Analysis of different equations of undrained shear strength estimations using Atterberg Limits on Pontianak Soft Clay

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Many researchers have done study to estimate the value of undrained shear strength for fine grained soils like clay or silt. Determining of undrained shear strength and compressibility parameters in laboratory are really tedious and time consuming. Therefore, a correlation between undrained shear strength and Atterberg limits is useful for restraint of testing number and costs. Central tendency parameters such as an average, deviation standard and coefficient of variation are performed to analyze the data of soft clay in Pontianak, Indonesia. Based on analysis that undrained shear strength coincides with 50 percentile of distribution data meanwhile undrained compressive strength is around twice of cohesion for testing using unconfined pressure. This relationship is the most familiar equation. Moreover, undrained shear strength using mean value is more realistic for correlation between undrained shear strength and Atterberg limits on some equations from previous findings.

Keywords and phrases: undrained shear strength, Atterberg limit, cohesion, soft clay.

Introduction

Undrained shear strength is a very important parameter in engineering. Undrained shear strength is a parameter to the bearing capacity of soil that could bear on it. Some laboratory tests needed to obtain these values are expensive and time consuming, while soil properties like moisture content and Atterberg limits can be performed faster and cheaper.

Literature Review

Bearing capacity for subsoil can be stated in some parameters. Several sources come from research in which correlations of the parameters were proposed. Atterberg limits can be employed to get bearing capacity of subsoil. By using regression analysis and central tendency parameters in statistical analysis we can obtain a correlation.

Bearing Capacity of Subsoil

There are some approaches to know bearing capacity of subsoil. Undrained shear strength shows capability or bearing capacity of soil. Relations between undrained shear strength of soil (s_u) and undrained cohesion (c_u) in the case without confining pressure called unconfined compressive strength (q_u), have been proposed by some previous research results and used as subgrade failure criteria for pavement design as depicted in Table 1.

Table 1. Undrained unconfined compressive strength.

Researchers and/or Sources	Equation
Giroud and Noiray (1981)	$q_u = 3.14\ c_u$
Barenberg (1992)	$q_u = 3\ c_u$
Philips (1987)	$q_u = 2.8\ c_u$
Rodin (1965)	$q_u = 3.14\ c_u$
Roadex III (2008)	$q_u = 4\ c_u$

Soil consistency can be estimated using value of unconfined compressive strength (Terzhagi &Peck, 1967) as shown in Table 2.

Table 2. Soil consistency.

Consistency	q_u (kPa)
Very soft	< 24
Soft	24 – < 48
Medium	48 – < 96
Stiff	96 – < 192
Very stiff	192 – < 383
Firm	> 383

Mohr-Coulomb equation gives a linear correlation between normal stress and shear stress. This line as criteria of Mohr-Coulomb failure is shown below:

$$s = c + \sigma \tan \phi \qquad (1)$$

where: s — shear stress (kPa);
σ — normal stress (kPa);
c — cohesion (kPa);
ϕ — internal friction angle (°).

Unconfined compressive strength test in which confining pressure is equal to zero, shear shear strength (s) is independent from confining pressure (σ_3), so that:

$$s = \sigma_1/2 = q_u/2 = c \qquad (2)$$

where: σ_1 — vertical stress (kPa);
q_u — unconfined compressive strength (kPa).

Undrained Shear Strength and Atterberg Limits

The ratio of undrained shear strength of clay to overburden stress by many researchers have been correlated to Atterberg limits. Results for this ratio (s_u/σ_i) are often defined by in the following equations. Normally consolidated clay with a plasticity index of more than 5%, Skempton [8] gives a linear relationship for this ratio value to the value of plasticity index.

$$s_u/\sigma_i = 0.11 + 0.0037\ I_p \qquad (3)$$

Bjerrum and Simons [2] present a power equation for correlation between undrained shear strength to plasticity index.

$$s_u/\sigma_i = 0.045\ Ip^{\ 0.5} \qquad (4)$$

In addition, Bjerrum and Simons [2] also present another equation between this ratio to liquid index, I_L which $I_L = (W_n — W_p)/(W_L — W_p)$ and consistency index, $I_C = (W_L — W_n)/W_L — W_p)$

$$s_u/\sigma_i = 0.18\ /\ I_L^{\ 0.5} \qquad (5)$$

Karlsson and Viberg [5] present a linear equation for correlation for undrained shear strength and liquid limit.

$$s_u/\sigma_i = 0.005\ W_L \qquad (6)$$

where: W_L — liquid limit (%);
I_p — plasticity index (%);

Table 3. Data from soil investigation of expand runway project at Supadio Airport.

STA	No. of sample	Depth (m)	γ (kN/m3)	σ i (kPa)	Wn (%)	W_L (%)	Wp (%)	Ip (%)	I_L	Ic	qu (kPa)	c (kPa)	ϕ (°)	su (kPa)	qu/c
0+072	1	9.30	14.23	132.34	75.79	35.23	25.81	9.42	5.31	-4.31	42.40	12.60	5.03	24.25	3.37
	2	20.30	14.33	290.90	49.85	49.38	26.38	23.00	1.02	-0.02	41.19	14.40	6.90	49.60	2.86
0+303	1	3.30	15.43	50.92	69.17	48.79	30.64	18.15	2.12	-1.12	20.50	10.00	4.34	13.86	2.05
	2	12.30	18.25	224.48	34.86	24.90	14.66	10.24	1.97	-0.97	65.50	15.60	10.59	57.57	4.20
0+639	1	9.30	15.64	145.45	68.83	48.29	26.23	22.06	1.93	-0.93	18.60	9.70	3.66	19.00	1.92
	2	20.30	15.66	317.90	60.38	62.46	32.79	29.67	0.93	0.07	39.50	12.00	7.46	53.63	3.29
1+054	1	3.30	14.74	48.64	81.55	48.73	27.53	21.20	2.55	-1.55	16.00	9.70	6.90	15.59	1.65
	2	12.30	15.82	194.59	71.80	54.89	28.97	25.92	1.65	-0.65	14.10	8.30	4.29	22.90	1.70
1+435	1	6.30	16.33	102.88	66.96	51.52	26.64	24.88	1.62	-0.62	22.00	9.70	5.54	19.68	2.27
	2	15.30	15.88	242.96	66.00	49.12	29.55	19.57	1.86	-0.86	32.30	13.90	6.84	43.04	2.32
1+773	1	9.30	16.03	149.08	90.96	53.44	26.86	26.58	2.41	-1.41	20.60	13.00	6.84	30.88	1.58
	2	18.30	18.54	339.28	35.59	22.62	19.08	3.54	4.66	-3.66	21.50	9.70	3.66	31.40	2.22
2+106	1	3.30	14.23	46.96	86.42	54.02	38.44	15.58	3.08	-2.08	8.40	10.50	3.03	12.99	0.80
	2	12.30	16.21	199.38	49.59	55.13	29.11	26.02	0.79	0.21	14.40	11.40	5.60	30.95	1.26
2+250	1	6.30	14.4	90.72	83.01	52.27	28.50	23.77	2.29	-1.29	11.40	8.30	3.77	14.28	1.37
	2	15.30	17.19	263.01	43.83	46.46	25.55	20.91	0.87	0.13	42.40	19.80	13.66	83.72	2.14
2+400	1	12.30	13.93	171.34	56.82	17.10	14.62	2.48	17.02	-16.02	16.70	7.20	3.72	18.34	2.32
	2	20.30	14.64	297.19	50.14	46.16	25.71	20.45	1.19	-0.19	13.60	9.70	3.09	25.74	1.40
2+550	1	9.30	16.87	156.89	54.10	51.96	24.87	27.09	1.08	-0.08	10.80	12.40	4.40	24.47	0.87
	2	18.30	16.28	297.92	52.31	39.08	22.91	16.17	1.82	-0.82	21.80	10.80	7.46	49.81	2.02
	Max	20.30	18.54	339.28	90.96	62.46	38.44	29.67	17.02	0.21	65.50	19.80	13.66	83.72	4.20
	Min	3.30	13.93	46.96	34.86	17.10	14.62	2.48	0.79	-16.02	8.40	7.20	3.03	12.99	0.80
	Avg	11.85	15.73	188.14	62.40	45.58	26.24	19.34	2.81	-1.81	24.68	**11.44**	5.84	**32.08**	**2.08**

s_u — undrained shear strength (kPa);
σ_i — overburden shear (kPa).

Central Tendency Parameters

Parameters of central tendency consist of the average value, coefficient of variation and standard deviation [1]. These parameters are relatively very familiar in calculating and analyzing of data and making conclusion afterwards.

Average: $X = \Sigma Xi / n$ (7)

Standard deviation for sample:

$$\sigma = [\Sigma (Xi - X)^2/(n - 1)]^{0.5} \quad (8)$$

Coefficient of variation: $Cov = \sigma/X$ (9)

Regression analysis also will be performed to give comparison from analysis using central tendency parameter as mentioned above.

Data Collecting

Main data for calculation and analysis came from soil investigation of the expand runway project of Supadio airport in Pontianak, Indonesia. There are 10 boreholes along at left and right of existing runway having 2.250 m length and 30 m width. It will be extended 2.550 m length and 45 m width. Data of soil investigation for the project is shown in Table 3.

Discussion for Analysis Results

Data shown in Table 3 above will be analyzed more detail in the next paragraph. Herein bearing capacity of subsoil namely unconfined compressive strength and undrained shear strength are important parameters.

Bearing Capacity of Subsoil

Bearing capacity of subsoil can be expressed by using undrained shear strength. Value of undrained shear strength without confining pressure is equal to unconfined compressive strength. This value is theoretically twice as big as cohesion. The rightmost column of Table 3 shows that the ratio unconfined compressive strength to cohesion taken from laboratory testing vary from 0.8 to 4.2 and arithmetic average is 2.08 while average of cohesion is 11.44 kPa. Based on the data above we obtain the arithmetic average value of unconfined compressive strength as bearing capacity of this subsoil to be 23.8 kPa which it is 2.08 times as much as cohesion. For value of unconfined compressive strength around 23,8 kPa can be classified as very soft soil and almost soft soil. Standard deviation for this value is 0.85 kPa and coefficient of variation is 40.86 percent.

By using descending dispersion of data for ratio of q_u/c and taking 50 percentile (mean value) we obtain the ratio of 2.05 and this value is very close with previous arithmetic average value of 2.08. It is moderate value. Nevertheless, for pavements design purpose in the case of bearing capacity of subsoil, it is usual for representative value of a segment road to be taken 75 percentile as design value should be even 90 percentile.

Table 4. Dispersion for value of q_u/c.

q_u / c	Dispersion
4.20	
3.37	
3.29	15 percentile
2.86	
2.32	25 percentile
2.32	
2.27	
2.22	
2.14	
2.05	50 percentile
2.02	
1.92	
1.70	
1.65	
1.58	75 percentile
1.40	
1.37	85 percentile
1.26	90 percentile
0.87	
0.80	

Undrained Shear Strength

Ratio between undrained shear strength and overburden stress can be correlated with Atterberg limits as described on the equation 3 through equation 6. In the presented four equations linear correlation and power function are used.

Table 5 shows that average value of ratio for 50-percentile is equal to 0.18 kPa. This value agrees with Skempton's equation with value of 0.182 kPa and little different with Bjerrum-Simons' first equation round of 0.192 kPa. These values are different from Karlsson--Viberg's equation and Bjerrum-Simons' second equation round of 0.228 kPa and 0.274 kPa respectively. Bjerrum--Simons' second equation gives higher result than the others [2, 5, 8]. Meanwhile value of s/σ_i when using Mohr-Coulomb's equation gives 0.186 kPa. In this calculation, shear strength is used vertical stress (overburden stress) as normal stress because internal friction angle is very small.

From the findings as shown in Table 5, it can be seen that the ratio s/σ_i coming from Skempton's equation is the best fit with 50 percentile of laboratory test. By using Mohr-Coulomb's equation a good enough estimation can be also made, but results from other equations are poorer.

Table 5. Ratio undrained shear strength to overburden stress.

Location	Percentile of value s/σ_i						Mohr-Coulomb	Skempton	Bjerrum-Simons	Karlsson-Viberg	Bjerrum-Simons **
	15	25	50	75	85	90	s/σ_i	s/σ_i	s/σ_i	s/σ_i	s/σ_i
0+072	0.284	0.200	0.177	0.137	0.137	0.109	0.183	0.145	0.138	0.176	0.415
	0.129	0.091	0.080	0.062	0.062	0.050	0.171	0.195	0.216	0.247	0.182
0+303	0.739	0.519	0.459	0.355	0.355	0.283	0.272	0.177	0.192	0.244	0.262
	0.168	0.118	0.104	0.080	0.080	0.064	0.256	0.148	0.144	0.125	0.253
0+639	0.259	0.182	0.161	0.124	0.124	0.099	0.131	0.192	0.211	0.241	0.250
	0.118	0.083	0.074	0.057	0.057	0.045	0.169	0.220	0.245	0.312	0.174
1+054	0.774	0.544	0.480	0.371	0.371	0.296	0.320	0.188	0.207	0.244	0.287
	0.193	0.136	0.120	0.093	0.093	0.074	0.118	0.206	0.229	0.274	0.231
1+435	0.366	0.257	0.227	0.176	0.176	0.140	0.191	0.202	0.224	0.258	0.229
	0.155	0.109	0.096	0.074	0.074	0.059	0.177	0.182	0.199	0.246	0.246
1+773	0.252	0.177	0.157	0.121	0.121	0.097	0.207	0.208	0.232	0.267	0.280
	0.111	0.078	0.069	0.053	0.053	0.042	0.093	0.123	0.085	0.113	0.389
2+106	0.801	0.563	0.498	0.385	0.385	0.307	0.277	0.168	0.178	0.270	0.316
	0.189	0.133	0.117	0.091	0.091	0.072	0.155	0.206	0.230	0.276	0.160
2+250	0.415	0.292	0.258	0.199	0.199	0.159	0.157	0.198	0.219	0.261	0.273
	0.143	0.101	0.089	0.069	0.069	0.055	0.318	0.187	0.206	0.232	0.168
2+400	0.220	0.154	0.136	0.105	0.105	0.084	0.107	0.119	0.071	0.086	0.743
	0.127	0.089	0.079	0.061	0.061	0.048	0.087	0.186	0.203	0.231	0.197
2+550	0.240	0.169	0.149	0.115	0.115	0.092	0.156	0.210	0.234	0.260	0.187
	0.126	0.089	0.078	0.061	0.061	0.048	0.167	0.170	0.181	0.195	0.243
Max	0.801	0.563	0.498	0.385	0.385	0.307	0.320	0.220	0.245	0.312	0.743
Min	0.111	0.078	0.069	0.053	0.053	0.042	0.087	0.119	0.071	0.086	0.160
Average	0.290	0.204	**0.180**	0.139	0.139	0.111	**0.186**	**0.182**	**0.192**	**0.228**	**0.274**

Table 6. Corrected constant for equation.

Researchers	Existing equations	Equations from laboratory test	Equations from regression analysis
Skempton	s/σ_i = 0.11 + 0.0037 I_p	s/σ_i = 0.11 + 0.0037 I_p	s/σ_i = 0.37674 - 0.00856 I_p
Bjerrum-Simons	$s/\sigma_i = 0.045\ I_p^{0.5}$	$s/\sigma_i = 0.0422\ I_p^{0.5}$	$s/\sigma_i = 0.38124 / I_p^{0.30086}$
Karlsson-Viberg	s/σ_i = 0.005 W_L	s/σ_i = 0.004 W_L	s/σ_i = 0.331 - 0.0026247 W_L
Bjerrum-Simons**	$s/\sigma_i = 0.180 / I_L^{0.5}$	$s/\sigma_i = 0.118 / I_L^{0.5}$	$s/\sigma_i = 0.12144 / I_L^{0.425287}$

Equations using regression analysis as a comparison for previous method gives different equation. Figure 1(a) through 1(d) show trend line of linear and power function. Coefficient of determination (R^2) for four equations are very low correlation between dependent variable (s/σ_i) and independent variables of I_p,W_L, I_c respectively.

Table 6 gives new some equations from laboratory test and equations are resulted from regression analysis. When we look into the determination factor (R^2), we must collect more samples to fulfill sufficient data in other to achieve a certain significant level.

Conclusions

There are two main conclusions from analyzing the data. Firstly, undrained shear strength without the confined pressure or undrained unconfined compressive strength (q_u) for Pontianak soft soil is around almost twice of

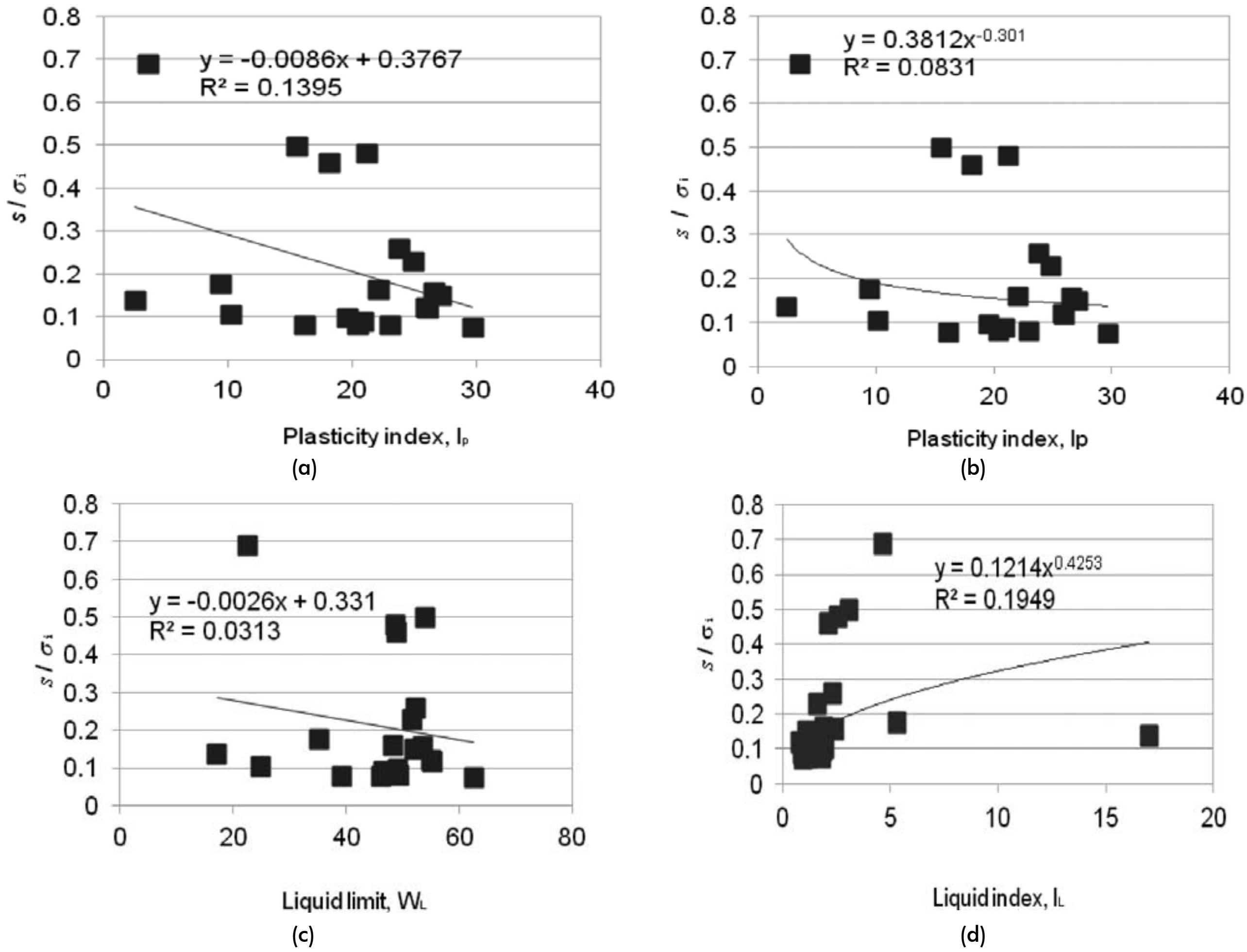

Fig. 1. Correlation using regression analysis, a) s/σ_i vs. I_p, b) s/σ_i vs. I_p, c) s/σ_i vs. W_L, d) s/σ_i vs. I_L.

its cohesion value. It is familiar correlation between the unconfined compressive strength and cohesion. This value coincides with 50-percentile of data distribution. Secondly, Skempton's equation shows the close correlation with this subsoil. Furthermore, when we want to estimate using regression analysis, large number of soil sample is needed to obtain a certain significant level and then for different site of subsoil can be done with the same procedure as described in this paper.

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