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On Nautical Observation Errors Evaluation

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ABSTRACT: Mathematical Theory of Evidence (MTE) enables upgrading models and solving crucial problems in many disciplines. MTE delivers new unique opportunity once one engages possibilistic concept. Since fuzziness is widely perceived as something that enables encoding knowledge thus models build upon fuzzy platforms accepts ones skill within given field. At the same time evidence combining scheme is a mechanism enabling enrichment initial data informative context. Therefore it can be exploited in many cases where uncertainty and lack of precision prevail. In nautical applications, for example, it can be used in order to handle data feature systematic and random deflections. Theoretical background was discussed and computer application was successfully implemented in order to cope with erroneous and uncertain data. Output of the application resulted in making a fix and a posteriori evaluating its quality. It was also proven that it can be useful for calibrating measurement appliances. Unique feature of the combination scheme proven by the author in his previous paper, enables identifying measurement systematic deflection. Based on the theorem the paper aims at further exploration of practical aspects of the problem. It concentrates on reduction of hypothesis frame reduction and random along with systematic errors identifications.

1 NAUTICAL EVIDENCE

Navigational evidence embrace results of observations as well as knowledge within discipline known as nautical science [3]. Observations mainly mean taking distances and or bearings. Horizontal angles are also taken from time to time. Results of observations are imprecise. It is widely assumed that any measurement contains systematic deflection along with random error. It is also assumed that random errors are governed by Gaussian distribution or take form of histogram that is empirical diagram of various tests outputs. Figure 1 shows two schemes of taking distance. Both presented cases marked as a) and b) differ with systematic defections. At presented example random deflections feature the same theoretical or empirical characteristics.

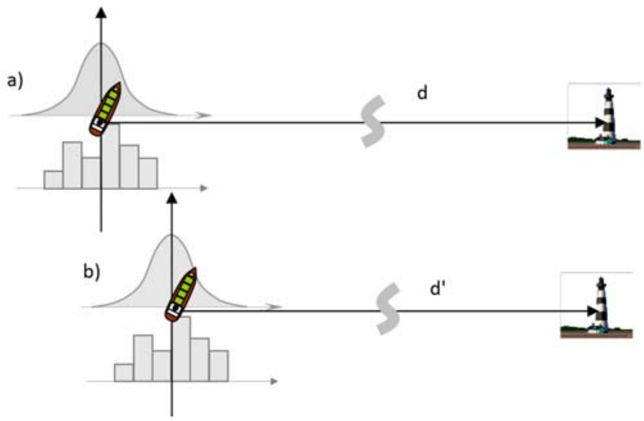

Figure 1. Result of taking distance is an imprecise value that is randomly and systematically distorted

Figure 2 presents situation in which two distances for two landmarks are taken. Distances were established for objects located at opposite sides from the observer position [9]. The scheme is usually followed in order to identify systematic deflection of the measuring appliance, which usually is a radar. Two circles being distance isolines projected on the chart should be tangent at collinear gradient directions unless they are distorted. Isolines are separated by certain distance once measurement deflections occurred. Breadth of resulted gap enable reasoning on kind of involved errors. In case when it is smaller than sum of both random distributions standard deviations: $\sigma_1+\sigma_2$ occurrence of a systematic error is very unlikely. Figure 2 presents situation for which the gap is bigger than sum of tripled deviations: $3\cdot\sigma_1+3\cdot\sigma_2$. It means that fixed deflection occurred and is to be identified. Systematic error is assumed as being the same for both observations provided taken with the same appliance. Therefore distance correction is estimated as half of the observed gap what is valid with assumption that random error is negligible.

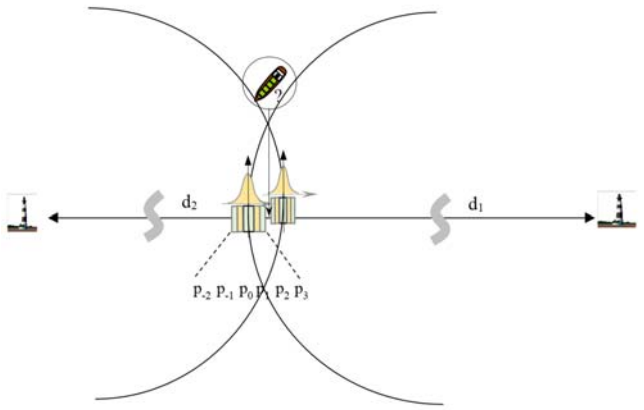

Figure 2. Graphical interpretation of two imprecise measurements, distorted with random and systematic errors, taken to objects at opposite directions

Seafarers know that mean error, related to standard deviation of the bell function, of a distance measured with radar variable range marker is a function of the obtained value and is said to be within the interval of [±1%; ±1.5%] of the measurement. Taken distance of 10 Nm is a random variable with mean error inside the range of [±1; ±1.5] cables. In figure 2 distance d_1 is assumed greater than d_2 (see also shapes of inserted distribution functions).

Standard deviation is one of the most important factor in observations accuracy evaluations. In practice its exact evaluation is rather impossible. Thus crisp valued mean errors of measurements are considered inadequate. Instead in recent navigation books (for example see [12]) measurement mean error is described as imprecise interval value usually as: $[\pm\sigma^-_d;\ \pm\sigma^+_d]$ (herein letter d denotes taken distance). Being interested in an isoline possible deflection one considers interval $[\pm m^-_d;\ \pm m^+_d]$ established along gradient direction. Since gradient module is equal to one for distance isoline both before mentioned parameters are of the same meaning. With fuzzy arithmetic notation [12] the latest can be rewritten as a quad $(-m^+_d;\ -m^-_d;\ +m^-_d;\ +m^+_d)$. It means fuzzy value with core of $[-m^-_d;\ +m^-_d]$ cables and support of $[-m^+_d;\ +m^+_d]$. In order to include such data into upgraded model one has to engage adequate formal apparatus.

2 IMPRECISE NAUTICAL EVIDENCE AND ITS ENCODING

In possibilistic approach uncertain evidence is represented with sets and masses of confidence attributed to these sets. Sets embrace relations between hypothesis and evidence spaces. Relations can be binary or fuzzy ones [1, 13, 14]. Fuzzy sets embrace grades expressing possibilities of belonging of consecutive hypothesis items to the sets related to each piece of evidence. Therefore appropriate relations between considered frames are encoded into evidence representation, which takes the form specified by formula (1).

$$\begin{aligned}\mathrm{m}(e_i) = \{&(\mu_{i1}(x_k),\ f(e_i \to \mu_{i1}(x_k))), \cdots,\\ &(\mu_{in}(x_k),\ f(e_i \to \mu_{in}(x_k)))\}\end{aligned} \tag{1}$$

What does hypothesis frame mean for navigator? What area should it cover in case of discrete version of the problem is considered? Hypothesis frame is to be considered as search space where for example true isoline, due its erroneous nature, is being located [5, 6]. It should also be perceived as a collection of chart points that represent fixed position of the ship. Results of combination of evidence related to two random variables that projected on the plane are separated by certain distance, were examined in previous paper by the author [10]. Like at figure 2 there were considered variables referred to isolines related to distances taken for two objects situated at opposite directions. It was assumed that variables could be distorted with systematic error apart from random one. Identifying permanent measurement shift is an important nautical issue.

Figure 3 shows histogram with probabilities of the true isoline being located in the vicinity of the observed one. The histogram was prepared based on bell function, width of each bin is equal to quarter of normalized standard deviation. Figures within each bin indicate probability that the true line related to the measurement is located within particular strip provided random error is taken into account. It should be noticed that due to discrepancies in statistical investigations regarding measurements distributions parameters presented bin limits are rather range valued [8, 9]. For normal distribution width of the search frame should be confined to $6\cdot m$, to six folded standard deviation of given isoline (see also figures related to extreme bins). It should be pointed that it is not always the case. Figure 2 shows search space limited to area located in between obtained distance isolines. In view of particular evidence related to distances taken for landmarks situated at opposite directions only half of the distribution is valid. The true location of the observer is to be located within shown gap. Only meaningless support can be attributed to contrary statement.

In presented application evidence representation consists of pairs [5]: fuzzy vectors $\mu_{i1}(x_k)$ representing locations of a set of each points $\{x_k\}$

within sets related to each piece of evidence – degrees of confidence assigned to these vectors $f(e_i \rightarrow \mu_{in}(x_k))$. Degrees of confidence reflect probability of true isoline being located within given strip area can be obtained from presented drawing (see figure 3).

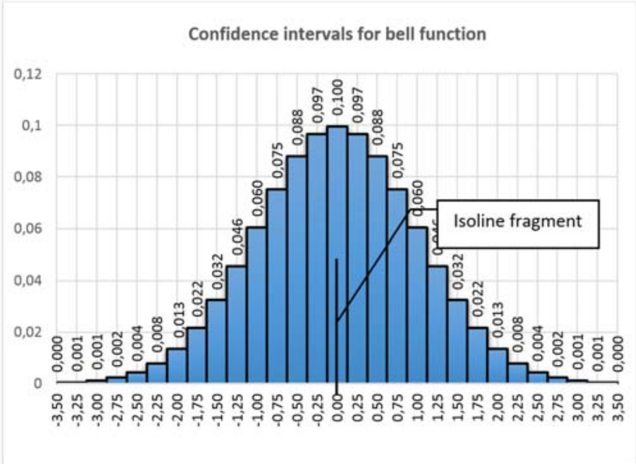

Figure 3. Histogram showing probabilities of the true isoline being located in the vicinity of the measured one

Strip areas are related to confidence intervals established for probability distribution functions and are assumed to be adjusted to the evidence at hand. For bell function they are quite often assumed as: half, single, double and triple of standard deviation. Figure 4 shows probabilities of the true isoline being located in the vicinity of the measured one. Bins width are assumed equal to a single unified standard deflection. First of each pair of figures placed within bins are those obtained for unconfined search frame. Then the space was reduced to range [-0.4; 4.8] of standard deviation. Second figure in each pair of numbers placed within bins are modified due to available evidence limitation.

Algorithm I

1 Sum up all probabilities attributed to ranges that are outside of the search frame. Include reduction of probabilities for partial inclusion (see extreme bins of included rectangle at figure 4)
2 All modified probabilities that are greater than zero divide by complement of the total calculated in step 1

Algorithm I guarantees that only focal elements are included into created belief structures [2]. Focal items are those with non zeroed masses assigned. Additionally total of all masses assigned to focal elements, without uncertainty, is one.

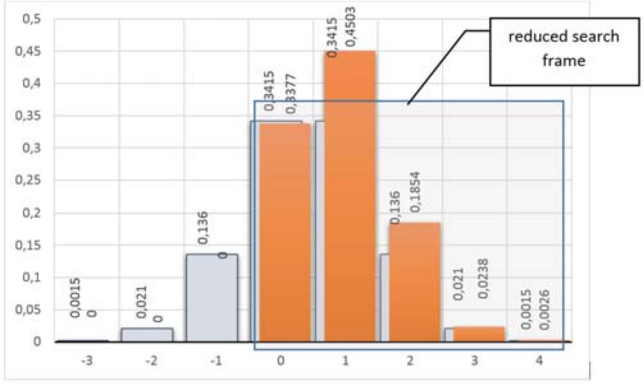

Figure 4. Histogram showing probabilities of the true isoline being located in the vicinity of the measured one and its modified version.

3 CASE STUDY

The concept of exploiting evidence that is meant as encoded facts and knowledge, in supporting decisions in navigation is based on measurement distributions and fuzziness. Introduced confidence intervals (see figure 3) define probabilities of true isolines being located within appropriate strips established along gradient directions. Modified probabilities are incorporated into belief assignments that enable the modelling of uncertain, imprecise data. Imprecision is due to random errors but systematic deflections occur quite often. This kind of error is to be identified and eliminated. The identification of a permanent measurement shift is an important practical nautical issue.

In this chapter considered are observations engaging two distances made for two objects situated at opposite directions as seen from the observer's position. Both observations resulted in isolines that are assumed to be distorted with random errors and include systematic deflection. Random errors distribution means are supposed to be within the range of ±1% of the measured distance. Possible limits of the estimated mean are within ±15% of their value. Data used in numerical experiment are gathered in table1.

Table 1. Summary of data used in numerical experiment

	observation 1	observation 2
distances	30 cables	50 cables
mean errors	0.3 cables	0.5 cables
mean error limits	[0.255; 0.345] cables	[0.425; 0.575] cables
subjective confidence evaluation	90%	80%
gap width (see figure 2) for case a)	0.58 cables	
gap width for case b)	3 cables	

Based upon presented nautical evidence navigator should reason on quality of measurements and possibly identify systematic deflection. He is supposed to answer two questions: what is the systematic error of the applied measuring device and how random error might affected his evaluation.

Figure 5 shows two examples in which pairs of observations made for two objects situated at opposite directions from ship position. Each of the observations is marked with small circular shape placed at abscissa axis that is assumed collinear with gradient directions. Observation's random error distribution are depicted with two bell functions that represent extreme value of assumed standard deviation (see also interval valued data in table 1). Shapes emphasising interval valued limits of mean error are also included. Search space was confined by both isolines, its discrete points represent true location of the vessel. Question which of them best represents the true location is resolved through reasoning base on results of evidence combination scheme.

Left hand side of illustrations placed in figure 5 presents situation in which gap between isolines is due to random errors. Case a) presents two observations for which systematic deflection should be rather excluded since gap between isolines is smaller than sum of mean errors. Statement is rather unlikely for right hand side case. The gap can be estimated as sum of three folded mean errors. Thus probability that systematic error was involved is rather high. In order to cover the isolines gap, consequently to create artificial free of systematic error case, mean errors were increased during iterative combination process. Final stage situation in which enlarged observations mean errors cover the gap as well as association result was presented at figure 6.

It should be stressed that figures 5 and 6 remain closely related. Based on results of combination illustrated at figure 6 (notice direct reference to case 5b) one can reason on solution to problem presented at figure 5a). Note that for the latest case location of true measurement in between extreme observations can be easily evaluated. Therefore one can reason on influence of random errors on final observations' evaluation as, for example, presented in right case 5b). Combination results are transferable for the two cases. Systematic error can be estimated as interval valued equal to observations gap mean distorted with random deflection. Herein the scheme of approach was exploited in order to demonstrate practical aspects of the methodology.

It was proven [10] that belief and plausibility measures that are calculated based on results of the combination of two pieces of evidence related to two random variables governed by Gaussian distributions with given approximate standard deviations for which appropriate isolines are separated with certain Euclidean distance (case 5a) and those obtained from association of evidence related to random variables governed by the same distributions with approximate standard deviations magnified by certain constant with isolines being separated with distance incremented with the same value (case 5b) are mutually dependent on this constant. The proposition was further exploited in order to calculate data included in table 2.

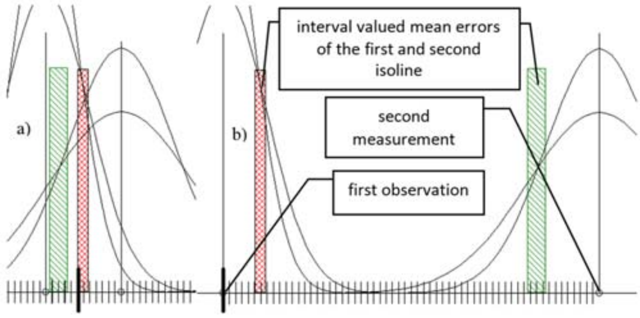

Figure 5. Two cases related to pairs of observations made for two objects situated at opposite directions

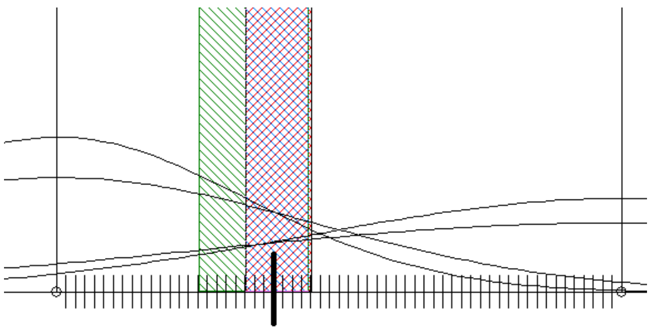

Figure 6. Case presented at figure 5b with proportionally enlarged observations mean errors

To practically prove above proposition, results of the combination of evidence related to two pairs of random variables represented by distances taken to different landmarks were examined. Example variables referred to isolines related to the distances taken for two objects located at counter bearings. Unlike second pair the first one was likely to remain free from systematic error. Further permanent error estimation was achieved with iterative imprecise evidence combination scheme. In each step proportional increment of isolines mean errors took place. Iterations stopped once maximum belief and plausibility measures are recorded for the same hypothesis point while the mass of inconsistency remained low (see data in table 2). Further looping results in decreasing of belief and plausibility measures.

Table 2. Summary of numerical experiment results

belief	hypothesis point number	plausibility	hypothesis point number	solution	inconsistency	mean error I[1])	mean error II[1])
-	-	0.157	1	0.05	0.714	0.30	0.50
0.131	23	0.710	23	1.15	0.003	1.18	1.95
0.098	23	0.699	23	1.15	0.001	1.30	2.15
0.064	23	0.690	23	1.15	0.001	1.37	2.26

[1]) iteratively increased values are presented

Figure 7 presents diagrams of plausibility values variations during iterative combination process. Bottom curve represents results of the initial stage of calculations. They refer to situation illustrated at figure 5b. Randomly distorted isolines approximately separated with doubled permanent deflection. More data regarding this situation are gathered in the first row of table 2. Row number one reads that uncertainty, in this case meant as inconsistency due to not overlapping evidence cases, is very high. Its value of 0.714 suggests rather contradictory data, one piece of evidence supports hypothesis points separated from those endorsed by the second one. Highest plausibility value indicates solution point that is located at the first isoline. Mean error of this measurement is smaller and assigned confidence is higher than for the second case (see data in table 1).

Three uppermost diagrams at figure 7 refer to last stages of iterative combination process. Combination scheme shows the same point of the hypothesis frame that is clearly distinguished as solution to the problem. Iterative association engaged incremented values of mean errors. From data gathered in table 2 one can notice that at the final stage of processing sum of increased mean distortions covers isolines intersection gap. For all last three cases uncertainty remains low, belief and plausibility are high and both these measures clearly indicate the same hypothesis point. The latest also mean that solution remains stable.

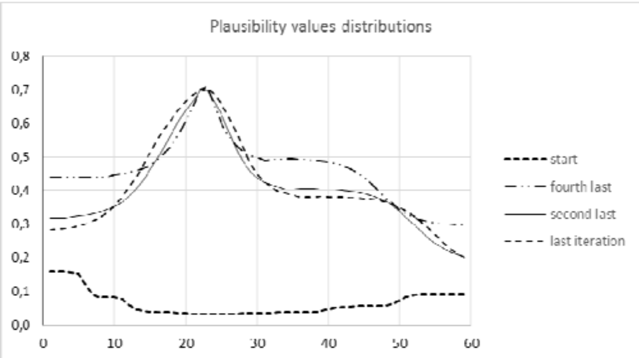

Figure 7. Diagrams showing plausibility values variations during iterative combination process

Benefits that can come out of the presented proposition were depicted within examples devoted to distance error analyses. Two pieces of evidence, one free from systematic error and another distorted with this kind of deflection, were associated. Results of combinations were confronted in order to mine for general practical aspects. Examination of the outcome empirically proves the correctness of the presented theorem and enables calibration of the nautical appliance. Utilization of the lemma for position fixing based upon multiple observations [7] taken with the same tool and possibly distorted with systematic error is straightforward. At first pair or pairs of observations enabling permanent shift indication should be selected. Constant should be extracted and further used for standard deviations of all observations adjustment. Modified evidence are to be encoded and combined afterwards.

Algorithm II

1. Assign initial data, evaluate approximate observations mean errors and their uncertainty (known discrepancies in their estimation)
2. Identify limits of the hypothesis frame and adjust probabilities for selected confidence intervals (see algorithm I)
3. Prepare belief structures, normalize[1] and combine them
4. Calculate total of inconsistency masses
5. Calculate belief, plausibility measures based on results of combination. Locate belief and plausibility maxima
6. Quit if belief and plausibility maxima refer to the same point, consistency is below required threshold and sum of modified mean errors covers isolines gap
7. Modify mean errors with given constant and go to step 3

Output generated by software implementing algorithm II for previously defined numerical example are presented in table 3, in which apart from constant *C* all data refer to cables as distance unit. Two distances for opposite locations objects were taken with medium class radar. Mean errors were estimated as: ±0.3 and ±0.5 cables respectively. Their possible random distortions were assumed to be ±15%. Presented data refer to four last iterations for which maximum of plausibility value remained high and referred to the same solution indicated value is 1.15. Collected data include mean errors multiplier *C* with calculated two random deflections δ_i along with interval valued systematic error. Based on introduced lemma for each multiplier random errors were estimated. The evaluation is based on proposition that enable migration to "free from systematic error case" (see both illustrations at figure 5). Please also note that direction of random shifts can not be indicated. Available evidence do not allow to state what signs of random deflection might be thus interval valued permanent errors were calculated taking into account both possible randomness directions (both negative and positive extreme values).

Table 3. Four last iterations results

C	δ_1	δ_2	gap width	S^-	S^+
3.933	0.292	0.470	0.763	1.119	1.881
4.133	0.278	0.448	0.726	1.137	1.863
4.333	0.265	0.427	0.692	1.154	1.846
4.567	0.252	0.405	0.657	1.172	1.828

Final results extrapolations for various gap's width are included in table 4. From data gathered in the first row of table 4 one can read that for isolines gap of 0.8 cables probability of an measurement being within the gap is 0.721. At the same time thanks to evidence combination (see data in table 2) solution for the case are random errors of 0.295 and 0.505 respectively. As can be seen from table 4 appropriate random errors tend to decrease but probability of particular case are getting smaller and smaller. Probability that

[1] General idea of normalisation was presented in [15], specificity of nautical applications in this respect was discussed by the author in [8]

systematic error is within range of [1.40;1.60] is reduced to 0.399.

Table 4. Final extrapolation results

std deviation ratio	distance	probability I	probability II	sum of probabilities	δ_{1i}	δ_{2i}	S^-	S^+
1	0.8	0.497	0.445	0.721	0.295	0.505	1.10	1.90
0.75	0.6	0.477	0.385	0.679	0.221	0.379	1.20	1.80
0.5	0.4	0.420	0.289	0.587	0.148	0.253	1.30	1.70
0.25	0.2	0.289	0.156	0.399	0.074	0.126	1.40	1.60

4 SUMMARY

Thanks to Mathematical Theory of Evidence approaches towards theoretical evaluation of tasks including imprecise data are to be reconsidered. One of such problem is indication and evaluation of a measurement systematic error. In nautical practice in order to calculate compass correction one has to know direction to a landmark or celestial body. Alternatively one has to make observations for objects that are located at opposite bearings. Application of Mathematical Theory of Evidence in order to reason on nautical appliance calibration was presented in the paper. At first range of hypothesis frame was reduced in order to guarantee correctness of a posteriori reasoning in selected nautical applications. Seafarers know where true measurement is supposed to be located. Observations are assumed to be made for landmarks situated at opposite sides are examples where such locations can be easily identified. Due to proposed reduction combination inconsistency mass remain small while belief and plausibility are relatively high. Usually high inconsistency mass indicates poor quality nautical evidence. Yet another reason for large conflicting mass is wrongly defined hypothesis frame, consequently it is not supported by evidence at hand.

In the second part of the paper proposition regarding unique feature of nautical evidence combination scheme was exploited. Statement regarding behaviour of the association process was presented and proven in recent paper delivered by the author. Theorem enables reasoning on random and systematic errors of observations made for objects situated at opposite sites as seen from observer's position.

In presented numerical example two distance observations distorted with random and systematic errors were considered. Obtained measurement data along with nautical knowledge were encoded into belief structures which were further iteratively combined. Iterations were quitted once stable solution was achieved. Given this solution reasoning regarding combination of systematic deflection free data was carried out. Thanks to MTE particular distance between isolines solely due to random errors could be achieved. It is identified by hypothesis point with the highest support measures in view of evidence at hand. It subsequently gives basement for random errors estimations and systematic deflection evaluation. Result fixed error appears interval valued, range of obtained values depends on required threshold probability. To estimate limits results of informal interpolation were introduced. For selected decreasing isolines gaps probability of the true isoline being located within were calculated. For each presented case the true location of the ship could be also estimated based on obtained results.

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