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The Case of Hidden Instantaneous Powers

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Summary: **This brief communication addresses the case of a dc nonlinear load supplied with a perfectly constant instantaneous power. A careful analysis shows that this condition does not provide optimum transfer of electric energy .**

Key words:
harmonics,
power definitions,
power factor

1. INTRODUCTION

The goal of this paper is to observe the structure of the instantaneous power measured at the terminals of a dc nonlinear load supplied with a distorted voltage:

$$v = V_0 + \sum_{h=1}^{N} \hat{V}_h \sin\left(h\omega t + \alpha_h\right) \tag{1}$$

and current:

$$i = I_0 + \sum_{h=1}^{N} \hat{I}_h \sin\left(h\omega t + \beta_h\right) \tag{2}$$

The peculiarity of this case consists in the fact that the instantaneous power:

$$p = vi = P \tag{3}$$

is not time-variable, but perfectly constant. In this case some observers may wrongly interpret such a condition as ideal and a superficial macroscopic examination of this case may lead to the conclusion that no power oscillations between load and the voltage source take place and no power factor compensation is required.

2. EXAMPLE

A numerical example, where P = 1000 W, will help explain the true nature of this case. The studied system is approximated by the first five harmonic voltages and current phasors (Table I), with the voltage spectrum determined from the expression $v = 1000/i$. The voltage, current and instantaneous power waveforms are shown in Figure 1. The first step is the computation of the rms voltage and current:

$$V = \sqrt{V_0^2 + \sum_{h=1}^{N} \hat{V}_h^2 / 2} = 130.7 \text{ V}$$

$$I = \sqrt{I_0^2 + \sum_{h=1}^{N} \hat{I}_h^2 / 2} = 10.45 \text{ A} \tag{4}$$

that yield the apparent power:

$$S = VI = 1365.90 \quad \text{VA} > P \tag{5}$$

and the power factor:

$$PF = P/S = 0.73 \tag{6}$$

A PF<1 indicates the presence of instantaneous nonactive powers. These are powers that oscillate between the load and the voltage source and their mean value is nil. Nevertheless, these powers cause additional power loss in the supplying line.

The detailed picture of the instantaneous powers is obtained from the product of the instantaneous voltage and current:

$$vi = V_0 I_0 + \sum_{h=1}^{N} v_h i_h + \sum_{\substack{m,n=1 \\ m \neq n}}^{N} v_m i_n \tag{7}$$

where $V_0 I_0 = P_0$ is the dc power,

$$\sum_{h=1}^{N} v_h i_h = \sum_{h=1}^{N} \left(P_h + p_{ih} + p_{qh}\right) \tag{8}$$

Table 1. Voltage and current phasors, and harmonic active powers

h	$\underline{\hat{V}}_h = \hat{V}_h / \underline{\alpha_h}$ (V)	$\underline{\hat{I}}_h = \hat{I}_h / \underline{\beta_h}$ (A)	P_h (W)
0	120	9.763	1171.56
1	60 / 60°	4.413 / –120°	–132.39
2	25 / 40°	0.506 / –33.350°	1.81
3	30 / 0°	2.281 / –174.43°	–34.05
4	12 / – 100°	1.443 / 9.46°	–2.88
5	10 / –90°	0.869 / 108.81°	–4.11
Total			P= 999.94

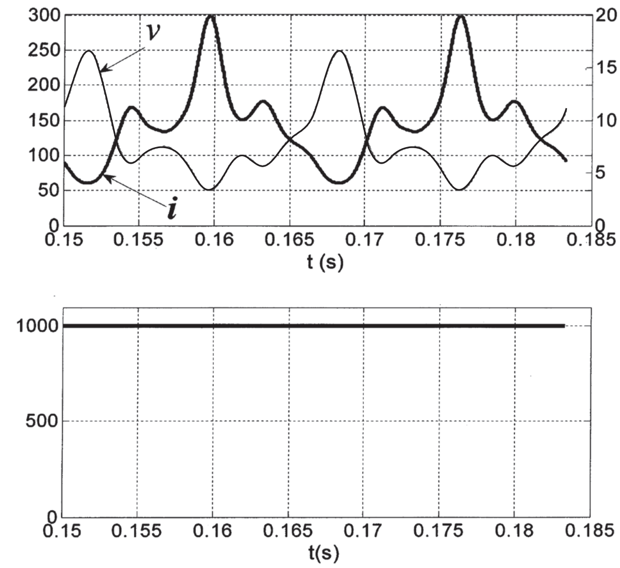

Fig. 1. Oscillograms: voltage v, current i and total instantaneous power p.

with the harmonic active power of order h:

$$P_h = \frac{\hat{V}_h \hat{I}_h}{2} cos(\vartheta_h) \text{ with } \vartheta_h = \alpha_h - \beta_h \quad (9)$$

followed by:

$$p_{ih} = -P_h \, cos(2h\omega t + 2\alpha_h) \quad (10)$$

the instantaneous intrinsic power [1], another oscillating nonactive component, always present when active power is present, and having the distinctive property that is not causing line power losses [2].

The last term in (8) is the instantaneous reactive harmonic power of order h:

$$p_{qh} = Q_h \, sin(2h\omega t + 2\alpha_h) \quad (11)$$

with the amplitude:

$$Q_h = \frac{\hat{V}_h \hat{I}_h}{2} sin(\vartheta_h) \quad (12)$$

and the sum.

The last term in (7) is a cross-product of voltages and currents:

$$\sum_{\substack{m,n=1\\m\neq n}}^{N} v_m i_n = V_0 \sum_{h=1}^{N} i_h + I_0 \sum_{h=1}^{N} v_h + \sum_{\substack{m,n=1\\m\neq n}}^{N} v_m i_n \quad (13)$$

Here we observe three instantaneous nonactive powers: Current Distortion Power:

$$V_0 \sum_{h=1}^{N} i_h = \sum_{h=1}^{N} \sqrt{2} \hat{D}_{Ih} \, sin(h\omega t + \beta_h); \quad D_{Ih} = \frac{V_0 \hat{I}_h}{\sqrt{2}} \quad (14)$$

and Harmonic Distortion Power:

$$I_0 \sum_{h=1}^{N} v_h = \sum_{h=1}^{N} \sqrt{2} D_{Vh} \, sin(h\omega t + \alpha_h); \quad D_{Vh} = \frac{I_0 \hat{V}_h}{\sqrt{2}} \quad (15)$$

Voltage Distortion Power:

$$\sum_{\substack{m,n=1\\m\neq n}}^{N} v_m i_n = \sum_{\substack{m,n=1\\m\neq n}}^{N} 2D_{mn} \, sin(m\omega t + \alpha_m) sin(n\omega t + \beta_m) \quad (16)$$

$$\hat{D}_{mn} = \frac{\hat{V}_m \hat{I}_n}{2}$$

It is possible to express the apparent power squared as follows:

$$S^2 = (VI)^2 = \left(V_0^2 + \sum_{h=1}^{N} V_h^2\right)\left(I_0^2 + \sum_{h=1}^{N} V_h^2\right) =$$

$$= P_0^2 + \sum_{h=1}^{N} P_h^2 + \sum_{h=1}^{N} Q_h^2 + \sum_{h=1}^{N} D_{Ih}^2 + \sum_{h=1}^{N} D_{Vh}^2 + \sum_{\substack{m,n=1\\m\neq n}}^{N} D_{mn}^2 \quad (17)$$

If the skin-effect is ignored, the power loss in the line with a total resistance R_S is:

$$\Delta P = R_s I^2 = \frac{R_s}{V^2} S^2 \quad (18)$$

Comparing (17) with (18) results that all the instantaneous powers, with the exception of the intrinsic powers, contribute to the conversion of electric in thermal energy, i.e. power losses.

To minimize the line losses the load current must be compensated to yield unity power factor. Defining an active current that has a waveform that is a perfect replica of the voltage wave, $i_a = Gv$ and choosing $G = P/V^2 = 0.0585$ S, results that the difference current, $i_N = i - i_a$ it is a pure nonactive current that must be compensated by means of an active filter. In this case the rms line current will be reduced from 10.45 A to 7.65 A. In Figure 2 are presented the voltage, current and total instantaneous power for the compensated system at unity power factor. Now it becomes evident that the uncompensated system, in spite of a perfectly constant instantaneous power, is not providing the best conditions for supplying line utilization.

3. CONCLUSION

The first impression of tranquility, of no power oscillations, is misleading. Hidden under the "veneer" of a perfect energy flow are energy oscillations that reduce the utilization of the supplying lines and contribute to additional power loss. This study dealt with one particular unusual condition that helped reveal the existence of energy oscillations.

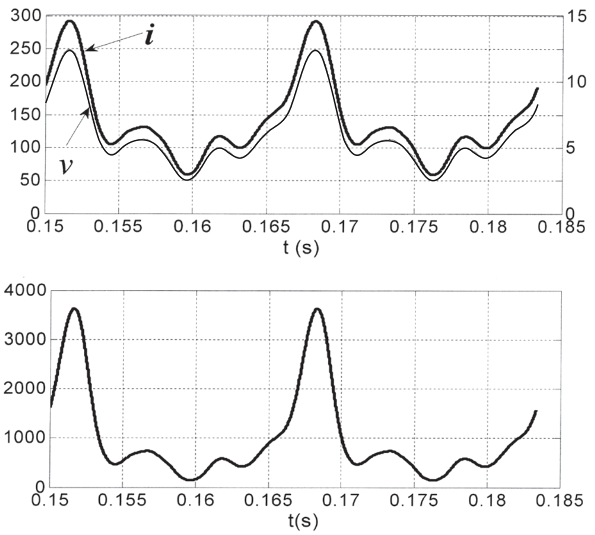

Fig. 2. Unity power factor: Oscillograms of voltage *v*, current *i* and total instantaneous power *p*.

4. REFERENCES

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