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Int. J. of Applied Mechanics and Engineering, 2013, vol.18, No.2, pp.589-597
DOI: 10.2478/ijame-2013-0036

Brief note

FREE VIBRATION ANALYSIS OF ISOTROPIC RECTANGULAR PLATES ON WINKLER FOUNDATION USING DIFFERENTIAL TRANSFORM METHOD

Y. KUMAR
Department of Mathematics
M.K. Government Degree College Ninowa
Farrukhabad-209 602, Uttar Pradesh, INDIA
E-mail: yaju_saini@yahoo.com

A differential transform method (DTM) is used to analyze free transverse vibrations of isotropic rectangular plates resting on a Winkler foundation. Two opposite edges of the plates are assumed to be simply supported. This semi-numerical-analytical technique converts the governing differential equation and boundary conditions into algebraic equations. Characteristic equations are obtained for three combinations of clamped, simply supported and free edge conditions on the other two edges, keeping one of them to be simply supported. Numerical results show the robustness and fast convergence of the method. Correctness of the results is shown by comparing with those obtained using other methods.

Key words: differential transform method, isotropic, rectangular, Winkler foundation.

1. Introduction

Rectangular plates are used as structural elements in various technological situations in different engineering fields, e.g., aeronautics, civil, marine, mechanical and optical engineering. Overall performance of a structure is highly influenced by the dynamic behavior of the plates. Vibrations of these plates have been analyzed using different techniques, namely, the finite element method (Venkateswara, 1974), the Superposition-Galerkin method (Gorman, 2000), the meshless method (Chen *et al.*, 2004), the differential quadrature method (Wang *et al.*, 2006), the singular convolution method (Omer *et al.*, 2010) and the Rayleigh-Ritz method (Kumar and Lal, 2011), etc. The differential transform method was introduced by Zhou (1986). This method was used by Arikoglu and Ozcol (2005) to find analytical solutions of differential or integro-differential equations. It has been successfully applied in the analysis of various vibration problems (Malik and Allali, 2000; Yalcin *et al.*, 2000; Attarnejad *et al.*, 2010 and Kacar *et al.*, 2011). Using this method the differential equation governing the free transverse vibrations of the plate and boundary conditions are transformed into algebraic equations which provide the characteristic equations. These characteristic equations are solved numerically to obtain first three natural frequencies. The results indicate that differential transform method is a reliable and fast converging method for the vibration analysis of rectangular plates.

2. Formulation of the problem

Consider an isotropic rectangular plate of uniform thickness h with the domain $0 \le x \le a, 0 \le y \le b$ in the $xy-$ plane, where a and b are the length and the breadth of the plate, respectively. The $x-$and $y-$ axes are taken along the edges of the plate and the axis of $z-$ is perpendicular to the $xy-$ plane. The middle surface is $z = 0$ and the origin is at one of the corners of the plate. The plate is resting on a Winkler foundation having the foundation modulus k_f.

The governing differential equation of motion of an isotropic rectangular plate resting on the Winkler foundation is given as follows

$$\frac{\partial^4 W}{\partial X^4} + 2\lambda^2 \frac{\partial^4 W}{\partial X^2 \partial Y^2} + \lambda^4 \frac{\partial^4 W}{\partial Y^4} + KW = \Omega^2 W, \qquad 0 \le X \le 1, \qquad 0 \le Y \le 1 \tag{2.1}$$

where $X = x/a, Y = y/b$, $\lambda = a/b$, W is the transverse displacement at (X,Y), $K = \left(k_f\, a^4 / D\right)$ is the foundation modulus parameter and $\Omega^2 = \dfrac{12\left(1-\nu^2\right)\rho a^4 \omega^2}{Eh^3}$ is the frequency parameter.

Assuming the two opposite edges $Y = 0$ and $Y = 1$ to be simply supported, the deflection function can be expressed as follows

$$W = \bar{W}(X)\sin(m\pi Y) \tag{2.2}$$

where m is an integer.

Substitution of Eq.(2.2) into Eq.(2.1) leads to

$$\frac{d^4 \bar{W}}{dX^4} - 2m^2\pi^2\lambda^2 \frac{d^4 \bar{W}}{dX^2} - \left(\Omega^2 - m^4\pi^4\lambda^4 - K\right)\bar{W} = 0. \tag{2.3}$$

3. Boundary conditions

Three boundary conditions, namely, SC, SS and SF have been considered where the first letter represents the boundary condition at the edge $X = 0$ and second one at the edge $X = 1$. Here *C* stands for a clamped edge, *S* for a simply supported and *F* for a free edge. The edges $Y = 0$ and $Y = 1$ are assumed to be simply supported. The conditions that should be satisfied at the edges $X = 0$ and $X = 1$ are

$$\bar{W} = \frac{d\bar{W}}{dX} = 0, \qquad \text{for clamped edge} \tag{3.1}$$

$$\bar{W} = \frac{d^2\bar{W}}{dX^2} - \upsilon\left(m^2\pi^2\lambda^2\right)\bar{W} = 0, \qquad \text{for simply supported edge} \tag{3.2}$$

$$\frac{d^2\bar{W}}{dX^2} - \upsilon\left(m^2\pi^2\lambda^2\right)\bar{W} = 0, \qquad \frac{d^3\bar{W}}{dX^3} - (2-\upsilon)\left(m^2\pi^2\lambda^2\right)\frac{d\bar{W}}{dX} = 0. \quad \text{for free edge} \tag{3.3}$$

4. The solution

The differential transform method (DTM) is based on the Taylor series expansion. The Taylor series expansion of a function $f(x)$ may be written as

$$f(x) = \sum_{r=0}^{\infty} \left(x - x_0\right)^r F_r$$

where $F_r = \dfrac{1}{r!}\left(\dfrac{d^r f}{dx^r}\right)_{x=x_0}$ is called the r^{th} order differential transform of $f(x)$ about a point $x = x_0$.

Differential transforms of some of the fundamental functions are given in Tab.1.

Table 1

S. No.	Function	Differential Transform
1	$f(x)+g(x)$	$F_r + G_r$
2	$af(x)$	aF_r
3	$f(x)g(x)$	$\sum_{r=0}^{k} F_r G_{k-r}$
4	$\dfrac{d^k f}{dx^k}$	$\dfrac{(r+k)!}{r!}F_{r+k}$
5	ax^k	$a\delta_{r-k}$, $\delta_{r-k} = \begin{cases} 1, r = k \\ 0, r \neq k. \end{cases}$
6	$\sin(ax+b)$	$\dfrac{a^r}{r!}F_r \sin(\pi r/2+b)$
7	$\cos(ax+b)$	$\dfrac{a^r}{r!}F_r \cos(\pi r/2+b)$
8	e^{ax}	$\dfrac{a^r}{r!}$
9	$\int_{x_0}^{x} f(t)dt$	$\dfrac{F(r-1)}{r}, r \geq 1, F(0) = 0$

Taking the differential transform of Eq.(2.3) at $X_0 = 0$ using Tab.1, we get

$$\frac{r+4!}{r!}\overline{W}_{r+4} - 2m^2\pi^2\lambda^2\frac{r+2!}{r!}\overline{W}_{r+2} - \left(\Omega^2 - m^4\pi^4\lambda^4 - K\right)\overline{W}_r = 0. \tag{4.1}$$

The boundary conditions (3.1)-(3.3) may be transformed as follows

$$\begin{gathered}\sum_{r=0}^{\infty}(X-X_0)^r\overline{W}_r = 0, \\ \sum_{r=0}^{\infty} r(X-X_0)^{r-1}\overline{W}_r = 0\end{gathered} \tag{4.2}$$

$$\begin{gathered}\sum_{r=0}^{\infty}\left(X-X_0\right)^r\overline{W}_r = 0, \\ \sum_{r=0}^{\infty} r(r-1)\left(X-X_0\right)^{r-2}\overline{W}_r = 0\end{gathered} \tag{4.3}$$

and

$$\sum_{r=0}^{\infty} r(r-1)\left(X-X_0\right)^{r-2}\bar{W}_r-\upsilon\left(m^2\pi^2\lambda^2\right)\sum_{r=0}^{\infty}\left(X-X_0\right)^r\bar{W}_r=0,$$

$$\sum_{r=0}^{\infty} r(r-1)(r-2)\left(X-X_0\right)^{r-3}\bar{W}_r-(2-\upsilon)\left(m^2\pi^2\lambda^2\right)\sum_{r=0}^{\infty} r\left(X-X_0\right)^{r-1}\bar{W}_r=0. \tag{4.4}$$

The general solution (Malik and Allali, 2000) of Eq.(2.3) can be written as

$$\bar{W}_r=\frac{\left(d_1R_1^{\ r}+d_2R_2^{\ r}\right)}{r!},\qquad r=0,1,2,3,.........,\infty, \tag{4.5}$$

where $R_1^{\ 2}=m^2\pi^2\lambda^2+\sqrt{\Omega^2-K}$, $\quad R_2^{\ 2}=m^2\pi^2\lambda^2-\sqrt{\Omega^2-K}$.

Rewriting Eq.(4.5) as

$$\bar{W}_{2r}=\frac{\left(d_1R_1^{\ 2r}+d_2R_2^{\ 2r}\right)}{2r!}, \tag{4.6}$$

$$\bar{W}_{2r+1}=\frac{\left(d_3R_1^{\ 2r+1}+d_4R_2^{\ 2r+1}\right)}{(2r+1)!}. \tag{4.7}$$

Case 1: Simply supported at $X=0$ and clamped at $X=1$

Let $X_0=0$. At $X=0$, Eq.(4.3) becomes

$$\bar{W}_0+0\bar{W}_1+0\bar{W}_2+0\bar{W}_3+0\bar{W}_4+0\bar{W}_5+..................=0,$$

$$0\bar{W}_0+0\bar{W}_1+\bar{W}_2+0\bar{W}_3+0\bar{W}_4+0\bar{W}_5+..................=0, \tag{4.8}$$

i.e., $\bar{W}_0=\bar{W}_2=0.$ Using these values in Eq.(4.5), we get $d_1=d_2=0.$ It implies $\bar{W}_{2r}=0.$

For $r=0$ and $1,$ Eq.(4.5) gives

$$\bar{W}_1=d_3R_1+d_4R_2,\qquad 6\bar{W}_3=d_3R_1^{\ 3}+d_4R_2^{\ 3},$$

and $\quad d_3R_1=\left(6\bar{W}_3-R_2^{\ 2}\bar{W}_1\right)\dfrac{\lambda^2}{2\Omega},\qquad d_4R_2=\left(R_1^{\ 2}\bar{W}_1-6\bar{W}_3\right)\dfrac{\lambda^2}{2\Omega}.$

Now, Eq.(4.7) becomes

$$\bar{W}_{2r+1}=\frac{\lambda^2}{2\Omega}\left[\frac{\left(\left(6\bar{W}_3-R_2^2\bar{W}_1\right)R_1^{2r}+\left(R_1^2\bar{W}_1-6\bar{W}_3\right)R_2^{2r}\right)}{\left(2r+1\right)!}\right]=$$

$$=\frac{\lambda^2}{2\Omega}\left[\frac{\left(R_1^2R_1^{2r}-R_2^2R_1^{2r}\right)\bar{W}_1+6\left(R_1^{2r}-R_2^{2r}\right)\bar{W}_3}{\left(2r+1\right)!}\right]. \tag{4.9}$$

At $X=1$, Eqs (4.2) become

$$\bar{W}_1+\bar{W}_3+\bar{W}_5+..................=0,$$

$$\bar{W}_1+3\bar{W}_3+5\bar{W}_5+..................=0,$$

$$\sum_{r=0}^{\infty}\left[\frac{\left(R_1^2R_2^{2r}-R_2^2R_1^{2r}\right)}{(2r+1)!}\right]\bar{W}_1+6\sum_{r=0}^{\infty}\left[\frac{\left(R_1^{2r}-R_2^{2r}\right)}{(2r+1)!}\right]\bar{W}_3=0, \tag{4.10}$$

$$\sum_{r=0}^{\infty}\left[\frac{\left(R_1^2R_2^{2r}-R_2^2R_1^{2r}\right)}{(2r)!}\right]\bar{W}_1+6\sum_{r=0}^{\infty}\left[\frac{\left(R_1^{2r}-R_2^{2r}\right)}{(2r)!}\right]\bar{W}_3=0. \tag{4.11}$$

The characteristic equation is obtained from the non-trivial condition

$$\begin{vmatrix} \displaystyle\sum_{r=0}^{\infty}\frac{\left(R_1^2R_2^{2r}-R_2^2R_1^{2r}\right)}{(2r+1)!} & \displaystyle 6\sum_{r=0}^{\infty}\frac{\left(R_1^{2r}-R_2^{2r}\right)}{(2r+1)!} \\ \displaystyle\sum_{r=0}^{\infty}\frac{\left(R_1^2R_2^{2r}-R_2^2R_1^{2r}\right)}{(2r)!} & \displaystyle 6\sum_{r=0}^{\infty}\frac{\left(R_1^{2r}-R_2^{2r}\right)}{(2r)!} \end{vmatrix}=0. \tag{4.12}$$

Case 2: Simply supported at $X=0$ and simply supported at $X=1$

After taking the differential transform, Eqs (4.3) at $X=1$ can be written as follows

$$\bar{W}_1+\bar{W}_3+\bar{W}_5+..................=0,$$

$$0\bar{W}_1+6\bar{W}_3+20\bar{W}_5+..................=0.$$

The characteristic equation becomes

$$\begin{vmatrix} \displaystyle\sum_{r=0}^{\infty}\frac{\left(R_1^2R_2^{2r}-R_2^2R_1^{2r}\right)}{(2r+1)!} & \displaystyle 6\sum_{r=0}^{\infty}\frac{\left(R_1^{2r}-R_2^{2r}\right)}{(2r+1)!} \\ \displaystyle\sum_{r=1}^{\infty}\frac{\left(R_1^2R_2^{2r}-R_2^2R_1^{2r}\right)}{(2r-1)!} & \displaystyle 6\sum_{r=1}^{\infty}\frac{\left(R_1^{2r}-R_2^{2r}\right)}{(2r-1)!} \end{vmatrix}=0. \tag{4.13}$$

Case 3: Simply supported at $X=0$ and free at $X=1$

The differential transform of Eqs (4.4) at $X=1$ leads to

$$(-q_1)\bar{W}_1+(6-q_1)\bar{W}_3+(20-q_1)\bar{W}_5+..................=0,$$

$$(-q_2)\bar{W}_1+3(2-q_2)\bar{W}_3+5(12-q_2)\bar{W}_5+..................=0,$$

where $\quad q_1=\upsilon\left(m^2\pi^2\lambda^2\right), \qquad q_2=(2-\upsilon)\left(m^2\pi^2\lambda^2\right)$

and the characteristic equation is given as follows

$$\begin{vmatrix} \sum_{r=0}^{\infty} \dfrac{\left((2*r+1)(2*r)-q_1\right)\left(R_1^{\,2}R_2^{\,2r}-R_2^{\,2}R_1^{\,2r}\right)}{(2r+1)!} & \sum_{r=0}^{\infty} \dfrac{\left((2*r+1)(2*r)-q_1\right)\left(R_1^{\,2r}-R_2^{\,2r}\right)}{(2r+1)!} \\ \sum_{r=0}^{\infty} \dfrac{\left((2*r)(2*r-1)-q_2\right)\left(R_1^{\,2}R_2^{\,2r}-R_2^{\,2}R_1^{\,2r}\right)}{(2r)!} & \sum_{r=0}^{\infty} \dfrac{\left((2*r)(2*r-1)-q_2\right)\left(R_1^{\,2r}-R_2^{\,2r}\right)}{(2r)!} \end{vmatrix} = 0. \quad (4.14)$$

The displacement function for SC plate

$$\bar{W}(X) = \sum_{r=0}^{N} (X - X_0)^r \bar{W}_r,$$

$$\bar{W}(X) = \sum_{r=0}^{N} X^{2r} \bar{W}_{2r} + \sum_{r=0}^{N} X^{2r+1} \bar{W}_{2r+1},$$

$$\bar{W}(X) = \sum_{r=0}^{N} X^{2r+1} \bar{W}_{2r+1} = \frac{\lambda^2}{2\Omega} \sum_{r=0}^{N} X^{2r+1} \left[\frac{(R_1^{\,2}R_2^{\,2r}-R_2^{\,2}R_1^{\,2r})}{(2r+1)!} \bar{W}_1 + 6\frac{(R_1^{\,2r}-R_2^{\,2r})}{(2r+1)!} \bar{W}_3 \right],$$

$$= \frac{\lambda^2}{2\Omega} \sum_{r=0}^{N} X^{2r+1} \frac{(R_1^{\,2}R_2^{\,2r}-R_2^{\,2}R_1^{\,2r})}{(2r+1)!} \bar{W}_1 + 6\frac{\lambda^2}{2\Omega} \sum_{r=0}^{N} X^{2r+1} \frac{(R_1^{\,2r}-R_2^{\,2r})}{(2r+1)!} \bar{W}_3, \quad (4.15)$$

$$= \frac{\lambda^2}{2\Omega} \sum_{r=0}^{N} X^{2r+1} \frac{(R_1^{\,2}R_2^{\,2r}-R_2^{\,2}R_1^{\,2r})}{(2r+1)!} \bar{W}_1 - 6\frac{\lambda^2}{2\Omega} \left(\sum_{r=0}^{N} X^{2r+1} \frac{(R_1^{\,2r}-R_2^{\,2r})}{(2r+1)!}) \right) \left(\frac{\sum_{r=0}^{N} \frac{(R_1^{\,2}R_2^{\,2r}-R_2^{\,2}R_1^{\,2r})}{(2r+1)!}}{6\sum_{r=0}^{N} \frac{(R_1^{\,2r}-R_2^{\,2r})}{(2r+1)!}} \right) \bar{W}_1.$$

The displacement function for SS plate

$$\bar{W}(X) = \frac{\lambda^2}{2\Omega} \sum_{r=0}^{N} X^{2r+1} \frac{(R_1^2 R_2^{2r} - R_2^2 R_1^{2r})}{(2r+1)!} \bar{W}_1 + 6\frac{\lambda^2}{2\Omega} \sum_{r=0}^{N} X^{2r+1} \frac{(R_1^{2r} - R_2^{2r})}{(2r+1)!} \bar{W}_3, \quad (4.16)$$

$$= \frac{\lambda^2}{2\Omega} \sum_{r=0}^{N} X^{2r+1} \frac{\left(R_1^2 R_2^{2r} - R_2^2 R_1^{2r}\right)}{(2r+1)!} \bar{W}_1 - 6\frac{\lambda^2}{2\Omega} \left(\sum_{r=0}^{N} X^{2r+1} \frac{(R_1^{2r} - R_2^{2r})}{(2r+1)!} \right) \left(\frac{\sum_{r=0}^{N} \frac{\left(R_1^2 R_2^{2r} - R_2^2 R_1^{2r}\right)}{(2r-1)!}}{6\sum_{r=0}^{N} \frac{\left(R_1^{2r} - R_2^{2r}\right)}{(2r-1)!}} \right) \bar{W}_1.$$

Displacement function for SF plate

$$\overline{W}(X)=\frac{\lambda^2}{2\Omega}\sum_{r=0}^{N}X^{2r+1}\frac{\left(R_1^{\,2}R_2^{\,2r}-R_2^{\,2}R_1^{\,2r}\right)}{(2r+1)!}\overline{W}_1+6\frac{\lambda^2}{2\Omega}\sum_{r=0}^{N}X^{2r+1}\frac{\left(R_1^{\,2r}-R_2^{\,2r}\right)}{(2r+1)!}\overline{W}_3,$$

$$=\frac{\lambda^2}{2\Omega}\sum_{r=0}^{N}X^{2r+1}\frac{\left(R_1^{\,2}R_2^{\,2r}-R_2^{\,2}R_1^{\,2r}\right)}{(2r+1)!}\overline{W}_1-\frac{\lambda^2}{2\Omega}\left(\sum_{r=0}^{N}X^{2r+1}\frac{\left(R_1^{\,2r}-R_2^{\,2r}\right)}{(2r+1)!}\right)*temp*\overline{W}_1,$$

$$temp=\frac{\displaystyle\sum_{r=0}^{N}\frac{\left((2*r)(2*r-1)-q_2\right)\left(R_1^{\,2}R_2^{\,2r}-R_2^{\,2}R_1^{\,2r}\right)}{(2r)!}}{\displaystyle\sum_{r=0}^{N}\frac{\left((2*r)(2*r-1)-q_2\right)\left(R_1^{\,2r}-R_2^{\,2r}\right)}{(2r)!}}. \tag{4.17}$$

To obtain the values of the frequency parameter Ω, the characteristic Eqs (4.12)-(4.14) have been solved with the help of a computer program developed in C++ using the bisection method for different values of the aspect ratio λ (=*a/b*) and foundation modulus parameter *K*. This program was run for different values of *N* until we get the first three values of the frequency parameter Ω correct to four decimal places and the value of *N* has been taken as 21. In this study, $m=1$ and $\upsilon=0.3$ have been fixed. The values of the frequency parameter Ω for different combinations of the aspect ratio and foundation modulus parameter are presented in Tab.2. It is concluded that the value of the frequency parameter decreases in the order SC>SS>SF. Further, it increases with the increasing values of the foundation modulus parameter *K* and aspect ratio *a/b*. A comparison of results for plates without foundation is given in Tab.3 which shows fast convergence of the method. Moreover, the results are better than those obtained by Bhat *et al.* (1990) and Bambill *et al.* (2000). Further, mode shapes may be drawn using Eqs (4.15)-(4.17).

Table 2

		SC			SS			SF	
		$\lambda=a/b$							
K	**Mode**	***0.5***	***1.0***	***2.0***	***0.5***	***1.0***	***2.0***	***1.0***	***2.0***
100	**I**	*20.0097*	*25.6739*	*52.6330*	*15.8809*	*22.1277*	*50.3510*	*15.3795*	*42.3930*
	II	*53.0490*	*59.4928*	*86.7130*	*43.1213*	*50.3510*	*79.5876*	*29.5028*	*59.9061*
	III	*106.9470*	*113.6690*	*141.2000*	*91.8399*	*99.2014*	*128.6940*	*62.6637*	*95.0114*
200	**I**	*22.3694*	*27.5526*	*53.5746*	*18.7671*	*24.2824*	*51.3344*	*18.3447*	*43.5564*
	II	*53.9833*	*60.3274*	*87.2877*	*44.2657*	*51.3345*	*80.2134*	*31.1515*	*60.7349*
	III	*107.4140*	*114.1080*	*141.5540*	*92.3827*	*99.7041*	*129.0820*	*63.4566*	*95.5362*
300	**I**	*24.5028*	*29.3112*	*54.4998*	*21.2650*	*26.2610*	*52.2994*	*20.8933*	*44.6897*
	II	*54.9017*	*61.1506*	*87.8586*	*45.3812*	*52.2994*	*80.8343*	*32.7172*	*61.5527*
	III	*107.8780*	*114.5450*	*141.9070*	*92.9223*	*100.2040*	*129.4690*	*64.2397*	*96.0581*
400	**I**	*26.4649*	*30.9701*	*55.4096*	*23.4990*	*28.1005*	*53.2469*	*23.1631*	*45.7948*
	II	*55.8050*	*61.9628*	*88.4259*	*46.4699*	*53.2468*	*81.4505*	*34.2113*	*62.3597*
	III	*108.3410*	*114.9810*	*142.2580*	*93.4589*	*100.7020*	*129.8540*	*65.0133*	*96.5773*
500	**I**	*28.2912*	*32.5446*	*56.3048*	*25.5382*	*29.8268*	*54.1778*	*25.2295*	*46.8739*
	II	*56.6938*	*62.7646*	*88.9896*	*47.5337*	*54.1777*	*82.0621*	*35.6429*	*63.1564*
	III	*108.8010*	*115.4150*	*142.6100*	*93.9923*	*101.1970*	*130.2390*	*65.7779*	*97.0936*

Table 3 $(a/b=1, K=0)$				
Boundary conditions	Reference	Mode		
		I	II	III
SC	Bhat *et al.* (1990)	*23.6463*	*58.6465*	*113.5220*
	Present	***23.6463***	***58.6464***	***113.2280***
SS	Bhat *et al.* (1990)	*19.7392*	*49.3481*	*99.3042*
	Present	***19.7392***	***49.3480***	***98.6960***
SF	Bambill *et al.* (2000)	*11.7195*	-	-
	Present	***11.6845***	***27.7563***	***61.8606***

5. Conclusions

The applicability of the differential transform method to analyze free transverse vibrations of isotropic rectangular plates of uniform thickness resting on a Winkler foundation is shown. The two opposite edges of the plate are assumed to be simply supported. Three combinations of boundary conditions, namely, simply supported, clamped and free have been taken on other two edges, keeping one of them simply supported. Characteristic equations have been obtained in the form of infinite series and have been solved numerically using a computer program developed in C++ to obtain natural frequencies. The results obtained show reliability and fast convergence of the method for rectangular plates.

Nomenclature

a – length of the plate
b – breadth of the plate
C – clamped edge
D – flexural rigidity
E – Young's modulus
F – free edge
h – thickness
K – foundation parameter
k_f – foundation modulus
m – an integer
S – simply supported edge
W – displacement
X, Y – non-dimensional variables
x, y, z – Cartesian coordinates
λ – aspect ratio
υ – Poisson ratio
Ω – frequency parameter
ω – radial frequency

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Received: December 21, 2011
Revised: November 24, 2012