Paper

Multi-Objective Evolutionary Optimization of Aperiodic Symmetrical Linear Arrays

Francesco Napoli[1], Lara Pajewski[2], Roberto Vescovo[3], and Marian Marciniak[4]

[1] *Italian National Agency for New Technologies, Energy and Sustainable Economic Development, Frascati, Italy*
[2] *Department of Information Engineering, Electronics and Telecommunications, Sapienza University of Rome, Rome, Italy*
[3] *Department of Engineering and Architecture, University of Trieste, Trieste, Italy*
[4] *National Institute of Telecommunications, Warsaw, Poland*

https://doi.org/10.26636/jtit.2017.118517

Abstract—In this paper, a multi-objective approach is applied to the design of aperiodic linear arrays of antennas. The adopted procedure is based on a standard Matlab implementation of the Controlled Elitist Non-Dominated Sorting Genetic Algorithm II. Broadside symmetrical arrays of isotropic radiators are considered with both uniform and non-uniform excitations. The work focuses on whether, and in which design conditions, the aperiodic solutions obtained by the adopted standard multi-objective evolutionary procedure can approximate or outperform the Pareto-optimal front for the uniform-spacing case computable by the Dolph-Chebyshev method.

Keywords—antenna array, Dolph-Chebyshev array, genetic algorithms.

1. Introduction

A deeper understanding of evolutionary mechanisms along with an increasing availability of computational resources, allow scientists to simulate natural evolution with computer programs and use it as a new paradigm for problem solving in physics and engineering.

Genetic algorithms (GAs) [1], [2] are robust population-based global-search stochastic iterative methods inspired by the concept of evolution by natural selection. GAs have been successfully applied in many areas to a wide range of engineering problems, and nowadays, they are largely accepted as useful optimization techniques. This is especially true for antenna array design [3], [4].

The typical array design problem consists in finding positions and weight coefficients of the array elements, so that the radiation pattern can satisfy a given set of design specifications [5]. The main design parameters are: gain and directivity, beamwidth (BW) and half-power beamwidth, sidelobe level (SLL), aperture, geometry, robustness, noise sensitivity, bandwidth, dynamic range of current excitations, input and output (radiated) power.

The synthesis techniques for linear arrays can be divided into two main categories: one dealing with uniformly spaced (periodic) arrays and the other with non-uniformly spaced (aperiodic) arrays. The former problem is, at least in some cases, analytically tractable [5], [6]. The latter problem is usually solved by numerical methods [7]–[14].

Aperiodic arrays are very attractive with respect to equally-spaced arrays [10]. A first reason is that the SLL may be improved over the –13.5 dB limit of uniform arrays, while keeping a uniform excitation (provided that a suited number of radiating elements is used and that the average spacing is approximately equal to or less than half wavelength). Second, when dealing with non-uniform excitations it is possible to reduce the amplitude tapering necessary to achieve a required SLL. Third, aperiodic arrays may be realized, in an assigned aperture, by using a reduced number of elements (thinned arrays) with a limited increase of BW and a significant reduction of the array cost. Fourth, by breaking the array periodicity it is possible to improve the bandwidth and reduce the grating lobes in the radiation pattern even if the average spacing is high. Fifth, the mutual coupling between adjacent elements can be reduced thanks to the aperiodicity and to the longer average spacing achievable.

Many numerical methods have been developed to face synthesis problems for aperiodic arrays, including techniques based on mathematical programming, such as constrained [8] and nonlinear [9] programming. Other numerical optimization strategies have also been proposed, based on the synthesis of a density-tapered distribution of uniformly excited elements [7], [12], or of a combined amplitude-density tapered distribution of non-uniformly excited elements [11], approximating a properly chosen continuous source. Stochastic global optimization techniques based on meta-heuristics, such as evolutionary algorithms (GAs, differential evolution), have been successfully applied to the antenna array design problem showing a high flexibility [3], [15]–[27]. In [28], a comparison is presented between different population-based optimization methods applied to the design of scannable circular antenna arrays: genetic algorithms, particle-swarm optimization and the differential evolution method are considered.

In the array design, it is often necessary to simultaneously satisfy two or more conflicting specifications, thus a trade-off between objectives must be found. This is the case where both SLL and BW must be minimized for a given number of array elements. In this case, as it is known, none of those parameters can be improved without worsening the other: if the radiated power is fixed, in the presence of a more directive main beam, there is also more radiation in undesired directions; smaller side lobes, instead, correspond to a larger main beam. Looking for the best trade-off between SLL and BW is a Multi-Objective Optimization Problem (MOOP). For this class of problems, GA-based methods are well-suited procedures, since they are conceived to handle more solutions at the same time [2]. Despite of their proven effectiveness with MOOPs, few research [17], [19]–[21], [23]–[25], [28], [29] treat the array design as an evolutionary MOOP, whereas the problem is often regarded as a single-objective optimization.

This paper deals with the problem of finding an optimal SLL-BW trade-off, for aperiodic linear arrays, using a GA-based technique. In particular, we aim to investigate whether, by using a standard Multi-Objective GA-based (MOGA) procedure, it is possible to synthesize aperiodic linear arrays with a better SLL-BW trade-off with respect to Dolph-Chebyshev [6] periodic arrays.

In [20], [21] Panduro *et al.* employed a standard MOGA procedure called Non-Dominated Sorting Genetic Algorithm II (NSGA-II) [30], to calculate the SLL-BW trade-off curves for linear arrays with uniform and non-uniform spacing. First, they validated NSGA-II in the uniform spacing case by comparing its non-dominated set against the Pareto front computed by using the Dolph-Chebyshev design method [6]. They observed that NSGA-II yields an effective approximation to the front, regardless of the number of array elements. Subsequently, with reference to the non-uniform spacing case they stated [20] that NSGA-II is able to find a non-dominated front that outperforms the Dolph-Chebyshev front for the same number of array elements. The authors considered both the case of non-uniform excitations [20] and that of uniform excitations [21]. They assumed as a benchmark the results obtained by applying the Dolph-Chebyshev method with a uniform half-wavelength spacing. We believe that a more appropriate benchmark be the optimal front for the periodic case, computed by applying the Dolph-Chebyshev method with optimum spacing, i.e. by using the maximum spacing that allows to prevent grating lobes in the physical domain. Accordingly, we here investigate whether by using a standard MOGA procedure it is possible to approximate or outperform the SLL-BW Pareto-optimal front for the uniform spacing case by means of evolved aperiodic solutions.

The paper is organized as follows. In Section 2 the setup of the MOGA procedure is presented and the adopted method is described in detail. Section 3 presents numerical results. In particular, we first recall the main results from our previous work [31] and analyze them more in depth. Subsequently, we further extend our study by using the adopted MOGA procedure to synthesize aperiodic arrays with uniform excitations. Moreover, the optimization of a third objective is performed: the power radiated by the sidelobes. Section 4 concludes the work.

2. The Method

The standard MOGA procedure employed in this work is a Matlab implementation of NSGA-II with Controlled Elitism (CE-NSGA-II). This algorithm shows a better convergence than the original NSGA-II [32]. We use the standard Matlab solver “gamultiobj” with the setup described in [31].

We consider aperiodic linear arrays of isotropic elements symmetrically arranged with respect to the array center, with a real symmetrical distribution of the excitations. Thus, the main beam will be directed to broadside. There are two reasons for this choice. First, symmetrical excitations make the design of the feed network and the compensation of the mutual coupling effects easier. Second, numerical results can be compared with broadside symmetrical Dolph-Chebyshev arrays.

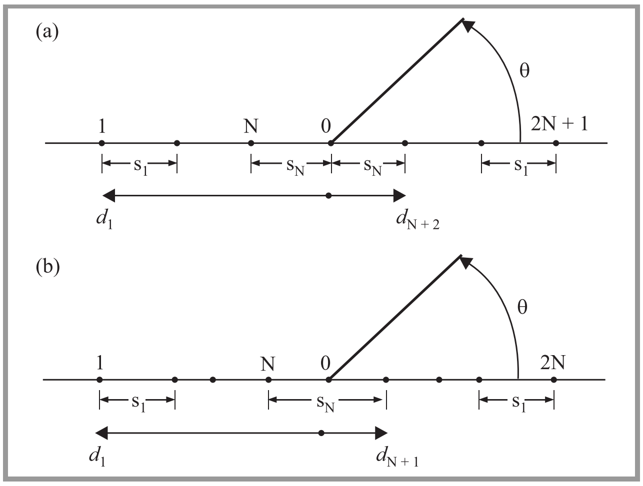

Fig. 1. Array geometry: (a) odd number $M = 2N+1$ of elements, (b) even number $M = 2N$ of elements.

Geometry and notations are described in Fig. 1. When the total number of isotropic elements is $M = 2N+1$ (Fig. 1a) the array factor is given by:

$$F(\theta, \mathbf{I}, \mathbf{D}) = \sum_{i=1}^{2N+1} I_i \, \mathrm{e}^{j\,\mathrm{sign}(i-(N+1))k d_i \cos\theta}, \qquad (1)$$

where $\mathrm{sign}(x) = 1$ if $x > 0$ and -1 otherwise, θ is the angle between the direction of observation and the array axis, $k = \frac{2\pi}{\lambda}$ is the wavenumber (λ being the radiation wavelength), $\mathbf{I} = [I_1, I_2, \ldots, I_{2N+1}]$ is the vector of current excitations, and $\mathbf{D} = [d_1, d_2, \ldots, d_{2N+1}]$ is the vector of distances between each array element and the array center. When the total number of isotropic elements is $M = 2N$ (Fig. 1b), the array factor is given by:

$$F(\theta, \mathbf{I}, \mathbf{D}) = \sum_{i=1}^{2N} I_i \, \mathrm{e}^{j\,\mathrm{sign}(i-N)k d_i \cos\theta}, \qquad (2)$$

where $\mathbf{I} = [I_1, I_2, \ldots, I_{2N}]$ is the vector of excitation currents and $\mathbf{D} = [d_1, d_2, \ldots, d_{2N}]$ is the vector of the distances between each array element and the array center.

In the genetic representation, each individual is specified by two vectors: the vector $\mathbf{I}$ of the current excitations and the vector $\mathbf{S} = [s_1, s_2, \ldots, s_{M-1}]$ of the separations between adjacent elements. By exploiting the array symmetry, the length of these two vectors can be roughly halved. More precisely, the two vectors can be re-defined as follows: $\mathbf{I} = [I_1, I_2, \ldots, I_N]$ if $M = 2N$, $\mathbf{I} = [I_1, I_2 \ldots, I_{N+1}]$ if $M = 2N+1$, and $\mathbf{S} = [s_1, s_2, \ldots, s_N]$ in both cases.

The array design problem is modeled as a bound constraint MOOP, namely as a minimization one.

We first deal with two design objectives: SLL and BW. In such a case, for the design of aperiodic non-uniformly excited linear arrays with M isotropic elements, we have the following formal MOOP statement: minimize (f_1, f_2) subject to $\mathbf{I} \in \Delta_1$ and $\mathbf{S} \in \Delta_2$, where the objective fitness functions are $f_1 = \text{SLL}$ and $f_2 = \text{BW}$, $\Delta_1 = [0,1]^L$ and $\Delta_2 = [0.5\lambda, s_{\max}]^L$, where $L = N$ if $M = 2N$ and $L = N+1$ if $M = 2N+1$. The $s_{\max}$ represents the maximum allowed separation between adjacent elements (also called, spacing). For what pertains the amplitude of the current excitations, as the input power is not assigned, only normalized amplitude values are of interest. Hence, we limit them to the dimensionless range $[0, 1]$. Moreover, to neglect mutual coupling effects, we bound the minimum spacing to 0.5λ.

Subsequently, an additional design objective is considered: the power radiated by the sidelobes (SLP) is minimized. The formal MOOP statement in this case becomes: minimize (f_1, f_2, f_3) subject to $\mathbf{I} \in \Delta_1$ and $\mathbf{S} \in \Delta_2$, where the objective fitness functions are $f_1 = \text{SLL}$, $f_2 = \text{BW}$ and $f_3 = \text{SLP}$.

3. Results and Discussion

The used benchmark is represented by the trade-off SLL-BW curves obtained by applying the Dolph-Chebyshev method with optimum period (in the following referred to as the Dolph-Chebyshev front).

It is apparent that for a fixed M, according to the Pareto criterion of dominance [2], the Dolph-Chebyshev front dominates the front of solutions adopted in [20], [21] and characterized by a half-wavelength period.

In [31], it was shown that the evolved non-dominated solutions from [20] do not outperform the right benchmark. Except for a bit inaccuracy due to the numerical nature of the method, the MOGA's non-dominated periodic solutions converge to optimum Dolph-Chebyshev solutions. The genetic procedure synthesizes approximations of the Dolph-Chebyshev front not only at objective value levels (SLL and BW) but also at design variable levels (currents and separations).

In [31], we also started the analysis of the non-uniform spacing case. The CE-NSGA-II was used to synthesize aperiodic non-uniformly excited linear arrays, for different values of M, in two test cases: separations belonging to the interval $[0.5\lambda, \lambda]$, as in [20], [21], and separations belonging to $[0.5\lambda, 5\lambda]$. The latter case was considered because aperiodic arrays can control the grating lobes in the radiation pattern, even if the average spacing is large. As a result, it was shown that the evolved non-dominated solutions can outperform Dolph-Chebyshev solutions when separations belong to $[0.5\lambda, 5\lambda]$, i.e. when the MOGA procedure can fully exploit the aperiodicity to control the grating lobes. Unfortunately, this happens only when SLL > –13 dB. It was also observed that, in the outperforming region, evolved fronts cross Pareto-optimal fronts with a higher number of elements. This behavior is known in the literature as array thinning [10], [33]. With an aperiodic array it is possible to obtain almost the same SLL-BW trade-off as a periodic array involving a higher number of elements.

Let us now study more in depth the non-uniform spacing case.

Figure 2 shows the evolved front obtained for aperiodic non-uniformly excited arrays, with $M = 12$ and separations belonging to the interval $[0.5\lambda, 5\lambda]$ (dotted line with circles). In the same figure, the Pareto-optimal front is reported (solid line, Dolph-Chebyshev). The evolved trade-off curve between SLL and BW was computed over data collected in 5 consecutive runs of the CE-NSGA-II. It can be appreciated that evolved solutions outperform Dolph-Chebyshev solutions when SLL is higher than nearly –13 dB as mentioned before, and that for lower SLL values the evolved front follows the Dolph-Chebyshev front (a worse approximation is observed for SLL lower than nearly –22 dB).

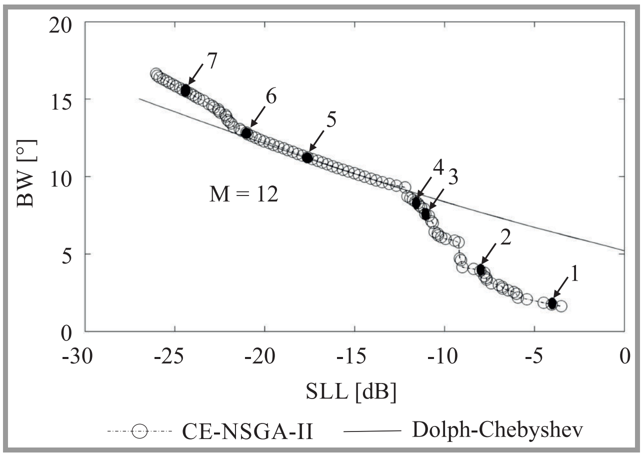

Fig. 2. SLL-BW trade-off curves computed over data collected in 5 consecutive runs of the CE-NSGA-II, with separations in $[0.5\lambda, 5\lambda]$ against the Dolph-Chebyshev front computed by means of the Dolph-Chebyshev method with optimum period.

For seven solutions of the evolved front, Fig. 3 shows array factors, excitation amplitudes, and spacing distributions. Each row refers to a different solution, identified by the relevant sampling number. The first column reports array factors, with angles in radians and amplitudes normalized to the peak value. The second column shows array current distributions, with amplitudes normalized to the maximum current. The third column shows array spacing distributions (normalized to λ). A horizontal solid line indicates a separation of one wavelength.

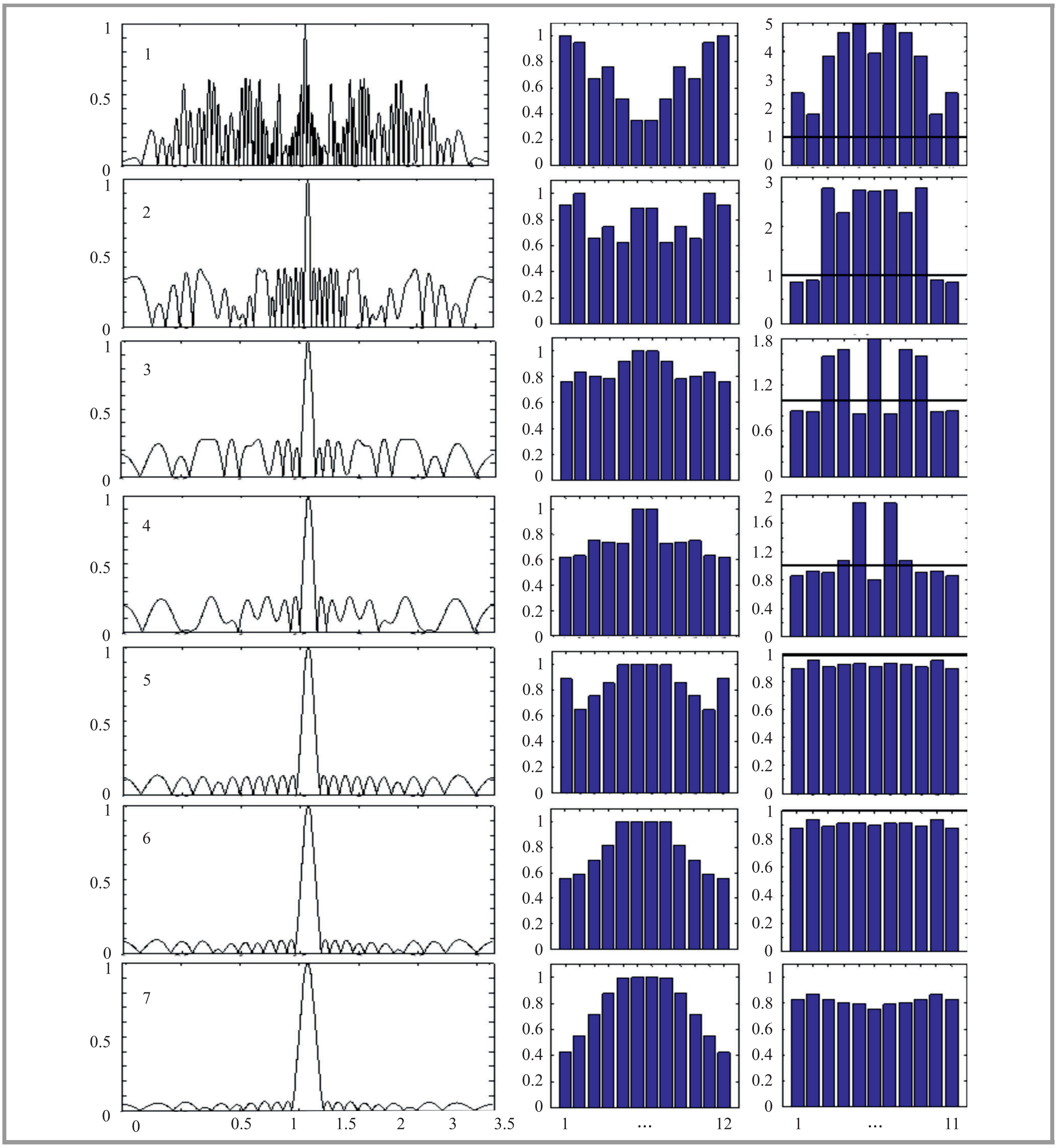

Fig. 3. Study of seven non-dominated solutions, sampled over the evolved front of Fig. 2. Each row refers to one of those solutions. The first column illustrates array factors (amplitudes, normalized to the peak value, as a function of angle in radians). The second column shows array current distributions, with amplitudes normalized to the maximum current. The third column shows array spacing distributions (normalized to λ). A horizontal solid line indicates a separation of one wavelength.

Observing rows 1 to 5 in Fig. 3, a transition can be noted from solutions with an aperture wider than the corresponding Dolph-Chebyshev solutions, to solutions with a comparable aperture. Solution 1 has all separations greater than one wavelength, and a number of zeros in the visible space much greater than the 22 zeros placed in the visible space by the Dolph-Chebyshev method. This result highlights the capability of aperiodic arrays of controlling the grating lobes in the visible space at the expense of an SLL value, which is only –4.23 dB. From row 1 to 4 a lowering of the side-lobes is observed as well as an increase of the beam width. Furthermore, a reduction of the number of zeros in the visible space occurs.

Solution 5 is an aperiodic approximation of a Dolph-Chebyshev array. The number of zeros in the visible space is 22, the side-lobe pattern is almost uniform, all separations are smaller than one wavelength, and the mean spacing is very close to the optimum period. From row 5 to 6 a transition occurs toward tapered current distributions, thus achieving lower SLL values.

Finally, the Solution 7 witnesses the deviation of the evolved front from the Dolph-Chebyshev one. The sidelobe pattern becomes more irregular, losing two zeros in visible space. The aperture gets smaller than the corresponding Dolph-Chebyshev solution, with an average separation shorter than the optimal period. The current distribution is still tapered to match the low SLLs. Thus, in this low SLLs region the adopted MOGA procedure reveals to be not able to efficiently exploit the array aperture, by putting all zeros in visible space to shrink the beam width.

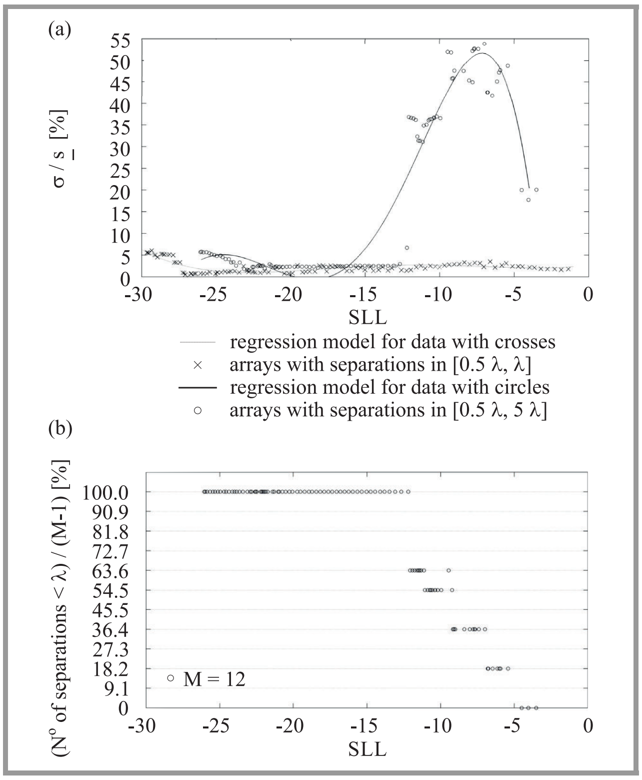

Fig. 4. Analysis of the distribution of separations obtained in the evolved solutions with $M = 12$ non-uniformly excited elements: (a) ratio between standard deviation and average spacing for each evolved solution as a function of SLL, (b) number of separations shorter than λ vs. SLL for evolved solutions with separations in $[0.5\lambda, 5\lambda]$.

The spacing distributions of the presented evolved non-dominated aperiodic solutions are further studied in Fig. 4. In Fig. 4a, the ratio $\frac{\sigma}{\bar{s}}$ between the standard deviation σ and the average separation $\bar{s}$ is plotted as a function of SLL for each evolved solution. Circles and crosses represent solutions with separations in $[0.5\lambda, 5\lambda]$ and $[0.5\lambda, \lambda]$, respectively. The corresponding regression models are in solid and dash-dotted line, respectively. This figure shows that the MOGA procedure fully exploits the aperiodicity of evolved arrays only in the region where the evolved front outperforms the Dolph-Chebyshev front. In Fig. 4b, the number of separations shorter than λ is plotted as a function of SLL, for evolved solutions with separations in $[0.5\lambda, 5\lambda]$. It can be argued that the outperforming occurs in a region of the SLL-BW plane where the MOGA procedure synthesizes a larger array aperture to shrink the BW, with separations longer than one wavelength. In fact, going towards SLLs smaller than –13 dB, we notice a transition of evolved solutions from arrays with all (or most) separations longer than one wavelength (right side of Fig. 4b) towards arrays with all separations shorter than one wavelength (left side of Fig. 4b). When SLL $<$ –13 dB, it is not possible to find aperiodic radiating configurations with apertures significantly larger than the corresponding Dolph-Chebyshev solutions, taking simultaneously under control the entrance of grating lobes in visible space.

Here a special case of symmetrical excitations is considered, which can highly simplify the design and realization of the feed network – the uniform current distribution. The CE-NSGA-II is used to synthesize aperiodic uniformly excited linear arrays for different values of M. The setup of the algorithm is the same as that of the previous experiment, except for the vector of current excitations, that now is $\mathbf{I} = [1]^L$ with L as in Section 2. The objectives are, once

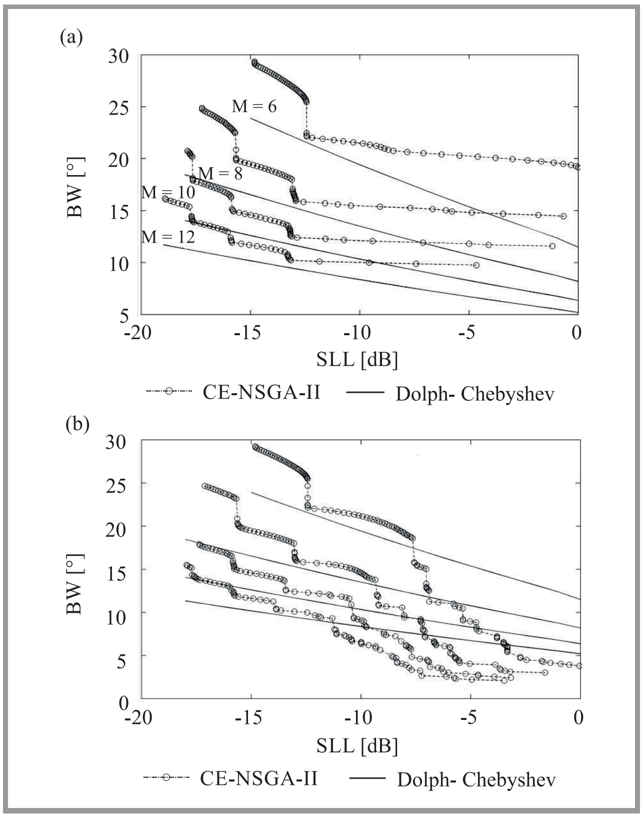

Fig. 5. SLL-BW trade-off curves, computed over data collected in 5 consecutive runs of CE-NSGA-II with separations in: (a) $[0.5\lambda, \lambda]$, (b) $[0.5\lambda, 5\lambda]$, against the Pareto-optimal fronts computed with the Dolph-Chebyshev method with optimum period.

again, SLL and BW. We distinguish two test cases: in the first case the separations belong to the interval $[0.5\lambda, \lambda]$, in the second one they belong to $[0.5\lambda, 5\lambda]$.

In Figs. 5a-b the numerical results obtained in the first and second test case are shown, respectively. In particular, in Fig. 5a the SLL-BW trade-off curves are plotted, computed over data collected in 5 consecutive runs of CE-NSGA-II (dash-dotted line) with separations in $[0.5\lambda, \lambda]$. The Pareto-optimal fronts computed with the Dolph-Chebyshev method with optimum period are also plotted (solid line). Figure 5b refers to the case where the separations are in the $[0.5\lambda, 5\lambda]$ interval.

As in the previous experiment, Fig. 5 shows that the presented evolved non-dominated solutions can outperform Dolph-Chebyshev solutions only when the separations belong to $[0.5\lambda, 5\lambda]$. Again, this happens in a region of the SLL-BW plane characterized by SLL > -13 dB. In both cases it is possible to observe an interesting partial overlapping of an evolved front with the Dolph-Chebyshev front relevant to solutions with slightly fewer elements.

This result is of practical interest, since it can be exploited to achieve the same array performances as non-uniformly excited arrays with a slightly greater number of radiating elements, but with uniform excitations. Moreover, when separations belong to $[0.5\lambda, 5\lambda]$, it can be noted an array-thinning effect in crossing points between evolved fronts and the Dolph-Chebyshev fronts with a higher number of elements (see Fig. 5b).

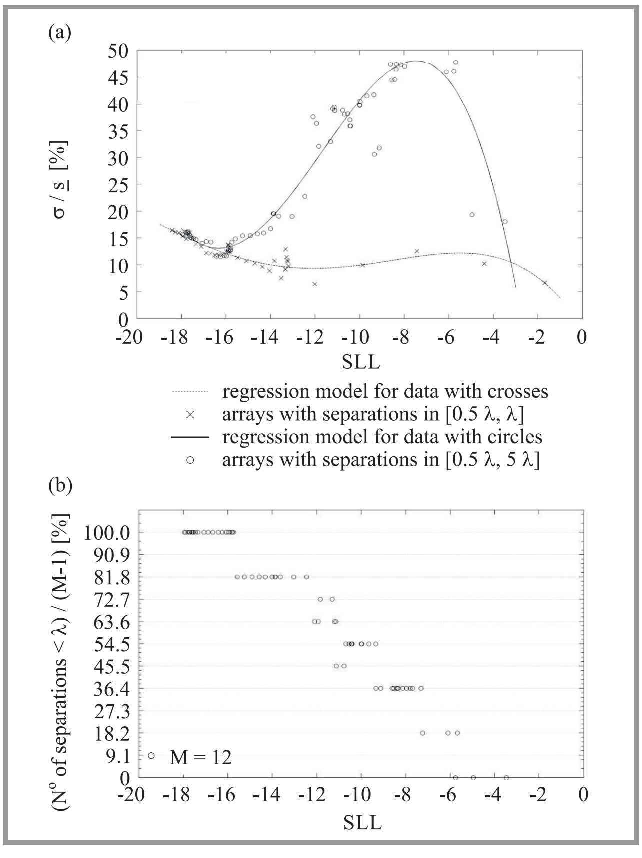

Fig. 6. Same as in Fig. 4 but with uniform excitations.

It is interesting to analyze the spacing distributions of presented evolved solutions. In Fig. 6, the same as in Fig. 4 is reported, but with uniform array excitations. It can be observed that the MOGA procedure yields solutions with a higher exploitation of aperiodicity with respect to the case studied in Fig. 4, with an enhancement of this behavior in the region where the evolved front outperforms the Dolph-Chebyshev fronts. Again, this happens for a selection of arrays with greater array apertures with respect to the corresponding Dolph-Chebyshev solutions.

Finally, we use CE-NSGA-II to synthesize aperiodic non-uniformly excited linear arrays with minimal radiated power in unwanted directions. For this purpose, the power radiated in sidelobes normalized to the power radiated in the main lobe is considered:

$$SLP = \frac{P_{SL}}{P_{ML}}, \tag{3}$$

where P_{SL} is the radiated power in side-lobes and P_{ML} is the radiated power in the main lobe. Therefore, the objectives to be minimized are now three: SLL, BW and SLP.

In Fig. 7a, non-dominated solutions with $M = 12$ are shown, computed over data collected in 5 consecutive runs of CE-NSGA-II with 3 objectives (circles). Some representative solutions obtained by the Dolph-Chebyshev method with optimum period are also reported (black triangles) for SLL = –5, –10, –15, –20, –25 and –30 dB. Figure 7b shows a comparison between the SLL-BW trade-off curves computed by CE-NSGA-II with 2 (dash-dotted line with black dots) and 3 (dash-dotted line with circles) objectives. The latter curve is the result of the application of the Pareto dominance criterion over data obtained from a projection of the tridimensional front (shown in Fig. 7a) on over the SLL-BW plane. For comparison, the Dolph-Chebyshev front is also reported (solid line).

It is apparent that the introduction of the third objective helps the MOGA procedure to converge towards fronts with a better spread [27]. In fact, the fronts are calculated over a wider range of SLLs. This is due to the synergy of the third objective (SLP) with the first one (SLL) for solutions approaching the Dolph-Chebyshev front, i.e. having almost the same BW.

In Fig. 8, a comparison is presented between the array factor of one evolved solution, sampled very close to the Dolph-Chebyshev front, and the array factor of the corresponding solution on the Dolph-Chebyshev front. The sidelobe radiation pattern of the evolved solution appears irregular and tapered to keep minimized the SLP. The evolved array considered in Fig. 8 has about the 33% of the SLP corresponding to a uniform sidelobe pattern.

Examining the current distributions, similar tapering strategies as in the Dolph-Chebyshev solutions are observed when SLL < -13 dB. In Fig. 9 the same as in Fig. 4 is reported, but with three objectives, and separations in the $[0.5\lambda, 5\lambda]$ interval. In the outperforming region, there is a greater exploitation of aperiodicity with respect to the solutions computed with two objectives (and with separations in $[0.5\lambda, 5\lambda]$). The higher aperiodicity in this SLL

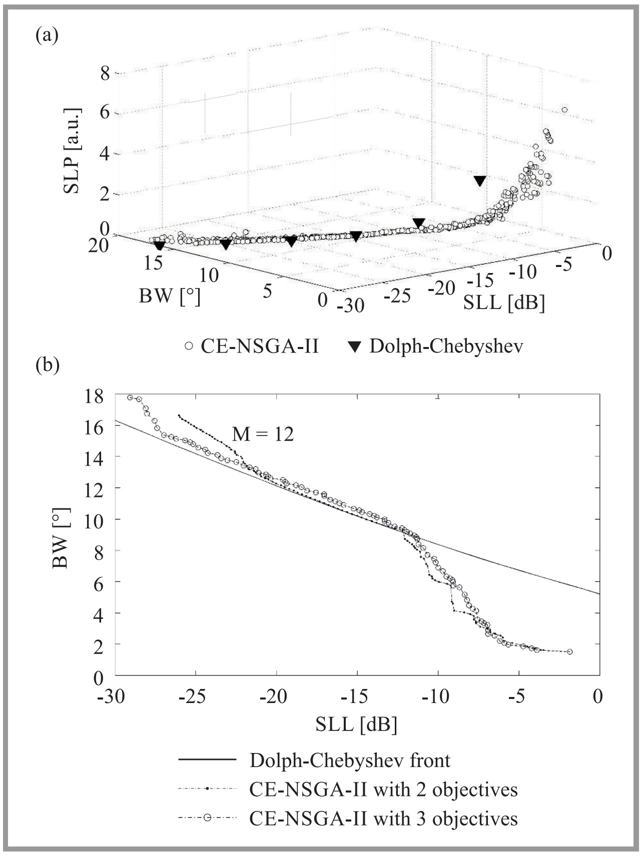

Fig. 7. Numerical result for: (a) non-dominated solutions with $M = 12$ computed over data collected in 5 consecutive runs of CE-NSGA-II with 3 objectives, (b) comparison of the SLL-BW trade-off curves computed by CE-NSGA-II with 2 and 3 objectives. The Dolph-Chebyshev front for $M = 12$ is also reported.

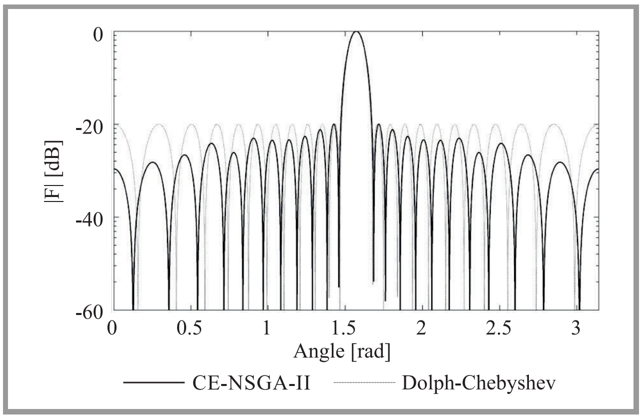

Fig. 8. Array factors for $M = 12$ and SLL = –20 dB of one evolved solution obtained by means of CE-NSGA-II working with three objectives (solid line) and of the corresponding Dolph-Chebyshev solution (dashdot line).

region is exploited by the MOGA procedure to minimize the radiated power in the reduced grating lobes, that are visible due to the larger array aperture.

These results confirm that now we can approximate the Dolph-Chebyshev front with aperiodic arrays of different nature, and precisely aperiodic linear arrays with optimal SLL-BW-SLP trade-off. It is worth to note that, as shown

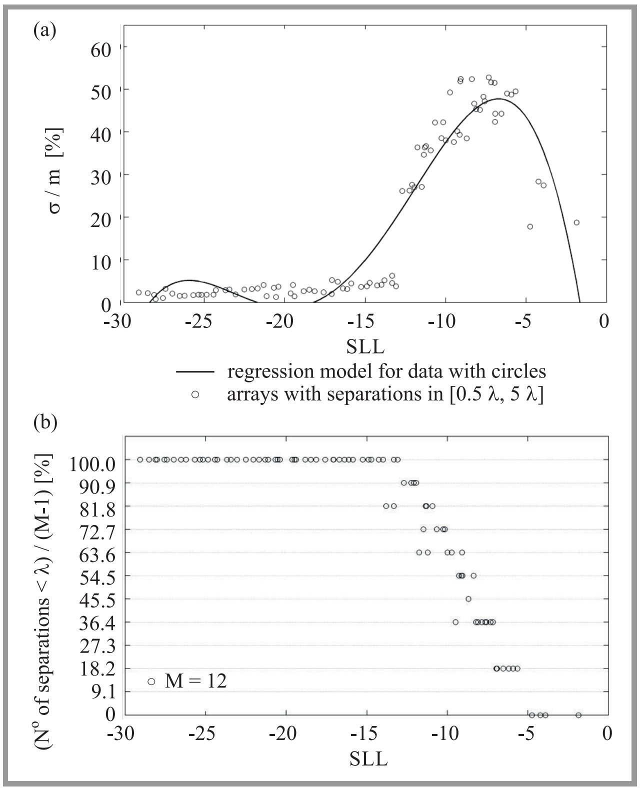

Fig. 9. Same as in Fig. 6 when the objectives are three and separations are in the $[0.5\lambda, 5\lambda]$ interval.

in Fig. 8a, the advantage over Dolph-Chebyshev solutions in the reduction of radiated power in unwanted directions gets smaller when SLL increases.

4. Conclusions

The main purpose of this work was to experimentally verify the capability of a standard Multi-Objective GA-based (MOGA) procedure to synthesize aperiodic linear arrays of antennas with a better trade-off between side-lobe level (SLL) and main beam width (BW) with respect to Dolph-Chebyshev arrays with optimum period. To this aim, we considered symmetrical broadside arrays with isotropic radiators, focusing on the problem of optimizing SLL and BW simultaneously.

The adopted procedure is based on a standard Matlab implementation of the so-called Controlled Elitist Non-Dominated Sorting Genetic Algorithm II (CE-NSGA-II).

First, we recalled numerical results obtained in [31] for aperiodic non-uniformly excited linear arrays. Those results furnish a better interpretation of CE-NSGA-II behavior, compared to results obtained by other authors in previous works, where Dolph-Chebyshev solutions with half-wavelength period were inappropriately utilized as a benchmark. We adopted as a benchmark the SLL-BW trade-off curves obtained by applying the Dolph-Chebyshev method with an optimum period, here defined as the Dolph-Chebyshev front. In the present paper, we demonstrated that the evolved aperiodic solutions obtained by the adopted MOGA procedure, as well as the aperiodic ones from [20], [21], cannot outperform Dolph-Chebyshev solutions. They only yield an aperiodic approximation of the Dolph-Chebyshev front, as long as the separations belong to the $[0.5\lambda, \lambda]$ interval.

As a further step, we allowed the MOGA procedure to fully exploit the capability of aperiodic arrays to take under control grating lobes in the visible space by considering a wider range of separations, i.e. the $[0.5\lambda, 5\lambda]$ interval. In this case, SLL-BW trade-off curves of evolved non-dominated solutions outperform the Dolph-Chebyshev front in a narrow region of the SLL-BW plane characterized by SLL $>$ –13 dB. We also analyzed results with respect to the spacing distribution: a higher exploitation of the aperiodicity was observed by the adopted MOGA procedure in the region where the Dolph-Chebyshev front is outperformed. In other regions, as long as the evolved front follows the Dolph-Chebyshev front, the MOGA procedure provides aperiodic approximations of Dolph-Chebyshev solutions. The obtained results suggest that the evolved arrays can outperform the Dolph-Chebyshev front thanks to a greater aperture, which they exploit – by using a higher aperiodicity – to take the grating lobes under control. An extrapolation of the data set constituted by the first non-outperforming points of the evolved fronts with respect to the corresponding Dolph-Chebyshev fronts for each number of radiating elements considered in our previous experiments [31] (shown in Fig. 10 of the present paper) suggests that, by further increasing the number of radiating elements, this behavior can be exploited only until a SLL limit of about –13.2 dB.

Subsequently, we studied the attractive case of aperiodic arrays with uniform excitations. As in the previous experiment, the evolved non-dominated solutions outperform Dolph-Chebyshev solutions only when separations are longer than one wavelength, i.e. only when the adopted MOGA procedure can synthesize aperiodic arrays with a higher aperture and, thanks to a higher aperiodicity, achieve a better control of the grating lobes. Again, this happens in a region of the SLL-BW plane characterized by SLL $>$ –13 dB. In all other cases, the evolved non-dominated solutions belong to discontinuous fronts. We also observed an interesting overlapping of evolved fronts with Dolph-Chebyshev fronts with slightly fewer radiating elements, which could be interesting for practical applications. By analyzing the spacing distributions of the evolved non-dominated solutions, we noticed that the adopted MOGA procedure provides solutions with a higher level of aperiodicity with respect to the previous experiment, with an enhancement of such behavior in the region where evolved fronts outperform Dolph-Chebyshev fronts. This was to be expected because, by assuming a uniform current distribution, we have reduced the degrees of freedom of the problem, so that the optimizing procedure can exploit only the spacing distribution.

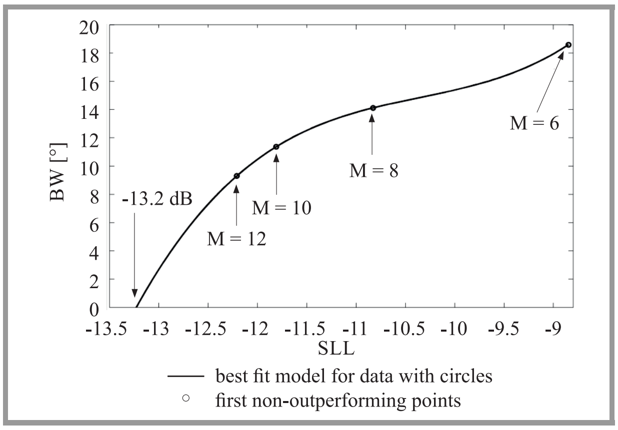

Fig. 10. Extrapolation analysis of the first non-outperforming points for different numbers of radiating elements. The data set is obtained by the evolutionary synthesis of aperiodic arrays with non-uniform excitations and with separations in the $[0.5\lambda, 5\lambda]$ interval. The figure shows (with circles) the first non-outperforming points for aperiodic arrays with $M = 6$, 8, 10, 12 radiating elements and (with a solid line) the extrapolation polynomial obtained by interpolation of the collected data.

In the last part of the paper, we extended the presented multi-objective approach and introduced a third objective: the minimization of the side-lobe power (SLP). We compared the obtained results with the previous ones, calculated with only two objectives (SLL and BW). The introduction of the third objective helps the MOGA procedure to converge to fronts with a better spread and to find new interesting solutions with irregular sidelobes, when SLL $<$ –13 dB, with radiation patterns different from those of Dolph-Chebyshev solutions, but very close to them in the SLL-BW plane.

In conclusion, the numerical results reported in this paper shed light on whether and in which design conditions, it is possible to synthesize, by using a standard MOGA procedure, aperiodic broadside symmetrical linear arrays with a SLL-BW trade-off competitive with optimal periodic Dolph-Chebyshev solutions.

Acknowledgements

This work benefited from the networking activities carried out in the COST (European Cooperation in Science and Technology) Actions MP0702 “Towards Functional Sub-Wavelength Photonic Structures” (cost-mp0702.nit.eu) and TU1208 “Civil engineering applications of Ground Penetrating Radar” (www.GPRadar.eu). The authors are grateful to COST (www.cost.eu) for funding and supporting the Actions MP0702 and TU1208.

References

[1] J. H. Holland, *Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control, and Artificial Intelligence*. Michigan: University of Michigan Press, 1975.

[2] A. E. Eiben and J. E. Smith, *Introduction to Evolutionary Computing*, Natural Computing Series. Heidelberg, Germany: Springer, 2003.

[3] R. Haupt, “Thinned arrays using genetic algorithms”, *IEEE Trans. on Antenn. and Propag.*, vol. 2, no. 7, pp. 712–715, 1994 (doi: 10.1109/8.299602).

[4] J. M. Johnson and Y. Rahmat-Samii, “Genetic algorithms in engineering electromagnetics”, *IEEE Antenn. and Propag. Mag.*, vol. 39, no. 4, pp. 7–21, 1997.

[5] C. Balanis, *Antenna Theory: Analysis and Design*, 3rd ed. Hoboken, New Jersey: Wiley, 2005.
[6] C. L. Dolph, "A current distribution for broadside arrays which optimizes the relationship between beam width and side-lobe level", *Proc. of the IRE*, vol. 34, no. 6, pp. 335–348, 1946 (doi: 10.1109/JRPROC.1946.225956).
[7] W. Doyle, "On approximating linear array factors", RAND Corporation Research Memoranda, Article ID RM-3530-PR, 1963.
[8] M. H. Er, S. L. Sim, and S. N. Koh, "Application of constrained optimization techniques to array pattern synthesis", *Sig. Process.*, vol. 34, no. 3, pp. 323–334, 1993.
[9] Y. C. Jiao, W. Y. Wei, L. W. Huang, and H. S. Wu, "A new low-side-lobe pattern synthesis technique for conformal arrays", *IEEE Trans. on Antenn. and Propag.*, vol. 41, no. 6, pp. 824–830, 1993.
[10] G. Toso, C. Mangenot, and A. G. Roederer, "Sparse and thinned arrays for multiple beam satellite applications", in *Proc. 29th ESA Antenna Worksh.*, Noordwijk, The Netherlands, 2007.
[11] P. Angeletti and G. Toso, "Aperiodic arrays for space applications: A combined amplitude/density synthesis approach", in *Proc. 3rd Eur. Conf. on Antenn. and Propag. EuCAP 2009*, Berlin, Germany, 2009.
[12] O. M. Bucci, M. D'Urso, T. Isernia, P. Angeletti, and G. Toso, "Deterministic synthesis of uniform amplitude sparse arrays via new density taper techniques", *IEEE Trans. on Antenn. and Propag.*, vol. 58, no. 6., pp. 1949–1958, 2010.
[13] G. Olivieri, M. Donelli, and A. Massa, "Linear array thinning exploiting almost difference sets", *IEEE Trans. on Antenn. and Propag.*, vol. 57, no. 12, pp. 3800–3812, 2009.
[14] G. Olivieri, P. Rocca, and A. Massa, "Analytic techniques for the design of non-regular arrays", in *Proc. IEEE Int. Symp. on Phased Array Syst. and Technol.*, Waltham, MA, USA, 2010 (doi: 10.1109/ARRAY.2010.5613281).
[15] K. K. Yan and Y. Lu, "Side lobe reduction in array-pattern synthesis using genetic algorithm", *IEEE Trans. on Antenn. and Propag.*, vol. 45. no. 7, pp. 1117–1122, 1997.
[16] F. J. Ares-Pena, J. A. Rodriguez-Gonzalez, E. Villanueva-Lopez, and S. R. Rengarajan, "Genetic algorithms in the design and optimization of antenna array patterns", *IEEE Trans. on Antenn. and Propag.*, vol. 47, no. 3, pp. 506–510, 1999.
[17] D. Ansell and E. J. Hughes, "Use of multi-objective genetic algorithms to optimize the excitation and subarray division of multifunction radar antennas", in *Proc. IEE Multifunc. Radar and Sonar Sensor Manag. Techniq.*, London, UK, 2001, vol. 8, pp. 1–4 (doi: 10.1049/ic:20010185).
[18] M. G. Bray, D. H. Werner, D. W. Boeringer, and D. W. Machuga, "Optimization of thinned aperiodic linear phased arrays using genetic algorithms to reduce grating lobes during scanning", *IEEE Trans. on Antenn. and Propag.*, vol. 50, no. 12, pp. 1732–1742. 2002.
[19] D. Ansell and E. J. Hughes, "Using multi-objective genetic algorithms to optimise the subarray partitions of conformal array antennas", *IEEE Trans. on Antenn. and Propag.*, vol. 52, pp. 151–155, 2004.
[20] M. A. Panduro, C. A. Brizuela, D. H. Covarrubias, and C. Lopez, "A multi-objective approach in the linear antenna array design", *Int. J. of Electron. and Commun.*, vol. 59, no. 4, pp. 205–212, 2004.
[21] M. A. Panduro, D. H. Covarrubias, C. A. Brizuela, and F. R. Marante, "A trade-off curve computation for linear antenna arrays using an evolutionary multi-objective approach", *Soft Comput.*, vol. 10, no. 2, pp. 125–131, 2005.
[22] K. Cheng, X. Yun, Z. He, and C. Han, "Synthesis of sparse planar arrays using a modified real genetic algorithm", *IEEE Trans. on Antenn. and Propag.*, vol. 55, no. 4, pp. 1067–1073, 2007.
[23] M. A. Panduro, C. A. Brizuela, and D. H. Covarrubias, "Design of electronically steerable linear arrays with evolutionary algorithms", *Appl. Soft Comput.*, vol. 8, no. 1, pp. 46–54, 2008 (doi: 10.1016/j.asoc.2006.10.011).
[24] M. A. Panduro, D. H. Covarrubias, and A. L. Mendez, "Design of phased antenna arrays using evolutionary optimization techniques", in *Advances in Evolutionary Algorithms*, W. Kosinski, Ed. Vienna, Austria: I-Tech Education and Publishing, 2008, pp. 361–376.
[25] F. Tokan and F. Gunes, "The multi-objective optimization of non-uniform linear phased arrays using the genetic algorithm", *Progr. in Electromag. Res. B*, vol. 17, pp. 135–151, 2009.
[26] S. Pal, B. Y. Qu, S. Das, and P. N. Suganthan, "Optimal synthesis of linear antenna arrays with multi-objective differential evolution", *Progr. in Electromag. Res. B*, vol. 21, pp. 87–111, 2010.
[27] S. Pal, S. Das, and A. Basak, "Design of time-modulated linear arrays with a multi-objective optimization approach", *Progr. in Electromag. Res. B*, vol. 23, pp. 83–107, 2010.
[28] M. A. Panduro, C. A. Brizuela, L. I. Balderas, and D. A. Acosta, "A comparison of genetic algorithms, particle swarm optimization and the differential evolution method for the design of scannable circular antenna arrays", *Progr. in Electromag. Res. B*, vol. 13, pp. 171–186, 2009.
[29] M. A. Panduro, C. A. Brizuela, and S. Jesus Garza, "A comparison of NSGA-II, DEMO, and EM-MOPSO for the multi-objective design of concentric rings antenna arrays", *J. of Electromag. Waves and Appl.*, vol. 27, no. 9, pp. 1110–1113, 2013.
[30] K. Deb, S. Agrawal, A. Pratap, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II", *IEEE Trans. on Evol. Comput.*, vol. 6, no. 2, pp. 182–197, 2002.
[31] F. Napoli, L. Pajewski, G. Schettini, and R. Vescovo, "On a multi-objective approach in the non-uniform symmetrical linear antenna array design", in *Proc. 5th Eur. Conf. on Antennas and Propag. EuCap 2011*, Rome, Italy, 2011.
[32] K. Deb and T. Goel, "Controlled elitist non-dominated sorting genetic algorithm for better convergence", in *Evolutionary Multi-Criterion Optimization*, A. Gaspar-Cunha, C. Henggeler Antunes, C. A. Coello Coello, Eds. *LNCS*, vol. 1993, pp. 67–81. Springer, 2001.
[33] X. Chen, Z. Luo, X. He, and L. Zhu, "Thinning and weighting of planar arrays by modified teaching-learning-based optimization algorithm", *J. of Electromag. Waves and Appl.*, vol. 28, no. 15, pp. 1924–1934, 2014.

Francesco Napoli holds the Ph.D. in Electromagnetics and a M.Sc. in Electronic Engineering, both from Roma Tre University, Italy. Since 2006 he is involved in the study of evolutionary algorithms and in their applications to electronic design automation. Since 2009 he is collaborating with the Department of Engineering of Roma Tre University and with the ENEA Research Center of Frascati for the RF heating of magnetically confined hot plasmas relevant for nuclear fusion. His current research interests include multi-objective evolutionary techniques applied to the design of electromagnetic devices, aperiodic arrays, traveling wave tubes, computational electromagnetics, mathematical modeling, machine learning, symbolic regression, complex system theory, plasma physics, and nonlinear plasma interactions of microwaves with artificial hot plasmas.

E-mail: ganimede55@hotmail.com
Italian National Agency for New Technologies, Energy and Sustainable Economic Development (ENEA)
Via Enrico Fermi 45
00044 Frascati (RM), Italy

Lara Pajewski, **Roberto Vescovo** – for biographies, see this issue, p. 29.

Marian Marciniak – for biography, see this issue, p. 35.