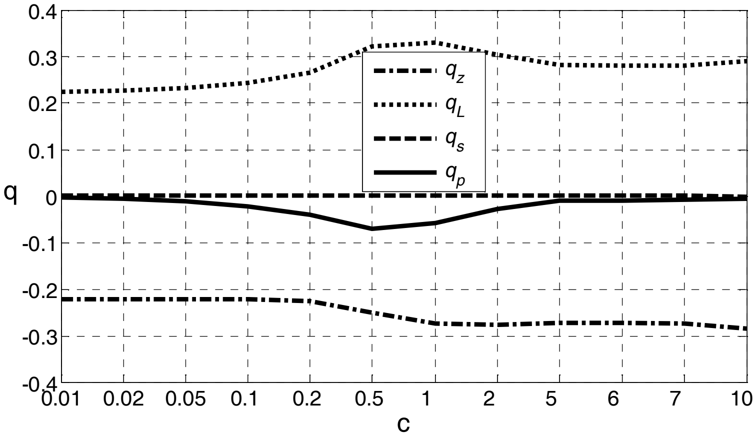

Fig. 18. Reactive power on the individual elements of circuit for different value of c

In the Figure 18 is shown that for $c = 0.5$ capacitive reactive power rectifier has a value of $q_p = -0.071$, which provides maximal value of reactive power compensation of inductive reactive power in circuit by the rectifier.

6. Conclusion

Presented model of the circuit constitutes the good and convenient model for analysis of influence of the non–linear load on mains and other loads connected to the network. Characteristics current – voltage observed on terminals AC the bridge are ambiguous, the reactive power of the rectifier is non – zero and has the negative sign like the power source or capacitive reactive power. For such case some compensation of the reactive inductive power is observed in the circuit.

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