

*turning, shaft with low rigidity, numerical simulation studies*

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## NUMERICAL SIMULATION STUDIES ON THE PROCESS OF MACHINING OF SHAFTS WITH LOW RIGIDITY

### Abstract

*The paper presents a computer program developed for the determination of the basic dynamic characteristics of the process of machining, numerical simulations of the dynamic system of the process of machining, and graphic presentations of the numerical simulations performed. Results of simulations of the runs of time and frequency characteristics of the process of machining with variable machining parameters are presented. High goodness-of-fit of the model with the actual process of turning was achieved.*

### 1. INTRODUCTION

The knowledge of the technical behaviour of machined parts during the production processes is very important both for the technologists and also for the manufacturing systems design purposes [3, 4, 7]. Therefore both modelling of the machining processes and the numerical studies are necessary and valuable [2, 12]. For the purpose of provided research in the area of low-rigidity shafts machining the program MATMOD was developed. The program allows the determination of the conditions and the basic dynamic characteristics of the machining process, numerical simulation of the dynamic system of the machining process, and graphic presentation of the characteristics of the numerical simulations performed [1].

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The paper presents the result of simulations of the runs of time and frequency characteristics of machining process performed at various values of machining parameters. The tests were performed for a shaft of steel C45, with dimensions of  $l_p = 280$  mm,  $d_p = 40$  mm. The variable input parameters were the rotation speed  $n_p = 700, 1100, 1500$  rev/min; feed  $f = 0.1, 0.25, 0.5$  mm/rev; depth of machining  $a_p = 1, 2, 3$  mm.

Based on the experimental study of the process of machining and on the results obtained in the simulation studies, conducted at the same parameters of the machining process that were applied in the experiment, a comparison was made of the experimental and simulation results. The value of relative deviations between the experimental data and those from the simulations,  $\delta$  (%), equalled 9.61%, which indicates very good fitting of the model with the actual run of the process of turning. The results obtained can be used as a starting point for further research in the area of design of systems of automated adjustment.

## 2. CHARACTERISATION OF COMPUTER PROGRAM MATMOD

For the needs of the study of the characteristics of the dynamic system the computer program *MATMOD* was developed [1, 10, 11].

The program realizes the following functions:

- determination of the conditions of the machining process after entering the parameters, of the machine tool, the object machined, and of the technological process,
- calculation, on the basis of models of the dynamic system of the machining process, of the basic dynamic characteristics (including operator transmittance), cutting force responses to step-wise change in feed rate, and frequency characteristics: amplitude and phase.

The program allows the performance of numerous numerical simulations of the dynamic system of the process of machining. It supports three kinds of fixing of the machines component, that have a significant effect on the final result of the computations: in lathe centres, in a chuck and a centre, and in two chucks. Depending on the kind of fixing, the program determines the maximum flexibility of the machined component in the radial direction. The program accepts input data for machined components differing in length. An additional advantage of the program is the possibility of defining the properties of the material machined by adding e.g. the values of  $E$  and  $R_m$ . Other input data for the computation process include also the values of the machine tool flexibility in the direction of coordinates  $X$ ,  $Y$  and  $Z$ , as well as the depth of machining, feed rate, tool cutting edge angle  $\kappa_r$  (on the range from  $45^0$  to  $90^0$ ) and the rotation speed of the machined component  $n_p$  (in the range from 100 to 2000 rev/min). After entering all the input data the program performs the computations.

The first to be calculated are the values of forces  $F_f$ ,  $F_c$  and  $F_p$ , and then the amplification coefficients of the machining process  $m_x$ ,  $m_y$ , and  $m_z$  with a change of feed rate and machining depth (these values can be treated as interference). The coefficient of relative dynamic stiffness  $B$  is determined, and then the operator transmittance in control, corresponding to the presented conditions, and the amplification coefficients of the component  $K_{ox}$ ,  $K_{oy}$ ,  $K_{oz}$  and retardation  $\tau$ . Therefore, the maximum value of coefficient  $B$  and operator transmittance at interference are determined again [10,11]:

$$G_{v,gi}(s) = \frac{\Delta F_i(s)}{\Delta v_f} = \frac{m_x h_{xx} G_\tau(s)}{s[(T_c s + 1)(T_{us}^2 s^2 + 2\xi T_{us} s + 1) + B_1 G_\tau(s) + n_y h_{yy}]}, \quad (1)$$

where:  $G_\tau(s) = 1 - e^{-s\tau}$ ,

$$B_1 = m_x h_x + K_{\kappa_r} m_y h_y.$$

Coefficients  $K_{bx}$ ,  $K_{by}$ ,  $K_{bz}$  and the values of coefficient  $B$  and retardation  $\tau$  are also calculated.

In the case of control, the operator transmittance of the approximated object is determined as follows:

$$G_o(s) = \frac{\Delta Y_o(s)}{\Delta v_f(s)} = \frac{K_o}{(T_1 s + 1)(T_2 s + 1)}. \quad (2)$$

After determining the operator transmittance, coefficients  $K_{ox}$ ,  $K_{oy}$ ,  $K_{oz}$  and time constants  $T_1$  and  $T_2$  are determined.

The final stage of the computational part of the program is the determination of the operator transmittance of the object at a variable material allowance  $b(s)$ , treated as an interference. In the initial phase the object transmittance is determined in accordance with specific rules. In the case of changes of the material allowance  $b(s)$  the operator transmittance of the object can assume the following form [10,11]:

$$G_{bFi}(s) = \frac{\Delta F_i(s)}{\Delta b_i(s)} = \frac{K_{bFi}(s)(1 - e^{-s\tau})}{s\tau[1 + B(1 - e^{-s\tau})]}, \quad (3)$$

whereas, coefficient  $B$  is determined from the relation:

$$B = \frac{m_x h_{xx} + K_{\kappa_r} m_y h_{yy}}{1 + n_y h_{yy}}. \quad (4)$$

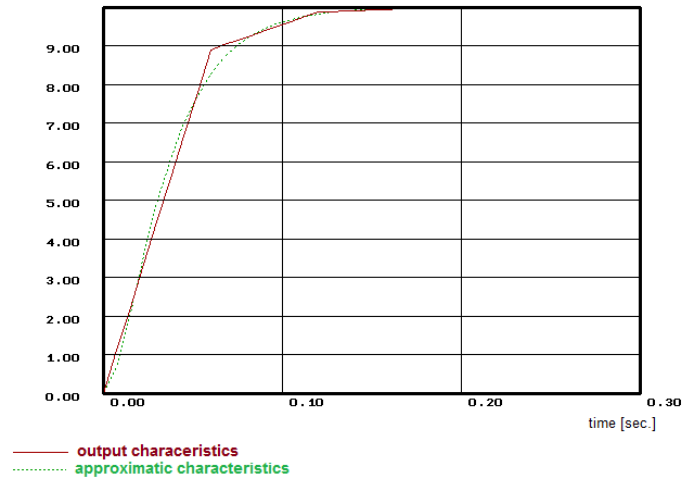
The program permits also the calculation of the values of elastic deformations relative to coordinates  $X, Y, Z$  and of the coefficients  $K_{ab} : K_{abx}, K_{aby}, K_{abz}$ . Equivalent values of the time constants  $T_1$  and  $T_2$  are determined, the coefficient of object amplification  $K_o$  and coefficient  $B$ . The last to be determined are the stabilised output value  $Y_{ust}$  and the maximum relative error of approximation.

The program permits also the presentation of graphic characteristics of numerical simulations.

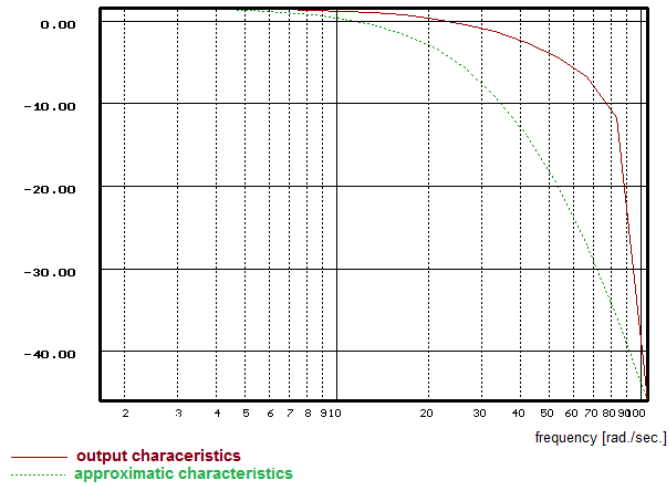
After starting the program, we need to enter the parameters of the machining process, the geometric parameters and the properties of the machined component. The limit values of the parameters (the minimum and the maximum values) are entered at the moment when the question about the value of a given parameter is displayed on the screen. The user can choose between the program options: calculation of response to unit step of feed and calculation of frequency characteristics. The graph of the amplitude frequency characteristics presents the relation of module  $A(\omega)$  of spectral transmittance  $G(j\omega)$  in the function of circular frequency  $\omega = 2\pi f$ , and that of the phase frequency characteristics –  $\varphi(\omega)$  in the function of circular frequency  $\omega = 2\pi f$ .

In the case of response to unit change of longitudinal feed, a graph of changes of machining force in time is displayed on the screen, for the accurate and the approximated models. Discrete values of the response (40 points) can be saved in the form of a table. It is also possible to save the complete information on data entered and on the results of computations of the parameters of the machining process. The program permits also to make copies of the graphic screen with responses to unit step of longitudinal feed.

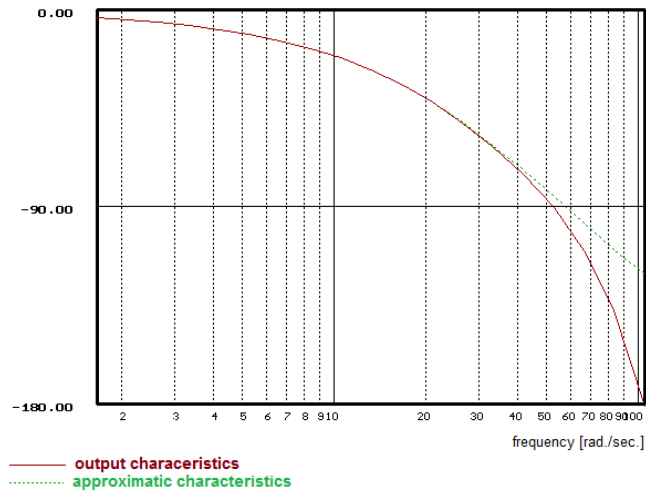
If we select the option of calculation of the amplitude and phase frequency characteristics, graphs of the amplitude and phase frequency characteristics of the accurate and approximated models will be displayed on the screen. Discrete values of the characteristics (20 points) can be saved in the form of a table. The frequency characteristics are presented in a semi-logarithmic scale. It is also possible to print a copy of the screen with the amplitude and phase characteristics.



**Fig. 1. Output time-characteristics (accurate and approximated) [source: own study]**



**Fig. 2. Amplitude-frequency characteristic [source: own study]**



**Fig. 3. Phase-frequency characteristic [source: own study]**

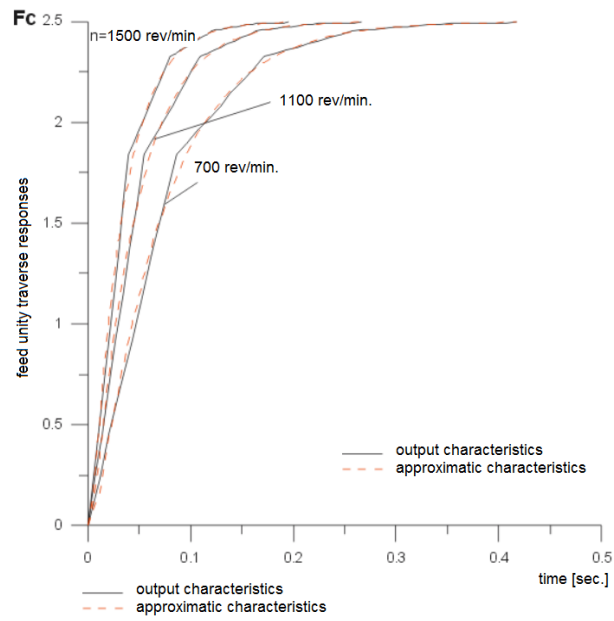
Examples of results of numerical studies performed in the program *MATMOD 1* are shown in Figs. 1, 2, 3 which present the process of machining at various parameters in the form of suitable characteristics.

### **3. NUMERICAL SIMULATION STUDIES ON THE DYNAMIC SYSTEM OF THE PROCESS OF MACHINING**

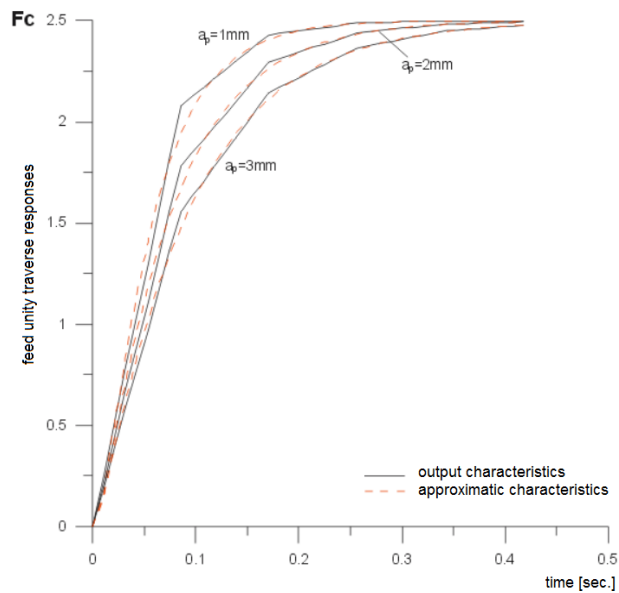
Simulations of the runs of time and frequency characteristics of the process of machining were conducted at various values of machining parameters.

The tests were performed for a shaft of steel C45, with dimensions of  $l_p = 280$  mm,  $d_p = 40$  mm. The variable input parameters were the rotation speed  $n_p = 700, 1100, 1500$  rev/min; feed  $f = 0.1, 0.25, 0.5$  mm/rev; depth of machining  $a_p = 1, 2, 3$  mm.

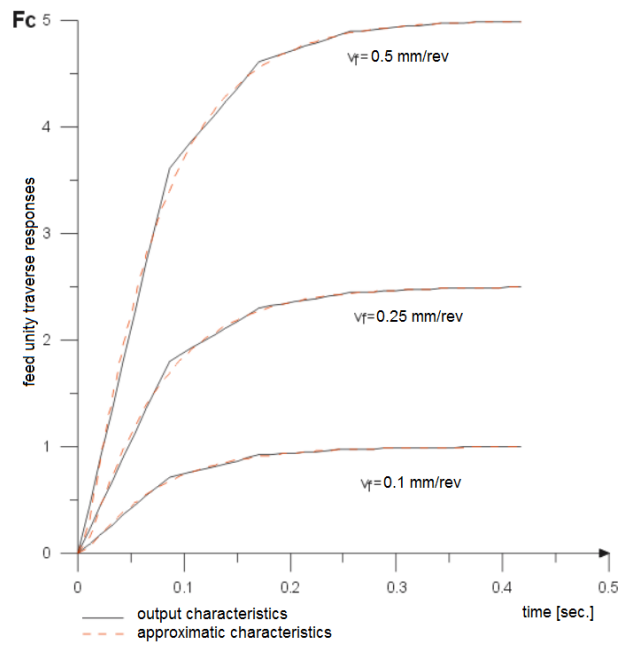
Results of simulations of the process of machining performed with the use of the program *MATMOD* are presented in graphic form in Figs. 4–8.



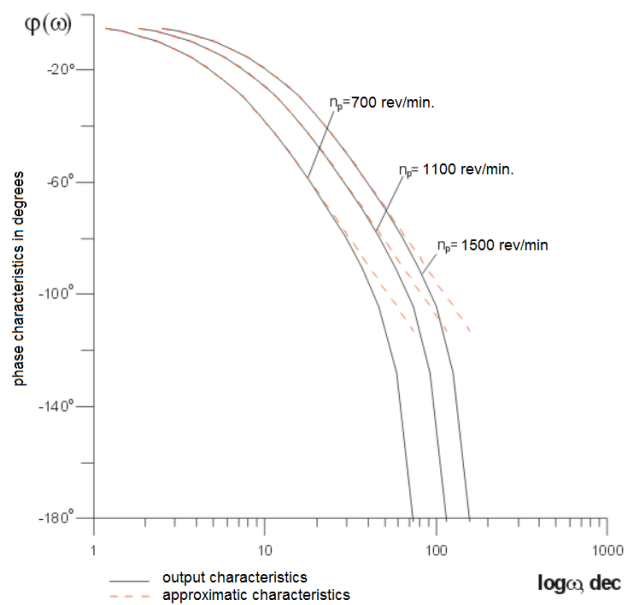
**Fig. 4. Machining force responses to unit step of feed at various rotary speeds [source: own study]**



**Fig. 5. Machining force responses to unit step of feed at various thicknesses of machined layer [source: own study]**

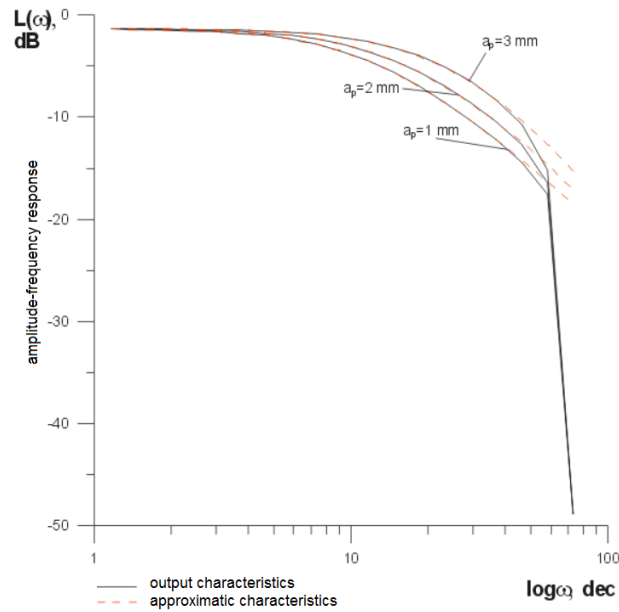


**Fig. 6. Machining force responses to unit step of feed at various speeds of machining [source: own study]**



**Fig. 7. Phase-frequency characteristics of machining process at various rotary speeds of machined component [source: own study]**





**Fig. 8. Amplitude-frequency characteristics of machining process at various thicknesses of machined layer [source: own study]**

Machining force responses at step-wise changes of feed rate, with varied conditions of the process of machining, are presented in Figs. 4–6. The phase-frequency and amplitude-frequency characteristics of the process of machining are presented in Figs. 7–8.

#### 4. ANALYSIS OF RESULTS OF SIMULATIONS

Based on the simulations conducted within the scope of the study it was observed that:

- a change of rotary speed causes a change in the time of stabilisation of the transient run,
- a change of the thickness of machined layer causes a corresponding change of response to unit step of feed, but at constant time intervals,
- a change of feed rate causes a change in the values of machining force at steady state in response to unit step of feed,
- changes in the phase characteristics are the most observable during changes of rotary speed, while in the case of changes of feed rate and thickness of machined layer the differences are small,
- changes in the amplitude characteristics can be observed mainly in the case of changes in the thickness of machined layer.

Based on the results of experimental tests of the process of machining [6, 8, 9] and on the results of the simulations, conducted at the same parameters of the machining process as those applied in the experiment, a comparative table of the experimental and simulation results was compiled (Tab. 1). Column 1 gives the successive numbers of experimental tests conducted on a test stand equipped with a lathe, measuring instruments and computer system for recording the changes of the parameters in the course of the experiment. Columns 2–7 specify the parameters of the process of machining applied in the successive experiments. Column 8 lists the time constants of the process, obtained in the experiment, and column 9 – the time constant calculated on the basis of the developed mathematical model. Column 10 gives the relative error of determination of the time constant on the basis of the model.

The experimental transition characteristics were approximated with the expression:

$$F_c(t) = F_{c0} [1 - \exp(-t/T_{1e})]$$

where:  $F_{c0}$  – determined value of an output value or of its gain,

$T_{1e}$  – equivalent time constant, determined on the basis of experimental transition run as the time after which an output value or its gain attains the level of 0.63 of the value in its steady state.

The values of  $T_{1e}$  from the experiment were compared with model-calculated values  $T_{1p}$ , defined as the time at which the calculated transition characteristic of the model, described by the relations presented above, attains 0.63 of the steady value.

Based on the values of relative deviations  $\delta$  (%) obtained between the experimental data and data from the simulations, a mean value of 9.61% was determined. In the case of such a small value, the divergences between the time constants of the model and the results of the experiment indicate a good fit of the model to the actual run of the process of turning. This fact can be used a starting point for further research in the area of design of automatic adjustment systems.

**Tab. 1. Parameters of machining versus the calculated and experimental time constants [source: own study]**

Item	$\tau$ , s	$\kappa_r$	$v_c$ , m/s	$a_p$ , mm	$a$	$F_{c0}$ , N	$T_{1e}$ , s	$T_{1p}$ , s	$\delta$ , %
1	0.48	45	0.8	2.0	0.2	900	0.59	0.55	7
2	0.48	45	0.8	3.0	0.2	1380	0.7	0.625	11
3	0.375	45	0.85	1.0	0.2	480	0.3	0.3	0
4	0.375	45	0.85	2.0	0.25	1140	0.46	0.43	7
5	0.375	45	0.85	3.0	0.2	1400	0.55	0.48	13
6	0.24	45	1.3	1.5	0.25	855	0.23	0.215	7
7	0.24	45	1.3	2.0	0.25	1150	0.34	0.28	18
8	0.24	45	1.3	3.0	0.2	1370	0.36	0.31	14
9	0.12	45	1.6	1.5	0.2	730	0.12	0.13	-8
10	0.12	45	1.6	3.0	0.2	1470	0.18	0.161	11
11	0.095	45	1.65	1.0	0.2	475	0.08	0.076	5
12	0.095	45	1.65	3.0	0.2	1475	0.15	0.13	13
13	0.095	45	1.65	5.0	0.1	1180	0.2	0.19	5
14	0.075	45	1.67	1.0	0.2	470	0.06	0.064	-7
15	0.075	45	1.67	2.0	0.2	981	0.11	0.09	18
16	0.075	45	1.67	3.0	0.2	1430	0.12	0.105	5
17	0.048	45	1.69	1.0	0.2	390	0.04	0.04	0
18	0.048	45	1.69	3.0	0.2	795	0.072	0.065	10
19	0.48	90	0.9	1.0	0.25	610	0.3	0.38	-26
20	0.48	90	0.9	2.0	0.25	1200	0.42	0.43	-2
21	0.19	90	1.33	2.0	0.25	1100	0.18	0.17	6
22	0.19	90	1.33	5.0	0.1	1270	0.32	0.37	-16
23	0.12	90	1.7	1.0	0.25	590	0.088	0.096	-9
24	0.12	90	1.7	3.0	0.25	1770	0.16	0.14	12
25	0.095	90	1.65	1.0	0.25	580	0.07	0.076	7
26	0.095	90	1.65	5.0	0.1	1200	0.16	0.18	13

## 5. CONCLUSIONS

The developed computer program for numerical studies of the process of machining permits the determination of the conditions and basic dynamic characteristics of the process of machining, the performance of numerical simulations of the dynamic system, and the presentation of graphic characteristics of the simulations performed.

The simulations performed permit the conclusion that:

- a change of rotary speed causes a change in the time of stabilisation of the transient run,
- a change of the thickness of machined layer causes a corresponding change of response to unit step of feed, at constant time intervals,

- a change of feed rate causes a change in the values of machining force at steady state,
- changes in the phase characteristics are the most observable during changes of rotary speed, while in the case of changes of feed rate and thickness of machined layer the differences are small,
- changes in the amplitude characteristics can be observed mainly in the case of changes in the thickness of machined layer.

The value of relative divergences between the experimental data and data from the simulations,  $\delta$  (%), equal to 9.61%, indicates a good fit of the model to the actual run of the process of turning. The results obtained can be used a starting point for further research in the field of design of automatic adjustment systems.

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