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## NUMERICAL MODEL OF THE 10 KVA TRANSFORMER WITH COPPER WINDINGS

**Abstract:** The paper describes a circuit model of a single phase transformer implemented in PSpice program. The computer model was prepared on the basis of the geometrical and electrical parameters of the 10 kVA transformer with copper windings. Model takes into account Jiles-Atherton equations to simulate nonlinearity of the magnetic core. The no-load, nominal load and short-circuit characteristics generated during the numerical analysis were compared to the measurement results acquired on the physical model.

**Keywords:** conventional transformer, computer simulation, PSpice model, Jiles-Atherton model

### 1. Introduction

A transformer is an electrical device that operates on the principle of magnetic induction. This device is used to convert electricity with a given input voltage into electricity with a different output voltage without the involvement of mechanical energy [1].

A single-phase transformer consists of two insulated electrical circuits, i.e. windings, and one common magnetic circuit, i.e. the core (Fig. 1). The parts of the core on which the primary and secondary windings are wound are called columns (limbs, legs). The remaining part of the core, i.e. the yoke, serves to reduce magnetic resistance for magnetic flux.

The windings that get energy from the grid are called primary windings, and the ones that give energy are called secondary windings. Copper or aluminum wires are most commonly used to build windings. However, intensive research is being conducted on transformers

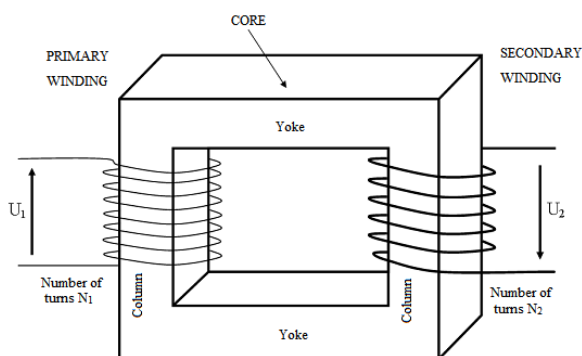


Fig. 1. Simplified structure of a single-phase column transformer

with windings made of tapes coated with high critical-temperature superconductor (high- $T_C$  superconductor or HTS). Such transformers

have higher performance compared to their traditional counterparts. At present, the biggest problem in the field of superconducting technology is the cost of the superconducting material [2-5].

### 2. Physical transformer model

The physical model of the conventional transformer was designed and built in the Laboratory of Superconductor Technology of the Electrotechnical Institute in Warsaw [4, 5].

Magnetic circuit of the conventional transformer was built of the magnetic core RZC-70/230-70 (wound and cut core) made of electrical steel ET114-27 with flux density  $B_{max}=1.75$  T at  $H_{max}=10$  A/cm and specific loss  $p=0.8$  W/kg at  $B=1$  T and  $f=50$  Hz. Both windings were made of 2.0 mm x 4.0 mm copper wire. It can be seen that windings were not optimized in sense of load losses. Fig. 2

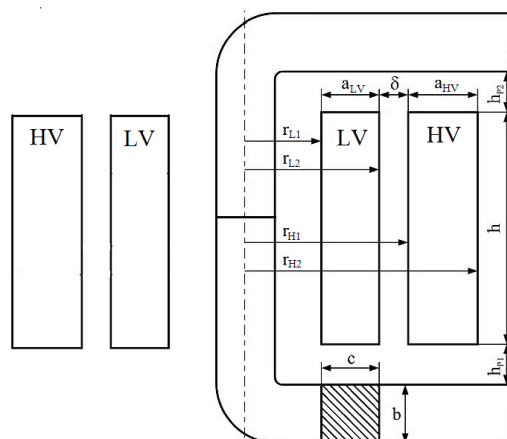


Fig. 2. Cross section of the 10 kVA transformer

shows the cross-section of the 10 kVA transformer with copper windings. Table I describes geometric dimensions and Table II presents its basic electrical parameters [4].

### 3. Numerical model of the transformer

Well-known circuit simulator PSpice was used to run simulations on the computer model of the conventional transformer. The computer model was developed on the basis of the electrical and geometrical parameters (Table I, Table II) of the physical model of the transformer with windings made of copper wire.

Table 1. Geometric Parameters of the 10 kVA Conventional Transformer [4]

Symbol	Dimension, mm	Symbol	Dimension, mm
$r_{L1}$	65.5	$a_{HV}$	8
$r_{L2}$	69.5	$h$	132
$r_{H1}$	77.7	$h_{P1}$	49
$r_{H2}$	85.7	$h_{P2}$	49
$\delta$	10	$b$	70
$a_{LV}$	4	$c$	70

Table 2. Parameters of the 10 kVA Conventional Transformer [4]

Parameter	Value
Number of phases	1
Rated power	10 kVA
Frequency	50 Hz
HV/LV winding voltage	230 V/115 V
HV/LV winding current	44 A/88 A
Core flux density	1.6 T
Core mass	27 kg
Relative short-circuit voltage	9%
Number of HV/LV winding turns	132/66

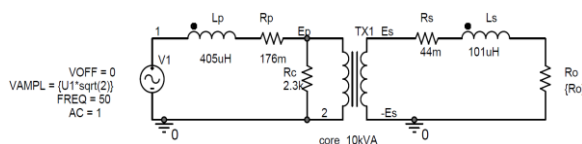


Fig. 3. Equivalent circuit of the conventional 10 kVA transformer

The equivalent circuit of the transformer (Fig. 3) consists of two subsystems. The first subsystem is a semi-ideal transformer TX1 with a magnetic core described by Jiles-Atherton model [6, 7]:

$$M_a = M_s \cdot \frac{H}{|H| + A}, \quad (1)$$

where:  $M_a$  – anhysteretic magnetization,  $H$  – magnetic field intensity (the magnetizing influence after air-gap correction),  $M_s$  – saturation magnetization,  $A$  – thermal energy parameter (shape factor of the normal magnetization curve) and

$$\frac{dM}{dH} = \frac{1}{1+C} \cdot \frac{(M_a - M)}{K} + \frac{C}{1+C} \cdot \frac{dM_a}{dH}, \quad (2)$$

where:  $M$  – magnetization,  $C$  – domain flexing parameter (elastic deformation factor),  $K$  – pinning energy per volume (drag, non-elastic deformation factor).

This system has full magnetic coupling of lossless windings and the magnetic core model with such geometrical parameters as: cross sectional area (AREA), cross-section fill ratio (PACK), average length of the magnetic flux path (PATH), air gap width (GAP), as well as the magnetic parameters of the ferromagnetic material: saturation magnetization (MS), shape factor of the basic magnetization curve (A), ratios of elastic (C) and non-elastic (K) magnetic domain wall deformations. Table III contains core model data. In the first approximation (case #1) parameter values were extracted from magnetization curve of ET114-27 steel.

The second subsystem consists of the lossy elements omitted in the first part. The longitudinal elements of the equivalent diagram are: stray-field inductances  $L_p$  and  $L_s$  of the primary and secondary winding, resistances  $R_p$  and  $R_s$  of the primary and secondary transformer winding. These values were calculated on the basis of copper wire dimensions and short-circuit voltage. The transverse element is the resistance  $R_c$ , which represents the eddy current loss in the magnetic core. It was roughly calculated using specific loss characteristic of ET114-27 steel [8].

### 4. The idle state simulation

The idle state test consisted of applying AC voltage to the terminals of the primary winding of the transformer and measuring the current taken by the transformer. High resistance of  $R_o=1 \text{ G}\Omega$  was connected to the secondary winding to imitate the no-load state during this simulation.

Fig. 4 compares the idle state curves in the physical and computer models of copper winding transformer. First core model

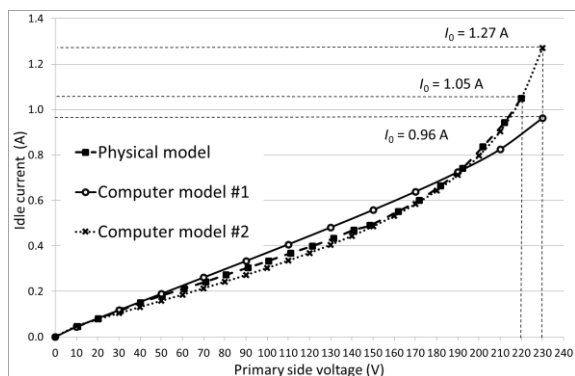


Fig. 4. Idle current vs. supply voltage

(computer model #1) gave inappropriate no-load characteristic. To deal with this situation, we adjusted values of thermal energy parameter  $A$  to the value 40 A/m and the air gap thickness  $GAP$  to 0.055 mm (computer model #2). Gap modification raises no questions – initial value was too high and cut-core parts actually had better fit. The need for shape factor modification may be explained by the temperature influence. The same magnetic core was used for testing superconducting windings and it is possible that it was exposed to liquid nitrogen at some point of experiments.

Table 3. Parameters of the Magnetic Core Model

Parameter	Description	Value
LEVEL	Model index	2
AREA	Core cross-section area	49 cm <sup>2</sup>
PACK	Core pack (stacking) factor	0.95
PATH	Average length of magnetic flux path	76.3 cm
GAP	Effective core air-gap length	0.1 mm (case #1)
		0.055 mm (case #2)
MS	Saturation magnetization	1.52 MA/m
A	Thermal energy parameter ( $B$ - $H$ curve shape factor)	10 A/m (case #1)
		40 A/m (case #2)
C	Domain flexing parameter	0.01
K	Domain anisotropy parameter	10 A/m

For unknown reasons, the laboratory tests of the physical model were executed only up to  $U_1=220$  V (rms) [4]. Computer model #2 is in a very good accordance with experimental data

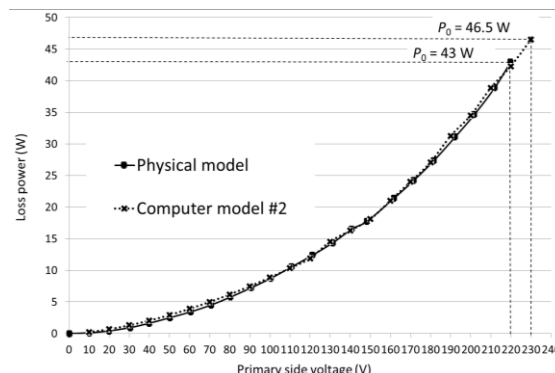


Fig. 6. Idle-state loss vs. supply voltage

and allows to extrapolate idle current value at nominal supply voltage  $U_1=230$  V as  $I_0=1.27$  A.

Fig. 5 compares the hysteresis loops of the transformer core for the computer model #1 and computer model #2. Due to the core packing factor assumed equal to 0.95, maximum flux density in both modelled cases is slightly over design level  $B=1.6$  T and it reaches  $B=1.68$  T. Strong influence of the  $A$  parameter of the core

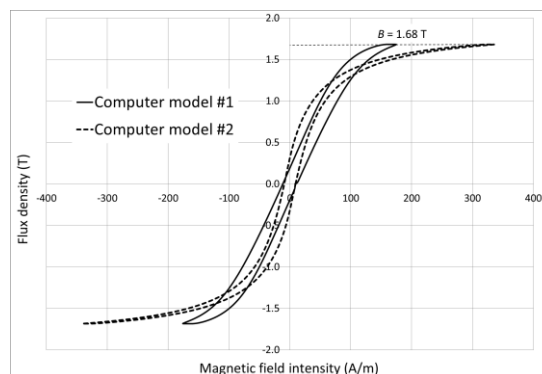


Fig. 5. Hysteresis loop of transformer core material model may be noticed.

When one is using internal PSpice variable  $H(\text{core})$  to draw hysteresis loop, special caution must be taken. In the presence of no-zero air gap, the mentioned above variable contains values equal to the resulting magnetomotive force divided by the average length of magnetic path (so-called “apparent  $H$ ”). To get correct  $H$  value for the ferromagnetic material, it is needed to solve magnetic circuit equation.

Fig. 6 depicts the loss characteristics in the idle state of the physical and computer model. Good agreement between physical and computer model #2 may be observed. Computer model #2 allows also to predict power loss at

nominal voltage  $U_1=230$  V as  $P=46.5$  W. Therefore, neglecting losses in windings, specific core loss value may be calculated as  $p=1.72$  W/kg at working flux density  $B=1.68$  T.

### 5. The nominal load-state simulation

The purpose of the load state simulation was to check the rated parameters and voltage regulation. Nominal load resistance of  $R_o=1.307$   $\Omega$  was connected to the secondary winding. Fig. 7 shows the primary ( $u_{HV}$ ) and secondary ( $u_{LV}$ ) voltage waveforms, while Fig. 8 illustrates corresponding current waveforms generated for the computer model #2.

The results of numerical analysis (Table IV) are close to the nominal values. Small differences are caused by the voltage regulation of value 6.52%. Nominal load test was not executed experimentally on the physical model [4, 5].

Table 4. Voltages and Currents in the Nominal Load State

Parameter	Computer model	Transformer design
Primary voltage $U_{HV}$	230 V	230 V
Secondary voltage $U_{LV}$	107.5 V	115 V
Primary current $I_{HV}$	41.5 A	44 A
Secondary current $I_{LV}$	82.6 A	88 A

### 6. Simulation of the short-circuit

The objective of simulating the short-circuit state in the computer model of the transformer was to acquire the characteristics of the winding current and total loss power to compare them with the laboratory measurements.

The short-circuit test consists of applying sinusoidal voltage to the primary winding of the transformer and measuring the current  $I_z$  and the power  $P_z$  drawn by the transformer. Secondary winding in this simulation was shorted by a very low resistance ( $R_o=1$   $\mu\Omega$ ).

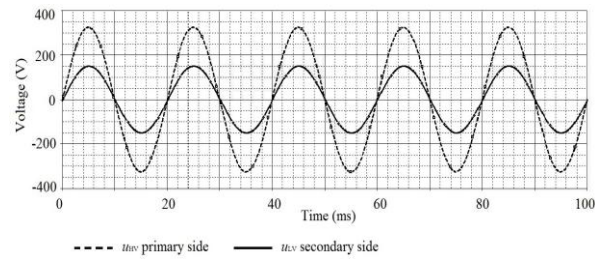


Fig. 7. Voltage of the primary ( $u_{HV}$ ) and secondary ( $u_{LV}$ ) winding in the computer model under nominal load

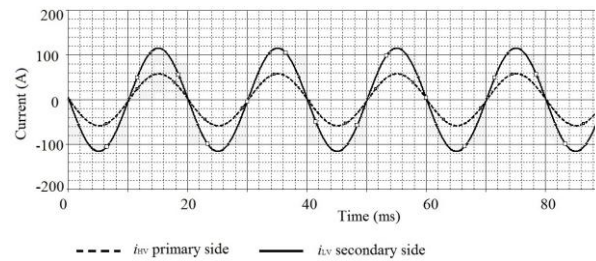


Fig. 8. Current of the primary ( $i_{HV}$ ) and secondary ( $i_{LV}$ ) winding in the computer model under nominal load

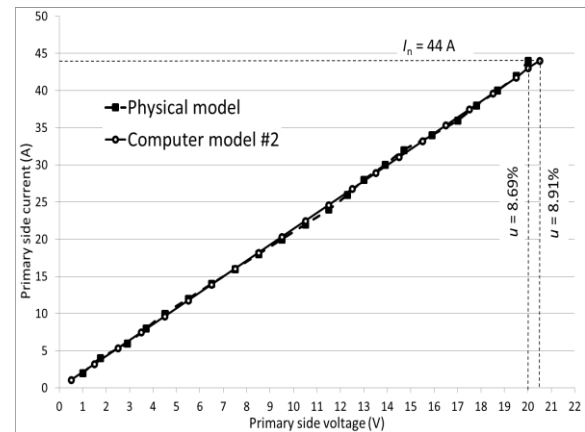


Fig. 9. Short-circuit current vs. supply voltage

Fig. 9 compares the characteristics of the short-circuit measurements for the physical model of the conventional transformer and computer simulation. The relative short-circuit voltage  $u_k$  from the numerical analysis is very similar to the results obtained at the laboratory. The relative short-circuit voltage for the computer model of the conventional transformer is  $u_k=8.91\%$  ( $U_1=20.5$  V), whereas for the physical model of the conventional transformer it equals  $u_k=8.69\%$  ( $U_1=20.0$  V). Both results are also comparable to the transformer design value  $u_k=9.0\%$  ( $U_1=20.7$  V).

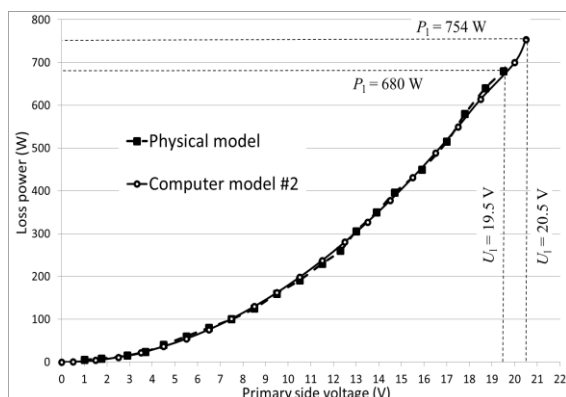


Fig. 10. Short-circuit power loss vs. supply voltage

Fig. 10 shows the comparative characteristics of the loss power during short-circuit test for the physical and computer models of the conventional transformer. Results are similar. Once again, due to unknown reasons, researchers conducting laboratory tests described in [4] stopped the loss measurement at  $U_1=19.5$  V despite the fact that the short-circuit voltage was determined earlier as 20.0 V. At this voltage, the loss level reaches  $P_z=680$  W for physical model. For the computer simulation, the loss level is equal  $P_z=754$  W at the nominal HV winding current  $I_1=44$  A ( $U_1=20.5$  V).

## 7. Conclusions

The PSpice program allows to create computer models of electric circuits containing components with non-linear magnetic cores. Jiles-Atherton model seems to be adequate to simulate any transformer operating at a fixed frequency. It must be noticed that this model does not take into account eddy-current loss. For precise analysis, an additional non-linear resistor with carefully crafted characteristic is needed to supplement transformer core model.

The waveforms generated using the numerical analysis were compared with the

results obtained in the laboratory during measurements on the physical model of the 10 kVA transformer with copper windings. Numerical results are very almost identical to the values acquired during the laboratory tests. All the noticeable differences come from the fact that laboratory experiments described in [4, 5] did not cover full supply ranges.

Electric circuit simulation seems to be a reliable tool for the analysis of non-linear electric devices. Presented transformer equivalent diagram will be extended by a model of windings made of high- $T_C$  superconductor coated tape to simulate HTS transformer behavior in transient states.

## 8. References

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