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Electrical schematics for 1D analysis of a bridge type MEMS capacitive switch

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In this paper, a robust method to analyse a simple model of a RF (Radio Frequency) MEMS (Micro Electro-Mechanical Systems) capacitive switch is depicted. The method is based on multi-physics phenomenons and interdependencies between different physics domains. A component, which enables to link variety of the physics domains is an electrical equivalent circuit. Such equivalent circuit has been created in LTSpice software, which is a freeware circuits simulator. This electrical equivalent schematic enables to run static, frequency domain, transient analysis. The presented method enables simultaneous simulations in an electrical, mechanical and electro-mechanical domain. Obtained results were verified by the comparison with the results of a commercial software, a scientific software and data from literature. The method, presented in the paper gives robust, accurate results in the one-dimensional analysis domain of simple mechanical resonators, which are simplified mechanical models of the RF MEMS capacitive switches.

KEYWORDS: RF MEMS capacitive switch, electrical equivalent schematic, multi-physics simulations

1. Introduction

On-going miniaturisation of electronic devices size is a chance and from the other point of view is a challenge for industry and research teams. The miniaturised electrical elements, cause, that new electrical products, which are created, based on them to be more commonly used and applications of such products are wider. At the same time miniaturisation is the challenge for engineers, designers and researchers, which need more accurate, more efficient methods of design and simulation of phenomenons at a micro scale.

The RF MEMS capacitive switch is a micro scale structure, typical size of such element is few microns up to one centimetre. The RF MEMS switches components combine mainly micro-mechanical and micro-electrical phenomenons. A typical RF MEMS capacitive switch consists of a thin metal membrane, which is also called a bridge, suspended over a central conductor and connected at both ends to a ground conductor of a CPW (coplanar waveguide). The bridge and the signal line of the CPW are two electrodes, which create a tunable capacitor. The capacitance between the two armatures is strongly dependent from the bridge displacement. The substrate of the RF MEMS capacitive switch is usually made from silicon. The main materials used in the construction of the MEMS devices are predominant of silicon and its compounds, silicon (di)oxide (SiO2) and silicon nitride (SixNy) [1].

All above mentioned parts of the typical RF MEMS capacitive switch are characterised by small dimensions, which effects in relatively small mass of these parts. Small mass is usually considered as advantage. One can generally observe, that smaller things are less affected by gravity, which allows faster speed and efficiency [2]. The RF MEMS capacitive switch uses mechanical movement to achieve a short circuit or an open circuit in a transmission line and can be used for microwave applications [3].

Applications for the MEMS are very wide, from washing machine in one's house, to the cell phone in one's pocket. These devices can be used in an automotive industry as for example accelerometers for air-bag systems [4], as navigation devices, in security systems, as wireless systems. The RF MEMS capacitive switches especially can be used in defence systems, radar systems, smart-phones, aerospace and RF wireless systems. The MEMS devices are replacing elements such as PIN diodes or switching field effect transistors (FET) [5].

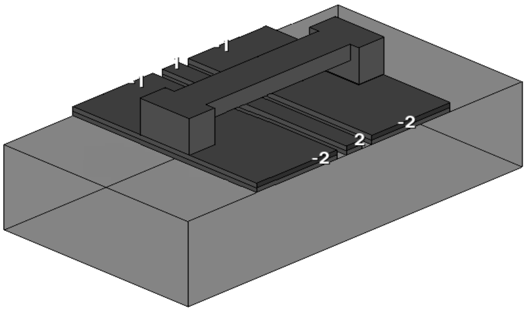


Fig. 1. 3D visualisation of the simple RF MEMS capacitive switch

The main goal of this paper is to correctly formulate multi-physics phenomenons to create efficient, accurate methods for simulating and modelling the RF MEMS capacitive switch devices. One of such an approach is a method, based on mechanical and electrical elements analogues, which enables to create an electrical equivalent circuit. Such circuit fully reproduces mechanical and electrical phenomenons occurring in the RF MEMS capacitive switch, and can be used to present and to solve coupled electromechanical problems. The electrical equivalent circuit was created in the LTSpice, based on algebraical, mathematical models but also using analogues elements and electrical equivalences [5]. Non-linear, behavioural current sources, which are defined in the LTSpice were also used [6]. These sources represent phenomena of an energy exchange between the mechanical and electrical fields, which indeed takes place inside of the real RF MEMS capacitive switch.

The DC analysis gives the characteristics of the bridge displacement with regard to the applied voltage, enabling to determine the static pull-in voltage. Transient analysis determines the time depending behaviour of the switch, giving information such as the dynamic pull-in voltage and switching time. RF analysis computes the frequency characteristics of scattering parameters, generated by the passing of the signal in the devices stable state. The novelty of the paper is the use of LTSpice software and the electrical equivalent circuit (EEC), obtaining a reduced order model of a RF MEMS device in the form of a Spice circuit.

2. Interdependencies of components

A knowledge of the basic rules and different mechanisms of the physical phenomenons is an essential, key issue to correctly model multi-physics problems. In physics and engineering area very often we can talk about similarities between phenomenons from different physics domains. In physics and a technical science we observe analogues, which take place between the various physics phenomenons, between various components. Thus very similar physical laws rule the phenomena or component. That means, that same or similar mathematical description can be used to depict of the behaviour of the component or to describe phenomenon. Using components analogues is a common practice in engineering science, which is strongly useful in modelling. Analogues are observed between electrical, mechanical, hydraulic, pneumatic components. Components analogy is also useful in analyses of MEMS devices. In MEMS devices occur mechanical phenomenon, heating phenomena fluidic phenomenon, electrical phenomenon, all these phenomenons and all components, which are subjects of these phenomenons can be depicted with the use of electrical equivalences. It is common practice to use components analogues and the EEC in MEMS devices modelling. This paper only debates Structural-Electrostatic coupled problems. The analogues between mechanical and electrical components were used, to model phenomenons, which occur in the RF MEMS capacitive switch. The final electrical equivalent circuit, which was created, was inspired by analogues presented in commercial software APLAC [7].

The essential analogy is to present forces as electrical currents [5]. For this purpose, arbitrary behavioural current sources are used. In the considered model, there are three forces modelled with the use of behavioural current sources. The Electrostatic, which is dependent from geometrical parameters and material parameters like in a typical capacitor. The electrostatic force also depends from the applied voltage between the armatures of the so formed capacitor and the displacement of the bridge, displacement that can be represented by a voltage. The elastic force, which value is equal to multiplication of spring constant and displacement (where the displacement is represented and proportional to proper U_z voltage).

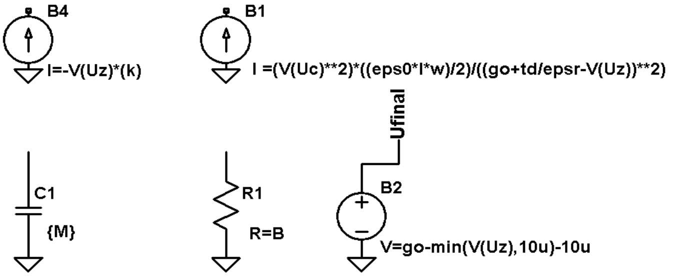


Fig. 2. Elements analogues B1-electrostatic force, B2-limiter, B4-elastic force, C1-mass, R1-damping coefficient

The resulting force directly depends from Uv voltage, that voltage represents the velocity. A damping force is also presented as an electrical current but in the presented approach not as current from current source but as current flowing through a resistor. Damping coefficient is included in the EEC as conductance, this conductance is achieved with the use of typical resistor. The mass in the depicted EEC is presented with the use of capacitor, the capacitance is equal to the mass. A voltage limited source is used in the EEC to include contact force, which appears when there is a physical contact between two plates.

3. Electromechanical problems in RF MEMS capacitive switch

The analysed structure of the mechanical resonator is presented in Fig. 3, representing a simplified model of an RF MEMS capacitive switch. Electrostatical and structural phenomenon dictates the behaviour of the switch, behaviour that is reproduced by using electrical components.

The structure is a mass-spring system, where *M* represents the mass of the bridge, *B* is damping coefficient and *K* spring constant (spring constant is dependent from bridge geometry and material properties).

The external voltage source Uc applied to the tunable capacitor plates causes an electrostatic field between the armatures resulting an attraction force of electrostatic nature. This force is responsible for the displacement of the upper plate (bridge). The electrostatic force F_{es} is described by formula [8]:

$$F_e = \frac{U_c^2}{2}\left(\frac{LW\varepsilon_0}{(g_0 + t_d/\varepsilon_r - z)^2}\right) \tag{1}$$

where: L - length, W - width, t_d - thickness of the isolating layer deposited on the bottom electrode, g_0 - gap between the bridge and pull-down electrode in state position, z - displacement from the up-state position.

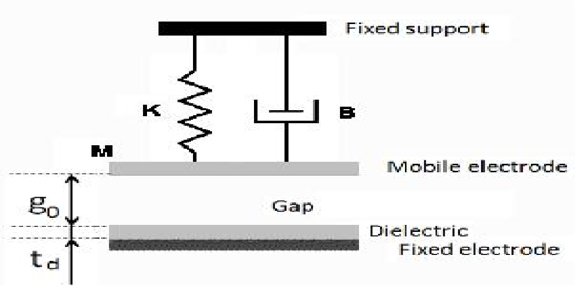


Fig. 3. The mechanical resonator with air gap and displacement limiter

The displacement of the bridge is strongly dependent from the force F_{es}, this relation can be depicted with the use of the Newtonian dynamic equation [8]:

$$M\frac{d^2z}{dt^2} + B\frac{dz}{dt} + Kz = F_e \tag{2}$$

The above formula represents a linear differential equation, which has been solved using Matlab to validate LTSpice results.

4. Electrical circuit in freeware software LTSpice

Based on the previously presented components and interdependencies between them the EEC in the LTSpice software is created. The essential component, which enables realization of the EEC in the LTSpice is BCS (behavioural current source) [9, 10]. The BCS is a device strongly dependent from the node voltage, which enables to create current-voltage dependencies. LTSpice gives the possibility to describe the behaviour of BCS using constant parameters and mathematical functions. These, above mentioned functionalities are enough to create the EEC, which is reliable imitation of the simplified model of the RF MEMS capacitive switch. On Fig. 4 the created EEC is presented, containing, the components interdependencies, depicted in the chapter two.

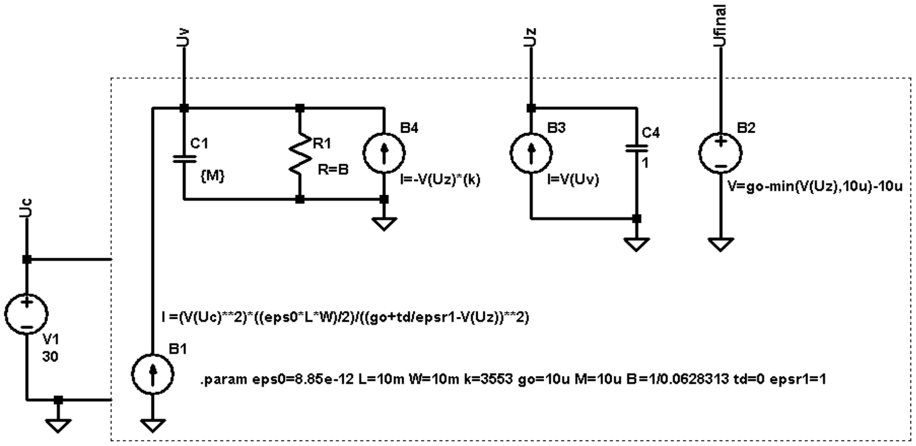


Fig. 4. The EEC of the RF MEMS capacitive switch in LTSpice

The proposed circuit contains three behavioural current sources, two voltage source, two capacitors and resistor [5]. Source B1 represents F_{es}, B4 represents the elastic force that forms in the spring, source B3 is used as gyrator to convert voltage Uv into current, which represents resultant force. The capacitor C4 is used to obtain relation between velocity and the displacement, which is represented as U_Z. The voltage source V1 is applied voltage and B2 is voltage limited source. The voltage Ufinal is the final voltage, which simulates displacement of upper plate and it is taking into account the limit of the gap between plates.

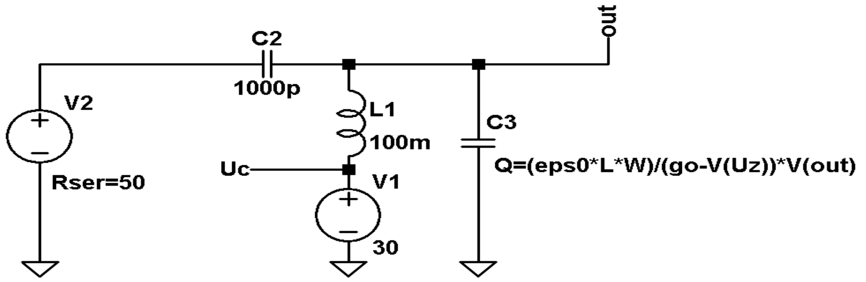


Fig. 5. The RF macromodel in LTSpice

The presented equivalent circuit requires from the potential user only to introduce geometrical and material input data of the structure [5]. To obtain the static characteristic, the DC analysis must be set and for the dynamic characteristic the time analysis must be set. All the parameters used in the description of the multi-physics macromodel had been scaled, obtaining the resulting voltage in μV representing the displacement in μm.

For the RF analysis already depicted EEC of the RF MEMS capacitive switch is not enough, it is necessary to use typical macromodel [10]. Such modified macromodel is presented on Fig. 5. Capacitor C3 is a tuning component dependent from voltage Uz (displacement), the value of its capacitance is given by the EEC from Fig. 4 In RF macromodel there is also voltage source V2 and the output “out”, these are RF ports used to simulate S-parameters values, C2 is a coupling capacitor for RF signals, V1 is the DC voltage source (Uc), that is isolated from RF signals with a inductor (L1).

5. Results validation

Results of the EEC and the RF macromodel from LTSpice, were compared to the APLAC data and to the Matlab software computations. The comparison was made on the example: mass of the bridge M = 10 µg, length of the bridge L = 10 m, width of the bridge W = 10 m, spring constant K = 3553 N/m, initial gap between the bridge and pull-down electrode g_0 = 10 µm, damping coefficient B = 15.91 Ns/m.

The second example has the same input data, the only difference is that the damping coefficient is not constant, is dependent from displacement z and gap g_0. Mathematically first example is described by formula (2) and the second example is described by following formula [8]:

$$M\frac{d^2z}{dt^2}+B(1.2-\frac{z}{g_0})^{-\frac{3}{2}}\frac{dz}{dt}+Kz=F_e \qquad (3)$$

The comparison of the results are presented on Fig. 6 - 13.

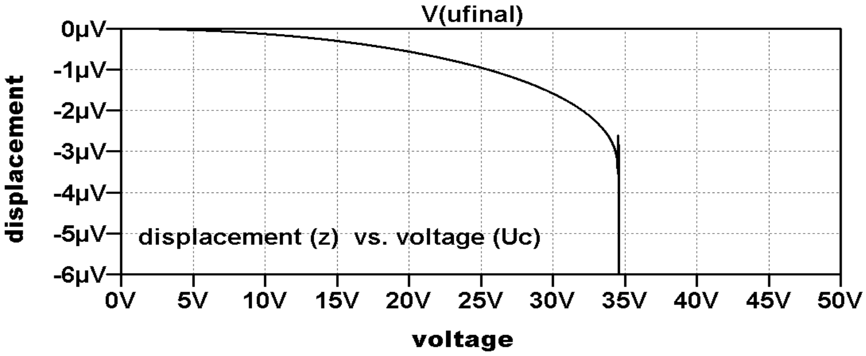


Fig. 6. Static characteristics made in LTSpice, example when B is constant

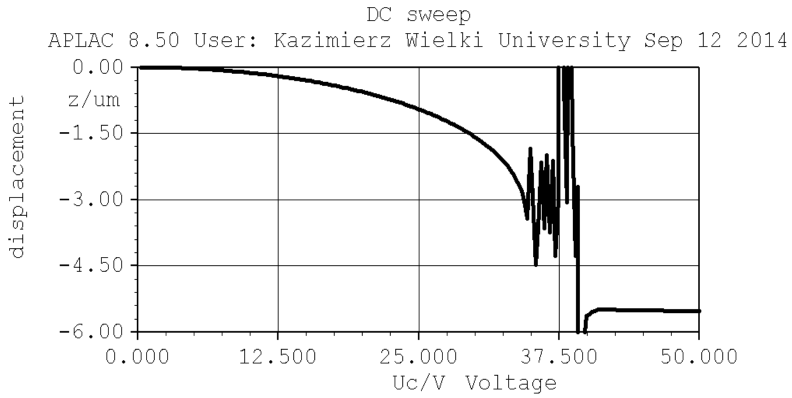


Fig. 7. Static characteristics made in APLAC, example when B is constant

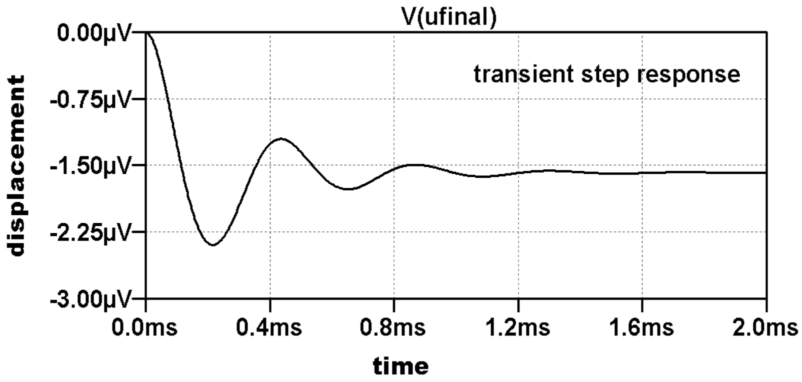


Fig. 8. Transient (time) characteristics made in LTSpice, example when B is constant

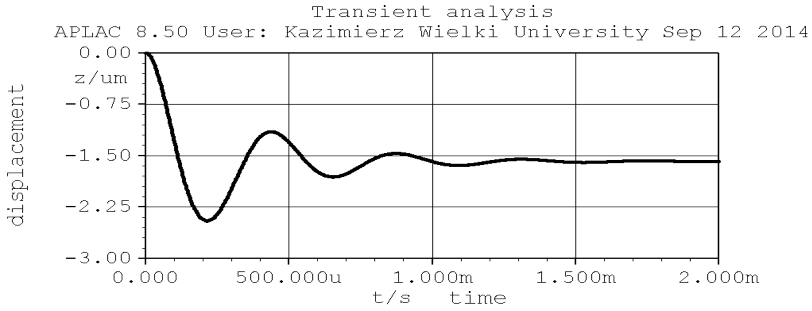


Fig. 9. Transient (time) characteristics made in APLAC, example when B is constant

On Fig. 6 - 7 static characteristics are presented. In LTSpice characteristic is more stable than in APLAC. On Fig. 8 - 9 transient characteristics are presented, they are almost identical. On Fig. 10 there is comparison of two examples when B is constant and when damping coefficient is changing.

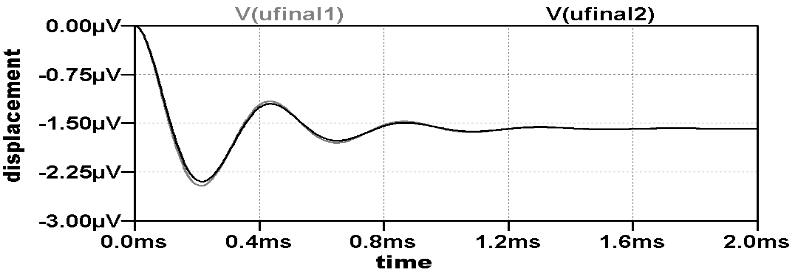


Fig. 10. Transient (time) characteristics made in LTSpice, comparison of two examples when B is constant V(ufinal1) and when damping coefficient is changing V(ufinal2)

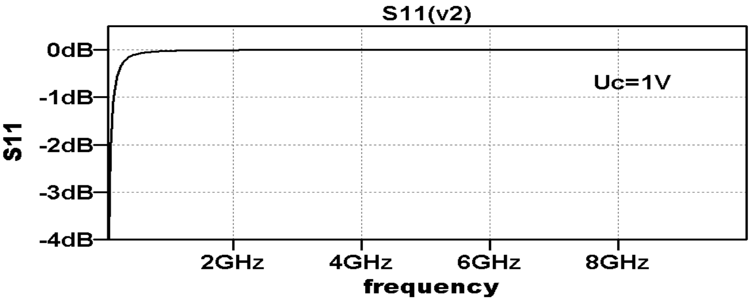


Fig. 11. The RF characteristics for S11 made in LTSpice, when Uc = 1 V

On Fig. 11 - 12 RF characteristics for S11 are presented the LTSpice and APLAC results are very similar (the shape of the curve and values). On Fig. 13 RF characteristics for S21 is presented.

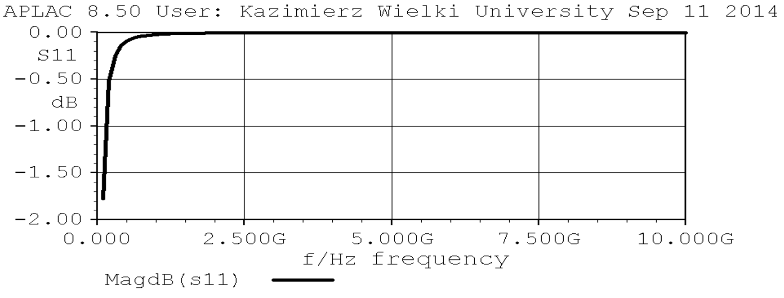


Fig. 12. The RF characteristics for S11 made in APLAC, when Uc = 1 V

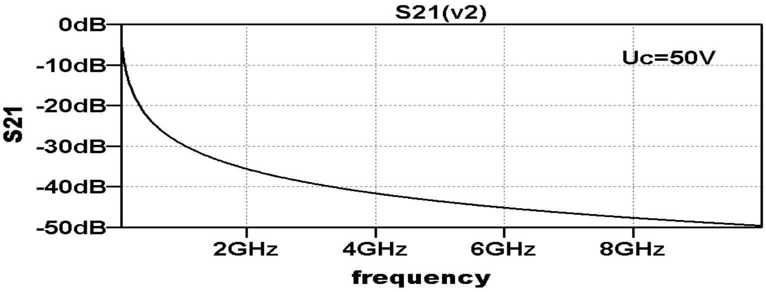


Fig. 13. The RF characteristics for S21 made in LTSpice, when Uc = 50 V

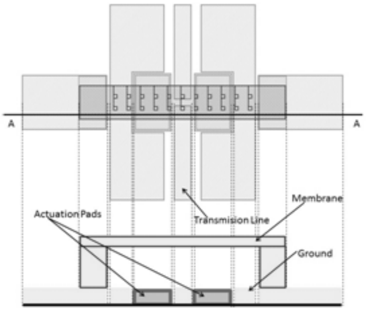
6. Information extraction of a MEMS switch

In this part we will give an example of how to approximate *K, M* and *B* parameters by using effective parameters k_{eff}, m_{eff} and b_{eff} from the numerical modeling and simulation. The analysed switch consists of two grounded conductors on which the membrane is placed, suspended over a transmission line, through which the RF signal passes. Fig. 14 shows the practical realization of this switch. The construction with two actuation pads is preferred to the classical one having the dielectric placed on the RF signal line, due to the fact that in this case the dielectric degradation problem that appears in down state is avoided [11].

Due to the symmetry, only half of the geometry is modeled. The signal line, the ground lines and the actuation pads are not relevant for the multiphysics models that focus on the switching between the up and down positions, when there is no signal passing through the signal lines. Also, to begin with, we considered a 2D model, which is easier to generate and simulate than 3D models, especially when doing parametric tests.

The parameterized geometry of the model is shown in Fig. 15. The geometry information is given in Table 1, whereas material properties used are: Poisson's ratio ν = 0.44, Elasticity (Young's) modulus E [Pa] $78 \cdot 10^9$, Mass density ρ [kg/m3] 19280, Relative permittivity ε_r of air 1 of dielectric 3.9. Only the information relevant for the simulations described is given.

For the multiphysics formulation the computational domain was divided in 2. The structural (MEC) domain includes only the membrane, its maximal limits being $x_{max} = L_m/2 + W_P$ and $y_{max} = H_{cpw} + H_{gap} + H_m$. A plane strain model is considered, in which the unknown is the 2D displacement vector u defined in every point of the computational MEC domain. The structural boundary conditions used are (Fig. 16): • symmetry conditions on the symmetry plane, that ensure that the displacement of the points on the symmetry plane are solely on the vertical axis i.e. $\mathbf{n} \cdot \mathbf{u} = 0$, where $\mathbf{n}$ is the normal at the boundary; • zero displacement along the line where the membrane is clamped **u** = 0; • the rest of the MEC boundary remain natural conditions (free movement).



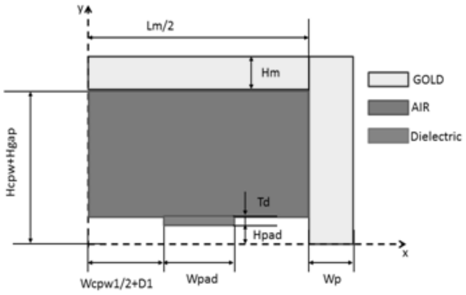


Fig. 14. The analyzed bridge type capacitive switch. The transversal section is not to scale

Fig. 15. Parameterized geometry of the switch

The electrostatic (ES) domain includes the dielectric and the air just under the membrane, its maximal limits being $x_{max} = L_m/2$ and $y_{max} = H_{cpw} + H_{gap} - H_{pad}$. The ES formulation is expressed with respect to the unknown V which is the electric scalar potential, defined in every point of the computational ES domain. Dirichlet boundary conditions are imposed as shown in Fig. 17, whereas the rest of the ES boundary is left with natural boundary conditions.

Table 1. Geometric parameters

Parameter	Value [µm]	Significance
L_m	910	Length of the membrane
W_m	200	Width of the membrane
H_m	2	Height of the membrane
H_{cpw}	0.6	Height of the signal line
H_{gap}	2.5	Height of the gap
H_{pad}	0.45	Height of the electrode
W_{cpw1}	100	Width of the signal line
W_{pad}	100	Width of the electrode
W_p	160	Width of the membrane support
D_1	30	Distance between the CPW and electrode
T_d	0.1	Height of the dielectric

The MEC and ES domains have two lines of common boundary (Fig. 18), where coupling conditions have to be imposed. Indeed, the electric field applied between the membrane and the electrode produces a force that makes the MEC domain to change its shape, and thus modify the ES domain and the electric field.

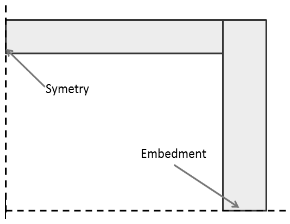


Fig. 16. MEC Boundary Conditions

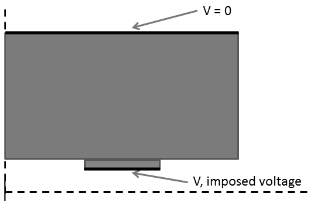


Fig. 17. ES Boundary Conditions

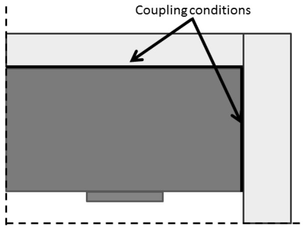


Fig. 18. MEC and ES fields are coupled along two lines

Several analysis types are useful to extract the switch behavior. The static simulation - in which the time dependencies are neglected – which gives information on the static pull-in voltage of the device. The modal analyses in which the eigenfrequencies that may appear after the excitations have vanished are computed. This analysis gives an indication on the useful time frame to use in a transient simulation. The transient simulation – which gives information about the devices switching behavior. Like it was shown before the behavior of this system depends on the values of the parameters *M*, *B* and *K*.

The effective stiffness coefficient k_{eff} is extracted from a set of static simulations, in which the applied voltage is increased from 0 to the pull-in voltage V_{pi}, identified when the output solution becomes unstable. For each applied voltage, we can define the stiffness coefficient as $K= F_{es}(z(V_0),V_0)/z(V_0)$, but we expect that this value is independent of the applied voltage V_0 only for small displacements. Fig. 19 shows the dependency of the displacement of a point from the middle of the membrane by the applied voltage. Fig. 20 shows the dependency of the stiffness coefficient w.r.t the applied voltage, compared with analytical value computed for a fixed-fixed beam [11].

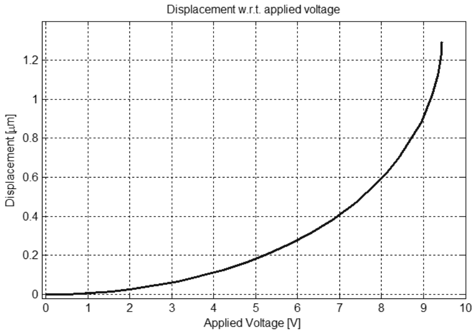


Fig. 19. Displacement of the center point of the membrane vs. the applied voltage

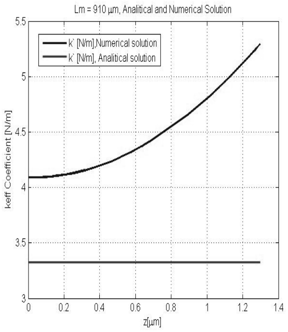


Fig. 20. Dependence of effective stiffness w.r.t the displacement. Analytical value is computed for a fixed-fixed beam

The pull-in voltage V_{pi} = 9.4 volts, the displacement of the membrane for V_{pi} is z_{max} = 1.3 μm. The effective stiffness coefficient is approximated by the medium value of the $k(V_{pi})$ characteristic divided by z_{max}, resulting k_{eff} = 4.4428 N/m.

The effective mass can be estimated as $m_{eff} = k_{eff}/\omega_0^2$, where ω_0 is the fundamental frequency obtained from the modal analysis. The first 5 eigenfrequencies from the modal analysis are (in Hz): 5256.757, 14469.75, 14989.44, 28473.67, 31371.01, resulting $m_{eff} = 4.1 \cdot 10^{-9}$ kg.

Finally, information about the effective damping coefficient can be obtained only after a transient simulation. In Fig.21 the displacement of the membrane w.r.t. time is plotted. Rewriting (2) as (4) we can extract b_{eff} using (5), where a represents the acceleration and v is the velocity.

$$\mathrm{Ma} + \mathrm{Bv} + \mathrm{Kz} = \mathrm{F}_{\mathrm{es}} \tag{4}$$

$$\mathrm{b}_{\mathrm{eff}} = (\mathrm{F}_{\mathrm{es}} - \mathrm{am}_{\mathrm{eff}} - \mathrm{zk}_{\mathrm{eff}}) / \mathrm{v} \tag{5}$$

The velocity is obtained by applying numerical derivation of displacement (Fig. 22). The acceleration it can be obtain from the numerical derivation of the velocity, or by using the second order derivation, from the displacement (Fig. 23). Fig. 24 represents the time variation of the electrostatic force.

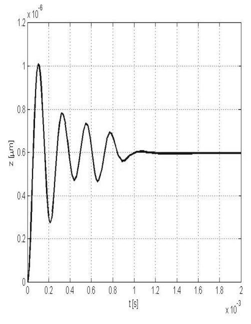


Fig. 21. Displacement

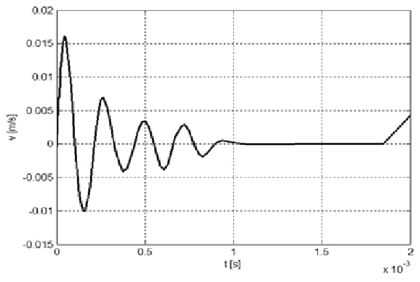


Fig. 22. Velocity

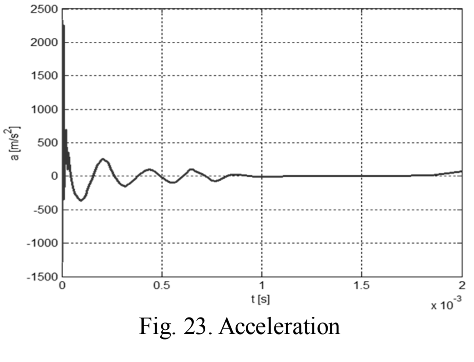


Fig. 23. Acceleration

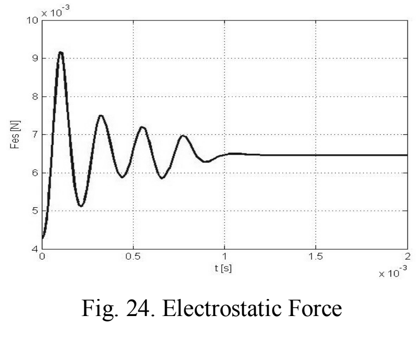


Fig. 24. Electrostatic Force

In Fig. 20 it can be seen that k_{eff} coefficient is linear only for small displacements. A better approximation it will be to consider a nonlinear component for the stiffness coefficient. Before determining the effective mass of the bridge a spectral analyses is required, to determine the contribution of each frequency to the effective mass.

7. Conclusions

In this paper, the EEC, which enables to conduct 1-D analyses of the simplified model of the RF MEMS capacitive switch has been presented. Series of characteristics have been obtained and these results were compared with the commercial software APLAC. This verification proved correctness, validity of the EEC, created in LTSpice. There is very good agreement between LTSpice and APLAC results for DC (static), transient and RF analysis.

The presented EEC can be treated as behavioural model of the RF MEMS capacitive switch, which reflects mechanical phenomenons, electrical phenomenons and interdependencies between them. The EEC was used as the base to create the RF macromodel and the RF (S-parameters) analyse. This analysis in the LTSpice software for the RF MEMS capacitive switch is the paper novelty. The presented reduced order model in the form of the Spice circuit and the results are useful to RF MEMS switches designers, more the circuit can be incorporated in EDA (Electronic Design Automation) software.

The last chapter of this paper presents an example on how effective coefficients k_{eff}, m_{eff} and b_{eff} can be extracted from the numerical modelling and simulations. Using this effective coefficients, for any bridge type RF MEMS switch, a reduced order macro-model can be created.

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