ISSN: 1231-4005
e-ISSN: 2354-0133
DOI: 10.5604/12314005.1216493

THE ROBOT JOINT LUBRICATION WITH ULTRA-THIN HYPERELASTIC SUPERFICIAL LAYERS

Krzysztof Wierzcholski

Technical University of Koszalin, Faculty of Technology and Education
Śniadeckich Street 2, 75-453 Koszalin, Poland
tel.: +48 943478344, fax: +48 943426753
e-mail: krzysztof.wierzcholski@wp.pl

Andrzej Miszczak

Gdynia Maritime University, Faculty of Marine Engineering
Morska Street 81-87, 81-225 Gdynia, Poland
tel.: +48 585586348, fax: +48 585586399
e-mail: miszczak@wm.am.gdynia.pl

Abstract

The surface of humanoid robots is more or less deformable metal and plastic replica of human body. An advanced humanoid robot has human like behaviour – it can talk, run, jump or climb stairs in a very similar way a human does. Hence follows that operation of construction of the robots artificial joints to be similar for biological joints activities. This fact requires applying proper as well corresponding soft solid materials, and specific lubricants. To the interesting phenomena belong the fact, that as well the surfaces of an articular cartilage human joint as the soft surfaces of the robot joints, coated with ultra-thin hyperelastic multi-layers, plays an important role in the surface active lubrication, relative small friction forces and wear during the human limb or robot body activities in the movement. The presence of the ultra-thin hyperelastic layers consisting the soft bearing materials including hyperelastic nano-particles during the robot bearing lubrication enables to indicate numerous positive effects among other the decreases the friction coefficient values. Therefore, the results obtained in this paper may be applicable during the joint-endo-prosthesis or artificial joint design in new humanoid robots, where instead cartilage and synovial fluid are applied new soft materials with active hyperelastic micro- and nano- particles. In this paper is shortly presented the mathematical model of hydrodynamic lubrication of thin boundary layer describing the robot joint. Mathematical model in 3D for lubricant consists of three equations of motion, continuity equation, conservation of energy equation and Young-Kelvin-Laplace equation describing the thin layer interfacial energy.

Keywords: *robot joint lubrication, various curvilinear hyperactive thin and soft layer shapes, basic hydrodynamic equations, physical models, humanoid robots.*

1. About humanoid robot slide bearing gap

After the Authors knowledge, the robots, including humanoid robots, can be found in many applications of various fields of technology, in particular connected with the land, marine and air transport. Humanoid robots are being developed to perform numerous tasks in the human environment. The main applications of humanoid robots are as follows: maintenance tasks of industrial plants, security services of home and office, human care, tele-operations of construction machines, cooperative work in the open air were explored and medical applications to provide assistance in rehabilitation and training.

Abovementioned humanoid robots require the unconventional constructions and non-classical materials of artificial joints, to make possible very complicated moves with accurate and precise actions during the work. Therefore, in the paper an attempt is made to elaborate and to describe the hydrodynamic process of such artificial joints in the form of hydrodynamic slide journal bearing.

The cooperating superficial surfaces of humanoid joint robot are coated with the 2 nm thick synthetically prepared soft and simultaneously hyperelastic layer (SHL) with amorphous shape contacting direct with the lubricant analogously with the membrane of living particles in human natural joints. The SHL forms a continuous barrier around the remaining cells of two cooperating body surfaces [1-8]. Abovementioned barrier keeps necessary oil ions, lubricin hyperelastic nano-particles where they are needed and prevents them from diffusing into areas where they should not be. On the superficial SHL are lying negatively charged ions strongly hydrated in synthetic prepared oil in presence of delivered electric field [7-8].

The typical, orthogonal, curvilinear, coordinates describing various shape of humanoid joints especially the ultra-thin hyperelastic membrane, rotational surfaces with monotone and non-monotone generating lines have usually cylindrical, spherical, conical, parabolic and hyperbolic rotational surfaces.

Irregularities of typical SHL surfaces have values from 1 to 5 micrometres but irregularities of new generation robot joint soft, plastic surfaces are more less and attain barely 1 micrometre. The Young modulus of elasticity of the prepared soft material lying on the robot joint surface attains values from 60 to 90 MPa. The Young modulus of elasticity E for robot joint material has value from 100 to 150 GPa, whereas for steel E=200 GPa [5]. Moreover, we have the robot load carrying capacity force denoted by the letter P and caused by the hydrodynamic pressure obtained from rotation motion of robot joint heads. The squeezing effects are considered too. In general, the senses and lines of forces R and P are the same, especially in the case of the squeezing, boosted squeezing and weeping lubrication. The load force W of the presented robot joint, has in general the reverse sense and the same line as the forces P and R. Hence, as well the carrying capacity force P as the repulsive force R counteracts and oppose the load force W [8].

2. The idea of future robot joints

Robot joint gaps are limited by the upper and lower soft&hyperelastic (SH) membrane and are filled with the proper prepared oil. Fig. 1 shows comparison between robot and human joint gaps. Here is visibly the repulsive force R caused by the negatively charged SH membrane in presence of specially prepared lubricant. Such charged surfaces are observed on the both external SH surfaces contacting with the lubricant fluid. The mechanism of robot surface lubrication with the various geometrical internal shapes coated with SH membrane requires to take in the presented considerations the curvilinear coordinates and non-Newtonian lubricants properties.

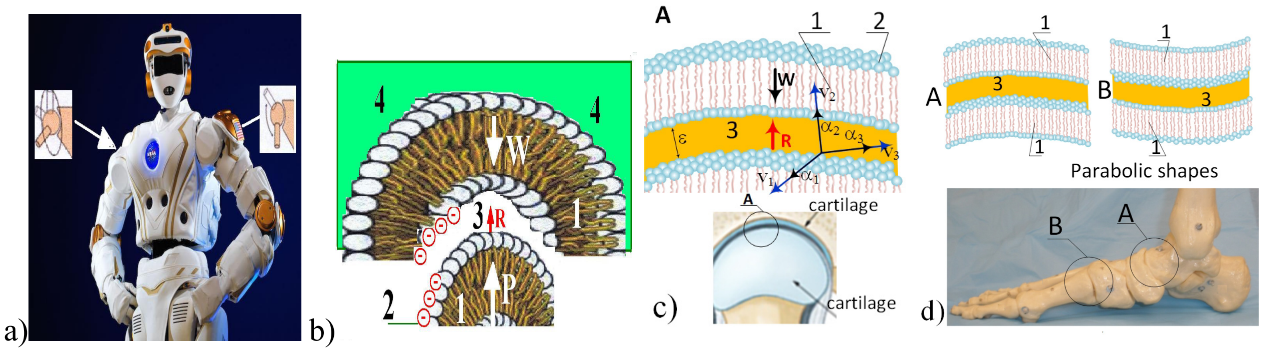

Fig. 1. The idea of future robot and human joints surfaces coated with the synthetic sophisticated phospholipid (SSPL) bilayer: a) Valkyrie-robot-to-go-to-mars [9]; b) robot joint surfaces, gap height 20-70μm (own elaboration); c) comparison of robot and human elliptical joint surfaces, gap height 30-120μm (own elaboration); d) comparison of parabolic food surfaces between robot and joints, gap height 20-60μm (own elaboration); Notations: 1-SSPL-bilayer, 2-hydrated sodium ion, 3-robot synthetic oil or synovial fluids, 4-robot-bearing-joint sleeve or human acetabulum

Transverse sections of curvilinear joint gaps surfaces presented in Fig. 1 have in general among other spherical, cylindrical, parabolic, hyperbolic shapes depended on the kind of normal (non used) and pathological (used) joint and depended of the joint surface irregularities and roughness.

3. The basic remarks about a new lubrication

Hereby are presented general remarks about a semi analytical method of solution of the asymmetrical, stationary, laminar, non-Newtonian lubricant flow lubrication between two rotational squeezing and deformable, curvilinear orthogonal movable humanoid robot SHL surfaces. The inertia forces of the synovial fluid are negligibly small because the speed of liquid particles in humanoid robot gap joint during the body motion attains values usually not larger than 3m/s. To include or exclude the direct influences of electric intensity field generated on the SHL membrane surface on the tribology properties and to indicate the indirect influence of electric intensity on the lubricant viscosity changes we neglect the body forces except the Lorentz forces. After own studies and scientific literature data it can be stated that the occurring usually in many robot not large temperature variations have influence on the tribology parameters during the SHL lubrication if and only if we consider the temperature influences on lubricant impurities consistence and pH ions concentration in lubricant and indirect on its viscosity effects. In case when the environment temperature of humanoid robots is significantly higher and attains value even 120°C, thus are observed influences always direct on the tribology parameters. Hence, the mathematical 3D model of SHL hydrodynamic humanoid robot joint lubrication includes always the conservation of energy equation and simultaneously Young-Kelvin-Laplace equation describing the interfacial energy in robot joint superficial layer. Here are neglected the convection energy terms, pressure dissipation energy terms. In addition, Joule heat terms are considered. The 3D fluid flow between two above mentioned solid robot surfaces in the electromagnetic field will be described in a vector form by the three equations of equilibrium of momentum, with a fluid continuity equation (1), and in scalar form by conservation of energy equation (2), and Young-Kelvin-Laplace equation (3). Mentioned equations are as follows [7]:

$$\mathrm{Div}\,\mathbf{S} + \rho_e \mathbf{E} = 0, \quad \mathrm{div}(\rho \mathbf{v}) = 0, \tag{1}$$

$$\mathrm{div}(\kappa\ \mathrm{grad}\ T) + \phi_F = J^2 / \sigma, \tag{2}$$

$$\gamma = \gamma_{max} + 2sR_g T \ln\left(\sqrt{\frac{K_a}{K_b}} + 1\right) - sR_g T \ln\left[\left(\frac{K_a}{a_H^+} + 1\right)\left(\frac{a_H^+}{K_b} + 1\right)\right], \tag{3}$$

where:

$\mathbf{S}$ – stress tensor in the lubricant [Pa],
$\mathbf{E}$ – electric intensity vector [V/m],
$\mathbf{J}$ – electric current density [A/m^2],
T – lubricant temperature [K],
ρ_ε – electric space charge in lubricant [C/m^3=As/m^3],
κ – thermal conductivity coefficient for lubricant [W/mK],
ϕ_F – dissipation of energy [W/m^3],
R_g – gas constant (8.3144598 J/Kmol),
A – the SHL surface [m^2],
ρ – lubricant density [kg/m^3],
σ – electrical conductivity of SHL [S/m],
γ – interfacial energy [J/m^2=N/m],
a_H – protons energy activity created in dissociated oil [J],
K_a – acid equilibrium constant (denotes how much energy is needed to stretch the SHL) [J],

K_b – base equilibrium constant (denotes how much energy is needed to bend or flex the SHL) [J],
γ_{max} – is the maximum interfacial energy connected with SHL,
$s=(N_A \cdot A)^{-1}$ – concentration of needed hyperelastic particles [mol/m^2],
$N_A=6.024\cdot 10^{23}$ – Avogadro number.

Due to the SHL-presence on the robot joints, considered lubricant has non-Newtonian especially pseudo-plastic properties. It will be shown that constitutive equations of power law type for abovementioned lubricant good describes the non-linear dependences between stresses and shear rates, whereas apparent viscosity and consistency coefficient depends mainly on the shear rate of lubricant flow, electrostatic field occurring on the SHL surface, porous material wettability of robot bearing materials, power hydrogen ion concentration from dissociated oil, and less from the temperature. For the lubricant in general the relationship between stress tensor **S** and displacement velocity tensor $2\mathbf{T_d}=\mathbf{A_1}$ i.e. constitutive equations are assumed in following form [4]:

$$\mathbf{S}= -p\boldsymbol{\delta} + \eta_p\mathbf{A_1}, \tag{4}$$

whereas unit tensor $\boldsymbol{\delta}$, strain tensor $\mathbf{A}_1$ have following components: δ_{ij}, Θ_{ij}. We denote: δ_{ij} – Kronecker Delta, p – pressure[Pa]. For non-Newtonian lubricant with power law type, the constitutive dependencies between the apparent viscosity η_p [Pas] have the following form [4]:

$$\eta_{pr} = 2^{n-1} m(n)\left|\frac{1}{2}\mathbf{I}_1^2(\Theta) - \mathbf{I}_2(\Theta)\right|^{\frac{n-1}{2}}, \quad \mathbf{I}_1(\Theta) = \Theta_{kk}, \quad \mathbf{I}_2(\Theta) = \frac{1}{2} e_{ijk} e_{imn} \Theta_{jm} \Theta_{kn}, \tag{5}$$

where: $\mathbf{I}_1$ [s^{-1}], $\mathbf{I}_2$ [s^{-2}] are the invariants of shear rate Θ_{ij} [s^{-1}], n – dimensionless flow index depended on needed oil additions in the form of hyperelastic nano-particles, m=m(n,p,H,T,We) – fluid consistency coefficient in Pas^n, e_{ijk} – tensor Levi-Civity, We-SHL wettability, pH – power hydrogen ion concentration. Relations between shear rate Θ_{ij} and synovial fluid velocity components v_i [m/s] are as follows [4, 5]:

$$\Theta_{ij} = \frac{1}{2}\left(v_{i|j} + v_{j|i}\right), \quad v_{i|j} \equiv \frac{1}{h_i}\left(\frac{\partial v_i}{\partial \alpha_j} - \frac{v_j}{h_i}\frac{\partial h_j}{\partial \alpha_i} + \delta_{ij}\sum_{k=1}^{3}\frac{v_k}{h_k}\frac{\partial h_j}{\partial \alpha_k}\right), \tag{6}$$

where h_i – Lame coefficients.

The SHL deformations and displacements in robots imply the not negligibly small gap changes. System of equations (1-6) is completed by electro-thermo-elasticity equations describing problem in stresses for two robot joint surfaces restricting the thin lubricant layer, to obtain robot surface displacement components in considered model. Additional system consists of the three partial differential equations in a one equation vector form (7). To this set of equations we add the heat conductivity equation in a robot joint body without heat sources and we obtain the following system [5]:

$$\mathrm{Div}\mathbf{S}^*(T) + \rho_e^*\mathbf{E}^* = 0, \ \mathrm{div}\left(\kappa^* \mathrm{grad}T^*\right) = 0. \tag{7}$$

The symbols with an asterisk are related to the body of robot joint. The hypo-elastic joint body solid material has the following properties: 1. The material specimen deforms reversibly i.e. removing the load gives returning to the initial shape. 2. The strain depends only on stress applied to it-it does not depend on the rate of loading. The stress is non-linear function on strain. 3. Isotropic features i.e. response of a material is independent of its orientation with respect to the loading direction. Hypo-elastic models of material feature are distinct from hyperelastic material models (or standard elasticity models) in that, except under special circumstances, they cannot be derived from a strain energy density function per unit solid material volume U [Pa]. In joint applications, non-linear behaviour material is often subjected to shear deformations characterized by $\mathbf{I}_2$ while stress varies linearly with volume changes characterized by $\mathbf{I}_1$. We have [4]:

$$U=\frac{1}{6}K\mathbf{I}_1^2(\varepsilon)+\frac{2n\tau_0^*\varepsilon_0}{n+1}\left(\frac{\mathbf{I}_2(\varepsilon)}{\varepsilon_0^2}\right)^{(n+1)/2n}. \tag{8}$$

We denote: K – bulk modulus [Pa], n – exponent in interval (from 0.6 to 1.2) as in the relationship (5), τ_0^* – characteristic value of stress [Pa], ε_0 – characteristic value of dimensionless strain. For large joint material deformations, the relations between stresses τ_{ij}^* [Pa] and strains can be formulated using function (8). The constitutive equation for a homogeneous, hyperelastic joint material occurring in humanoid robot bearing has the following form [4-5]:

$$\tau_{ij}^*=\frac{\partial U}{\partial\varepsilon_{ij}}=\frac{K}{3}\varepsilon_{kk}\delta_{ij}+\tau_0^*\left(\frac{\varepsilon_{ij}-\varepsilon_{kk}\delta_{ij}/3}{\varepsilon_0}\right)\left(\frac{\mathbf{I}_2(\varepsilon)}{\varepsilon_0^2}\right)^{(1-n)/2n}, \tag{9.1}$$

for i,j=1,2,3, where: δ_{ij} – unit Kronecker tensor component (δ_{ij} =1 for i=j) and (δ_{ij} =0 for i≠j).

For typical elastic, isotropic and non-isothermal bearing alloy properties valid the Duhamel Neumann constitutive relations between the stress tensor $\mathbf{S}^*$ with the components τ_{ij}^* and the strain tensor $\boldsymbol{\varepsilon}$ with dimensionless strain components ε_{ij} described in the following form:

$$\tau_{ij}^*=2G\varepsilon_{ij}+\left(\Lambda\varepsilon_{kk}-3\alpha_T KT^*\right)\delta_{ij}. \tag{9.2}$$

For simple elastic robot joint alloy body material we have: bulk modulus K=λ+2G/3 [Pa], shear modulus of cartilage G=E/[2(1+ϑ)] [Pa], dimensionless Poisson ratio ϑ, Λ=Eϑ/(1+ϑ)(1−2ϑ) [Pa], Young's Modulus E [Pa], cartilage thermal coefficient of linear expansion α_t [K^{-1}].

For soft material such as SHL with large deformations, the geometrical relation between dimensionless strain tensor components ε_{ij} and displacement vector components u_i[m] are as follows:

$$\varepsilon_{ij}=\frac{1}{2}\left(u_{i|j}+u_{j|i}\right)\quad,\quad u_{i|j}\equiv\frac{1}{h_i}\left(\frac{\partial u_i}{\partial\alpha_j}-\frac{u_j}{h_i}\frac{\partial h_j}{\partial\alpha_i}+\delta_{ij}\sum_{k=1}^{3}\frac{u_k}{h_k}\frac{\partial h_j}{\partial\alpha_k}\right). \tag{10}$$

For the system of equations (7-10) presenting humanoid robot joint cooperating surfaces coated with the SHL as the thin boundary joint superficial layer lying on the internal joint gap surfaces with already existing surface electric intensity charge, we to add Maxwell and Ohm equations in stationary electromagnetic field, reduced to the following form:

$$\nabla\times\mathbf{E}^*=0,\quad \nabla\times\mathbf{E}=0, \nabla\cdot\mathbf{J}=0,\ \mathbf{J}^*=\sigma^*\mathbf{E}^*, \mathbf{J}=\sigma\mathbf{E}, \tag{11}$$

where: σ – electrical conductivity coefficient [S/m], **J** – electric current density [A/m^2].

On the ground of the already indicated remarks due to the robot joint surface, it can be stated that the mechanism of lubrication for the various geometrical joint surface shapes coated with SHL membrane requires taking into account in presented system (7)-(11), the Lame coefficients proper to the curvilinear coordinates. Therefore, the Lame coefficients are derived in local curvilinear and orthogonal coordinate system (α_1,α_2,α_3) connected with the movable surfaces, where α_2 denotes the direction of gap height.

4. Apparent viscosity of the lubricant

Expanding in α_i (i=1,2,3) directions the basic equations (1)-(7) in orthogonal coordinate system (α_1,α_2,α_3), and taking into account layer boundary simplifications i.e. neglecting the negligibly small terms of order 0.001 presenting the quotient of characteristic joint gap height ε_T to the radius of curvature of robot joint surface with SHL membrane, denoting joint radial clearance ψ, we obtain the system of non-linear basic partial differential hydrodynamic equations describing the solutions of hydrodynamic humanoid robot joint lubrication with SHL in local curvilinear coordinates. We take into account for various shapes of joints, deformable joint gap height, where non-Newtonian, incompressible synovial fluid apparent viscosity $\eta_p(\alpha_1,\alpha_2,\alpha_3)$ changes in length α_1, width α_3 and gap- height directions α_2. From Y-KL equation (3) follows that the apparent viscosity of the non-Newtonian lubricant with presence of SHL-layer has the following form:

$$\eta_p(n,p_H,We,T,E)\equiv\frac{\gamma}{\delta_v\cdot v_0}\left(\sqrt{\left(\frac{\partial v_{11}}{\partial\alpha_{21}}\right)^2+\left(\frac{\partial v_{31}}{\partial\alpha_{21}}\right)^2}\right)^{n-1},\quad \gamma\equiv\gamma_{max}+kA^{-1}T\ln L, \tag{12}$$

where:

$$L_a\equiv\frac{K_a}{a_H^+},\ L_b\equiv\frac{a_H^+}{K_b},\ L_k\equiv L_aL_b,\quad (L_a+1)(L_b+1)>\left(\sqrt{L_k}+1\right)^2,\ 0<L\equiv\frac{\left(\sqrt{L_k}+1\right)^2}{(L_a+1)(L_b+1)}<1. \tag{13}$$

By virtue of equations (1),(2) we denote: Boltzaman constant $k=1.38054\cdot10^{-23}$ [J/K], A surface of lubricated area coated by SHL [m^2], T – average temperature of robot body [K], γ – interfacial energy [mJ/m^2], We wettability of SHL and porous robot joint material in interval from 30° for hydrophiling to 100° for hydrophobic properties, v_0 – characteristic value of linear velocity of lubricant in joint gap from 0.25 to 4.0 [m/s], δ_v – dimensionless coefficient introduced by author in range ($2<\delta_v<6$) determining the variations of concentration of nano-meter size hyperelastic nano-particles from $\delta_v=2$ for c_c=1 000 000 mol/mm^3 to c_c=100 mol/mm^3 and less for $\delta_v=6$ occurring in robot-synthetic lubricant. We denote: v_{11}, v_{31} – dimensionless velocity component in α_1, α_3 – curvilinear coordinate direction, $\alpha_{21}=\alpha_2/\varepsilon_0$, ε_0 – characteristic dimensional value of the robot joint gap height.

5. Discussion and conclusions

Presented in this paper, a new hydrodynamic model of robot joint lubrication enables to obtain numerous remarks referring as well the lubricant as joint solid material properties. Some of them, without adhesion forces, are illustrated in Tab. 1 for 0<Re<1.

1. Power Hydrogen ion concentration from about 4 to 10 in oil for constant temperature 310 K and constant wettability about 40° of porous soft joint material in lubricated robots, decreases the lubricant apparent viscosity from about 0.500 to 0.003Pas.

Mentioned corollary follows from the initial calculations illustrated in Fig. 2 and performed by virtue of the Authors formulae derived in this paper and obtained on the ground of presented a new model of humanoid robot joint lubrication.

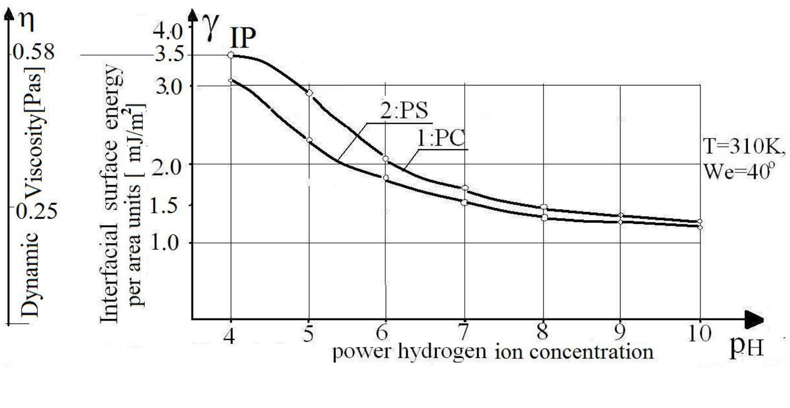

Fig. 2. Dynamic viscosity of the lubricant η and interfacial surface energy values γ vs. power hydrogen ion concentration pH in lubricant during the humanoid joint lubrication for constant wettability and temperature, IP – isoelectric point, PC – phpsphatidylcholine additions, PS – phosphatidylserine additions 10%

Tab. 1. Comparisons of main available operator parameters occurring in human joint and typical robot joints

Number	Parameter or phenomenon	Standard synovial fluid in natural sound human joint	Typical cartilage human sound joint	Typical oil for robot joint	Hypo-elastic soft solid body of robot joint	Hyper elastic solid body of robot joint
1	Apparent dynamic viscosity	0.03-0.50 Pas	no concern	0.05-0.40 Pas	no concern	no concern
2	Temperature	35-43°C	35-44°C	30-75°C	30-85°C	30-85°C
3	Viscosity changes with temperature increases as in versus no.2	decreases	no concern	decreases	no concern	no concern
4	Wettability of solid body (in range from 80° hydrophobic – to 30° hydrophilic)	no concern	70°-50°	no concern	80°-50°	85°-50°
5	Viscosity changes if grade of wettability decreases as in versus no.4	decreases for 4<pH<10	no concern	decreases for 4<pH<10	no concern	no concern
6	Hydrogen ion concentration pH	3.7<pH<12	no concern	3.7<pH<10 2<pH<3.7	no concern	no concern
7	Viscosity changes Viscosity with pH index increases as in versus no.6	decreases	no concern	decreases increases	no concern	no concern
8	Shear rate values Θ during the lubrication process; and how changes viscosity η if Θ increases?	5 $s^{-1}<\Theta<10^4$ s^{-1} viscosity [Pas] decreases but for pseudoplastic fluid only	no concern	5 $s^{-1}<\Theta<10^5$ s^{-1} viscosity[Pas] decreases for pseudoplastic fluid only	no concern	no concern
9	Young modulus of elasticity E or many hyper elasticity parameters	no concern	from 12 to 50 MPa	no concern	from 10 to 90 MPa	from 12 to 100 MPa
10	s-s: Stress-shear rate relations(fluid) s-s: Stress-strain relations (solid) s-d: Strain-displacement (solid)	s-s: non-linear	s-s: linear or not, s-d: linear	s-s: non-linear	s-s: linear or not, s-d: non-linear	s-s: non-linear, s-d: non-linear

2. The good known law for pseudoplastic liquids about viscosity decrements with shear rate increments valid for humanoid robot joints, bearing, endoprosthesis lubrication

From formula (13) follows that L function attains values in interval (0,1), hence term (T/A)lnL has always negative values. Thus, temperature increments are decreasing the numerator of the fraction defining the apparent viscosity (12) hence viscosity decreases. Analogously surface A increments denote increments of SH (soft hyperelastic) nano-particles concentration hence negative value (T/A)lnL decrease. Thus increases numerator in formula (12) of the fraction describing viscosity i.e. viscosity increases. Because velocity increments of oil flow denote shear rate increments, hence the denominator in formula (12) increases and fraction defining the viscosity decreases i.e. viscosity decreases. This fact completes the proof of corollary in point 2.

3. During the humanoid robot joint squeezing lubrication, the dynamic viscosity of specially prepared lubricant for robots, varies significantly crosswise the joint gap limited by the SHL and dynamic viscosity of the squeezing lubricant, increases intensive in the proximity of the SHL surface and attain in these places maximal values.

Proof: The humanoid robot joint gap height attains average values from 10 to 120 micrometre. On the ground of hydro mechanical laws, we deduce that oil velocity distribution during the squeezing lubrication flow in joint bearing gap has usually parabolic profile (shape) with minimal values in neighbourhood of superficial layer. From formula (12) follows, that in these places dynamic viscosity of oil attains larger and larger values because the denominator (fluid velocity) of the fraction presenting the dynamic viscosity has smaller and smaller values. Moreover, in neighbourhood of superficial layer we observe increases of SH nano particles concentration described by the small values of dimensionless coefficient δ_v. From formula (20) follows, that the small coefficient δ_v as the denominator of the fraction presenting the apparent viscosity implies large values of dynamic viscosity of the lubricant in indicated places. From mechanical laws follows that the places of maximal values of fluid velocity profiles coincide with the points where the synovial fluid dynamic viscosity attains the minimal values.

This fact completes the proof of corollary in point 3.

4. Increasing of the humanoid robot joint surfaces area coated by SH nano particles and assuming the constant temperature during the robot joint lubrication leads to the lubricant apparent dynamic viscosity increases.

Proof: From formula (13) follows that L function attains values in interval (0,1), hence term (T/A)lnL has always negative values. Thus, temperature increments are decreasing the numerator of the fraction defining the apparent viscosity (12) hence viscosity decreases. Analogously surface A increments denote increments of SH nano-particles concentration hence negative value (T/A)lnL decrease. Thus increases numerator in formula (12) of the fraction describing viscosity i.e. viscosity increases. This fact completes the proof in point 4.

References

[1] Andersen, O. S., Roger, E., et al., *Bilayer thickness and Membrane Protein Function: An Energetic Perspective,* Annular Review of Biophysics and Bimolecular Structure, 36 (1), pp. 107-130, 2014.

[2] Arkin, R. C., *Behaviour-Based Robotics*, MTP Press, Cambridge 1998.

[3] Asada, H., Slotine, J.-J. E., *Robot Analysis and Control*, Willey, Cambridge 1986.

[4] Carpenter, J., Davis, J., Erwin-Stewart, N., Lee, T., Bransford, J., Vye, N., *Gender representation in humanoid robots for domestic use,* International Journal of Social Robotics (special issue), 1(3), pp. 261-265, 2009.

[5] Chagnon, G., Rebouah, M., Favier, D., *Hyperelastic Energy Densities for Soft Biological Tissues*, Review, Journal of ElasticityM, *Evaluation of hyperelastic models for the non-linear and non-uniform high strain-rate mechanics*, Vol. 120, Is. 2, pp. 129-160, 2015.

[6] Deneveth, J. M., McLean, S. G., Arruda, E., *Cs of tibial cartilage*, J. of Biomech., Jun 21, 46(10), 1604-10, 2013.

[7] Mashaghi, A., et al., *Hydration strongly affects the molecular and electronic structure of membrane phospholipids*, J. Chem. Phys., 136, 114709, 2012.

[8] Pawlak, Z., Urbaniak, W., Hagner-Derengowska, M. W., *The probable explanation for the low friction of natural joints,* Cell Biochemistry and Biophysics, Vol. 71, 3, pp. 1615-1621, 2015.

[8] Pawlak, Z., Urbaniak, W., Gadomski, A., Yusuf, K. Q., Afara, I. O., Oloyede, A., *The role of lamellate phospholipid bilayers in lubrication of joints*, Acta of Bioengineering and Biomechanics, Vol. 14, No. 4, pp. 101-106, 2012.

[9] www.spectrum.ieee.org/automaton/robotics/humanoids/new-r5-valkyrie-robots, 06.2016.