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Reliability of production machines in the bakery industry – theoretical and practical issues

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ABSTRACT:

The aim of the article is to present probabilistic models, which were then used to analyse the reliability of production machines in the bakery industry. The author conducted research in the period from 2 January 2016 to 31 December 2018 regarding the measurement of reliability of a traditional production system based on a probabilistic concept. Due to the limited scope of the article, reliability calculations for a set of thermal and oil ovens with a graph of failure intensity in the years of 2006-2018 and the predicted distribution of intensity functions in the years of 2019-2026 have been presented.

Niezawodność maszyn produkcyjnych w branży piekarniczej – zagadnienia teoretyczne i praktyczne

Słowa kluczowe: niezawodność maszyn produkcyjnych, modele probabilistyczne

STRESZCZENIE:

Celem artykułu jest przedstawienie modeli probabilistycznych, które następnie zostały wykorzystane do analizy niezawodności maszyn produkcyjnych w branży piekarniczej. Autor prowadził badania w okresie 2016.01.02-2018.12.31 dotyczące opomiarowania niezawodności systemu produkcyjnego tradycyjnego, bazującego na koncepcji probabilistycznej. Z uwagi na ograniczony zakres artykułu zostały przedstawione obliczenia niezawodności dla zespołu pieców termo-olejowych z wykresem intensywności uszkodzeń w latach 2006-2018 oraz przewidywanym rozkładem funkcji intensywności w latach 2019-2026.

1. INTRODUCTION

Micro and small companies in the bakery industry (MSCBI) are crucial for economic development in Poland. They represent more than 95% (GUS 2019) of all operating bakeries in Poland. They are essential for the sustainable functioning of the economy and contribute to accelerating economic growth.

The fourth industrial revolution and the increased pressure of competitiveness on the part of the market have resulted in an unprecedented technical and technological progress, both at the level of technological machinery, auxiliary operations equipment, as well as the communication and control systems of the generating systems, becoming an important element having an impact on the direction of development of MSCBI.

The increasing level of competitiveness in the bakery industry causes a constant increase in demand for solutions increasing the reliability of all production processes (machines and equipment) of MSCBI, which allows for the improvement of the quality of manufactured products.

In order to analyse the reliability of production processes in the bakery industry for MSCBI, it was necessary to use target probabilistic models, such as: exponential distribution, Weibull distribution, gamma distribution, normal standardised distribution and normal logarithmic distribution.

2. RELIABILITY THEORY. BASIC PROBABILISTIC MODELS USED IN THE BAKERY INDUSTRY

The definition of reliability may cover different requirements described by the technical, economic and sociological characteristics of the facilities. The following types of reliability may be distinguished:

– technical reliability which takes into account technical characteristics;

– technical and economic reliability which takes into account technical and economic characteristics;

– global reliability which takes into account technical, economic and social characteristics of the facilities [4].

Reliability is understood as technical reliability. Reliability of the facility is its ability to meet the requirements set.

The value characterising the ability to meet the requirements may be the probability of meeting the requirements. Therefore, the following definition applies: “the reliability of the facility is the probability of the facility for meeting its requirements” [5]. When the requirement is that the facility be capable (efficient) within a range (0, t), which may be measured by time, amount of work performed, number of actions performed, distance travelled etc., then: “the reliability of a facility is the probability that this facility is capable (efficient) within a range (0, t)” or: “the reliability of a facility is the probability that the values of parameters specifying important properties of this facility will not exceed permissible limits under specified conditions of operation of the facility within a period of (0, t)” [1]. In probabilistic terms, the reliability of the facility R(t) at a given moment t is the probability $P(T \geq t)$ that its life T is greater than t, i.e. $R(t) = P(T \geq t)$. Life T can be expressed e.g. by time in [s], length in [km] etc. It means that each time R(t) is different [10].

In the following section of the paper, basic probabilistic models, such as exponential distribution, Weibull distribution, gamma distribution, normal standardised distribution and normal logarithmic distribution, are provided.

2.1 Exponential distribution

One of the simplest probabilistic models for life of a non-renewable facility is a random variable T whose failure intensity is constant, i.e. independent of the time $\lambda(t) = \lambda = const$ [4].

The main reliability parameters for exponential distribution are [12]:

1. Reliability function R(t); it is the probability of operation of the facility according to its intended purpose within the assumed time frame t. This function is determined by the formula: $R(t) = \exp(-\lambda t)$, $R = 0$.

2. Failure function F(t); it is the probability of incorrect operation, i.e. damage up to time t. This function may be expressed by the following formula: $F(t) = 1 - R(t)$.

Another notation of this function using the function formula from subsection a is as follows: $F(t) = e^{-\lambda(t)}$.

3. The average time of operation in accordance with the intended purpose of a particular component T_0 of the facility may be determined using the following formula:

$$T_0 = \frac{1}{\lambda}.$$

2.2 Weibull distribution

This distribution is a certain generalisation of the exponential distribution discussed above. It is extended by the parameter p which is an additional element. This distribution for positive values of random variable X is characterised by the adoption of a non-zero value. The main reliability parameters for this distribution are:

a) R(t) which is the reliability function defined by the following formula [3]:

$$R(t) = 1 - F(t) = e^{-\left(\frac{t}{a}\right)\cdot p},$$

b) F(t) which is the failure function. It is calculated using the formula:

$$F(t) = 1 - R(t) = 1 - = e^{-\left(\frac{t}{a}\right)\cdot p}.$$

2.3 Gamma distribution

The expected value of this distribution is described by the ratio of two parameters of this distribution. This value is calculated using the following formula:

$$E(T) = \frac{p}{b} \text{ [2]},$$

where p and b are the values resulting from the parameters of this distribution.

In a random variable T, there is a gamma distribution if it is possible to present a density function f(t) for a value t > 0 in the form of the following formula [12].

$$f(t) = \frac{b^n}{(p-1)!} t^{p-1} \exp(-bt).$$

The symbols b and p in this formula represent the distribution parameters and the value of each of them must be greater than zero, i.e. b > 0 and p > 0.

2.4 Normal distribution

In the range specified from - ∞ to ∞, the random variable X has a normal distribution and this is described symbolically as N (m, δ), where m is an expected value, hereinafter referred to as the mean value. On the other hand, the symbol δ means standard deviation. Both values in numerical terms are parameters of this distribution. The density function of this distribution is described by the following formula [11]:

$$f(x) = \frac{1}{\delta\sqrt{2\pi}} \exp[-\frac{(x-m)^2}{2\delta^2}] = \frac{\varphi(y)}{\delta}.$$

If you have such a formula, you can set a formula for distribution of a random variable. It is described using the formula presented below. The θ symbol indicates the value for the Laplace integral F(x) = φ(y) + 0.5 θ(y) [1].

2.5 Normal standardised distribution

This distribution is a variety of normal distribution where instead of a continuous random variable X a standardised variable U is used. This variable is determined by using the standardisation formula:

$$U = \frac{(x-m)}{\delta} \text{ [7]},$$

where m is the mean of the distribution, δ is the standard deviation and x is the numerical value of the variable.

The normal standardised distribution is described by means of u equal to zero and equal to unity. This distribution is symbolically presented as N (0, 1).

Using the probability density function for normal distribution and the values of the mean and variance parameters resulting from the normal standardised distribution, the density function of normal standardised distribution can be determined.

2.6 Normal logarithmic distribution

Normal logarithmic distribution may be used for reliability testing of objects whose damage is caused by progressively increasing fatigue cracks. The random variable T is subject to normal logarithmic distribution if the random variable X expressed by the normal logarithm from T is subject to normal distribution, i.e. X = lg T or the same in another notation X = ln T. Designation of the reliability function requires the use of the dependencies that occur between the X variable and the T variable, i.e. X = ln T. In this case the following equation applies: R(t) = 1 – P (T < t) = 1 – P (X < x) [9].

3. RELIABILITY ANALYSIS IN THE BAKERY INDUSTRY

Bread is the most important product in the bakery production process. It represents 75-80% of the daily production in MSCBI. The production process of the bread production line can be divided into numerous stages (Tab. 1).

Table 1 Traditional flowchart of bread production process (own tests)

Item	Bread production process
A1	Ingredient storage
A2	Delivery of ingredients to mixing station
A3	Measuring of ingredients
A4	Mixing of ingredients
A5	Transport of kneading trough
A6	Division of dough into chunks
A7	Rounding of chunks
A8	Preliminary proofing of chunks
A9	Stretching of chunks
A10	Laying of chunks on the tray + inspection
A11	Notching of chunks
A12	Tray handling
A13	Proofing
A14	Tray handling
A15	Wetting of chunks
A16	Baking
A17	Inspection
A18	Bread handling
A19	Cooling down of bread
A20	Slicing
A21	Packing to baskets + inspection
A22	Shipment

Source: Own study based on tests performed in the bakery X

Each MSCBI bakery uses dedicated machines which are the core of the production process of the bread production line. The machines in question must ensure: very good constant quality of bread with precise dough weight, high degree of repeatability of processes and high reliability related to their efficient and failure-free operation. The basic operating chart of individual machines consists of the following items: a silo unit, dosing systems, a spiral mixer unit, a mobile kneading trough unit, a kneading trough tippler unit, a suction and pumping divider unit, a conical rounder unit, an intermediate proofer unit, a dough stretcher unit, processes for placing of chunks on trays and trolleys, a proofing unit, a set of thermal and oil ovens, cooldown process, a set of slicers with clippers. The structure of the production system may sometimes be covered by process secrecy.
In the aforementioned reliability tests, probabilistic models, such as exponential distribution, Weibull distribution, gamma distribution, normal standardised distribution and normal logarithmic distribution, were used.
Table 2 shows the selected information concerning the operation of thermal and oil oven unit in the bakery X. This information was necessary to calculate the reliability for the ovens in question as well as to prepare a diagram of failure intensity.

Table 2 Information concerning the operation of thermal and oil oven unit in the bakery X (own tests)

Name of the machine unit	Thermal and oil oven unit
Daily operation time Annual operation time (2018)	7 hours per day 7 h × 312 days = 2184 h per year
List of diagnosed unit failures	Overall unit malfunction time (years)
– No evaporation	2011 (1 day / 4 hours) 2013 (1 day / 4 hours) 2014 (1 day / 4 hours) 2015 (1 day / 4 hours) 2016 (1 day / 4 hours) 2017 (1 day / 4 hours) 2018 (1 day / 4 hours)
– Cracking radiators inside the oven (oil leaks)	2009 (2-3 working days of maintenance) 2011 (2-3 working days of maintenance) 2013 (2-3 working days of maintenance) 2014 (2-3 working days of maintenance) 2015 (2-3 working days of maintenance) 2016 (2-3 working days of maintenance) 2017 (2-3 working days of maintenance) 2018 (2-3 working days of maintenance)
– Control electronics failure	2009 (1 day / 3 hours) 2011 (1 day / 3 hours) 2013 (1 day / 3 hours) 2014 (1 day / 3 hours) 2015 (1 day / 3 hours) 2016 (1 day / 3 hours) 2017 (1 day / 3 hours) 2018 (1 day / 3 hours)
Year of manufacture	2006
Symbol in the process	**ZP 1-10 (10 pcs)**
Expected duration of failure-free operation	10 years = 87,600 hours

Source: Own study based on tests performed in the bakery X

Below are the examples of reliability calculations for the thermal and oil oven unit performed during the tests in question.

3.1 Exponential distribution

t = number of years from start-up of the units until the end of 2018 × average number of working days a year × daily number of hours of operation of the thermal and oil oven unit

t = 13 × 312 × 7 = **28,392**

E(t) = expected duration of failure-free operation

$$E(t) = \frac{1}{\lambda}$$

$$\lambda = \frac{1}{87600} = 0.00001141552$$

$$R(t) = e^{-\lambda t} = e^{-0.00001141552*28392} \approx \mathbf{0.7231}$$

E(t) = mean operation time to failure on the basis of the obtained data

E(t) = 6552 (2009) + 10,920 (2011) + 15,288 (2013) + 17,472 (2014) + 19,656 (2015) + 21,840 (2016) + 24,024 (2017) + 26,208 (2018) / 8 = 141,960 / 8 = **17,745 (wh)**

$$\lambda = \frac{1}{17745} = 0.0000563539$$

$$R(t) = e^{-\lambda t} = e^{-0.0000563539*28392} \approx \mathbf{0.2019}$$

For exponential distribution of the reliability function R(t), the expected duration of failure-free operation of the thermal and oil oven unit is 72.31%. The probability of proper operation of the unit in this reliability function E(t) is 20.19%.

3.2 Weibull distribution

$$\boldsymbol{R(t) = e^{-\left\{\frac{t}{a}\right\}^{p}}}$$

p	E(t) Expected duration of failure-free operation	E(t) Mean operation time to failure on the basis of the obtained data
1	$R(t) = e^{-1} \approx 0.7232$	$R(t) = e^{-1} \approx 0.2019$
2	$R(t) = e^{-2} \approx 0.9003$	$R(t) = e^{-2} \approx 0.0773$
	$R(t) = e^{-3} \approx 0.9666$	$R(t) = e^{-3} \approx 0.0167$

For Weibull distribution of the reliability function R(t), the expected duration of failure-free operation of the thermal and oil oven unit is for indicators p-1 = 72.32%, p-2 = 90.03%, p-3 = 96.66%. The probability of proper operation of the unit in this reliability function E(t) is for indicators p-1 = 20.19%, p-2 = 7.73%, p-3 = 1.67%.

3.3 Gamma distribution

E(t) Expected duration of failure-free operation

$$E(T) = \frac{p}{b}$$

p = 1 $\quad 87{,}600 = \frac{1}{b} \quad$ b = 0.00001141552

$$R(t) = 1 - F(t) = \exp(-bt)\sum_{i=0}^{p-1}\frac{(bt)^i}{i!} = 1 - \exp(-bt)\sum_{i=p}^{\infty}\frac{(bt)^i}{i!}$$

$$R(t) = 1 - e^{(0.00001141552*28392)} * \left[\frac{(0.00001141552*28392)^1}{1}\right] \approx \mathbf{0.7656}$$

p = 2 $\quad 87{,}600 = \frac{2}{b} \quad$ b = 0.00002283105

$$R(t) = 1 - e^{(0.00002283104*28392)} * \left[\frac{(0.00002283104*28392)^1}{1} + \frac{(0.00002283104*28392)^2}{2}\right] \approx \mathbf{0.5511}$$

p = 3 $\quad 87{,}600 = \frac{3}{b} \quad$ b = 0.00003424656

$$R(t) = 1 - e^{(0.00003424656*28392)} * \left[\frac{(0.00003424656*28392)^1}{1} + \frac{(0.00003424656*28392)^2}{2} + \frac{(0.00003424656*20280)^3}{6}\right] \approx \mathbf{0.3955}$$

E(t) Mean operation time to failure on the basis of the obtained data

$$R(t) = 1 - F(t) = \exp(-bt)\sum_{i=0}^{p-1}\frac{(bt)^i}{i!} = 1 - \exp(-bt)\sum_{i=p}^{\infty}\frac{(bt)^i}{i!}$$

p = 1 $\quad 17{,}745 = \frac{1}{b} \quad$ b = 0.0000563539

$$R(t) = 1 - e^{(-0.0000563539*28392)} * \left[\frac{(0.0000563539*28392\,)^1}{1}\right] \approx \mathbf{0.6769}$$

p = 2 $\quad 17{,}745 = \frac{2}{b} \quad$ b = 0.0001127078

$$R(t) = 1 - e^{(0.0001127078*28392)} * \left[\frac{0.0001127078*28392^1}{1} + \frac{(0.00011270781*28392)^2}{2}\right] \approx \mathbf{0.6614}$$

p = 3 $\quad 17{,}745 = \frac{3}{b} \quad$ b = 0.0001690617

$$R(t) = 1 - e^{(0.0001690617*28392)} * \left[\frac{(0.0001690617*28392)^1}{1} + \frac{(0.0001690617*28392)^2}{2} + \frac{(0.0001690617*28392)^2}{2} \approx \mathbf{0.7150}\right.$$

For gamma distribution of the reliability function R(t), the expected duration of failure-free operation of the thermal and oil oven unit is for indicators p-1 = 76.56%, p-2 = 55.11%, p-3 = 39.55%. The probability of proper operation of the unit in this reliability function E(t) is for indicators p-1 = 67.69%, p-2 = 66.14%, p-3 = 71.50%.

3.4 Normal distribution

m = Expected duration of failure-free operation

$$\delta = \sqrt{\frac{\sum(x-m)^2}{n}} \approx \mathbf{74{,}131.30}$$

for n = 8, m = 87,600
x_1 = 6552, x_2 = 10,920, x_3 = 15,288, x_4 = 17,472, x_5 = 19,656, x_6 = 21,840, x_7 = 24,024, x_8 = 26,208

$$yt = \frac{t-t_0}{\delta} = \frac{28392-87600}{74131.30} \approx -\mathbf{0.7987}$$

$$y_0 = -\frac{t_0}{\delta} = -\frac{87600}{74131.30} \approx -\mathbf{1.1817}$$

$$R(t) = \frac{1-\phi(yt)}{1-\phi(y_0)} = \frac{1-(-0.7987)}{1-(-1.1817)} \approx \mathbf{0.8244}$$

m = Mean operation time to failure on the basis of the obtained data

$$\delta = \sqrt{\frac{\sum(x-m)^2}{n}} \approx \mathbf{6255.96}$$

for n = 8, m = 17,745
x_1 = 6552, x_2 = 10,920, x_3 = 15,288, x_4 = 17,472, x_5 = 19,656, x_6 = 21,840, x_7 = 24,024, x_8 = 26,208

$$yt = \frac{t-t_0}{\delta} = \frac{28392-17745}{6255.96} \approx \mathbf{1.7019}$$

$$y_0 = -\frac{t_0}{\delta} = -\frac{17745}{6255.96} \approx -\mathbf{2.8364}$$

$$R(t) = \frac{1-\phi(yt)}{1-\phi(y_0)} = \frac{1-(-1.7019)}{1-(-2.8364)} \approx \mathbf{0.7043}$$

For normal distribution of the reliability function R(t), the expected duration of failure-free operation of the thermal and oil oven unit is 82.44%. The probability of proper operation of the unit in this reliability function E(t) is 70.43%.

3.5 Normal logarithmic distribution

m = Expected duration of failure-free operation
for n = **8**, m = ln 87,600 **11.38**, x = ln 28,392 **10.25**
x_1 = ln 6552 8.7875; x_2 = ln 10,920 9.2984;
x_3 = ln 15 288 9,6348, x_4 = ln 17 472 9,7684;
x_5 = ln 19,656 9.8861; x_6 = ln 21,840 9.9915;
x_7 = ln 24,024 10.0868; x_8 = ln 26,208 10.1738;

$$\delta = \sqrt{\frac{\sum(x-m)^2}{n}} \approx \mathbf{1.7315}$$

$$yt = \frac{t-t_0}{\delta} = \frac{10.25-11.38}{1.7315} \approx -\mathbf{0.6526}$$

$$y_0 = -\frac{t_0}{\delta} = -\frac{11.38}{1.7315} \approx -\mathbf{6.5723}$$

$$R(t) = \frac{1-\phi(yt)}{1-\phi(y_0)} = \frac{1-(-0.6526)}{1-(-6.5723)} \approx \mathbf{0.2182}$$

m = Mean operation time to failure on the basis of the obtained data
dla n = **5** m = ln 17,745 ≈ **9,78** x = ln 28,392 **10.25**
x_1 = ln 6552 8.7875; x_2 = ln 10,920 9.2984;
x_3 = ln 15,288 9.6348, x_4 = ln 17,472 9.7684;
x_5 = ln 19,656 9.8861; x_6 = ln 21,840 9.9915;
x_7 = ln 24,024 10.0868; x_8 = ln 26,208 10.1738;

$$\delta = \sqrt{\frac{\sum(x-m)^2}{n}} \approx \mathbf{0.3940}$$

$$yt = \frac{t-t_0}{\delta} = \frac{10.25-9.78}{0.3940} \approx \mathbf{1.1929}$$

$$y_0 = -\frac{t_0}{\delta} = -\frac{9.78}{0.3940} \approx -\mathbf{24.8223}$$

$$R(t) = \frac{1-\phi(yt)}{1-\phi(y_0)} = \frac{1-(-1.1929)}{1-(-24.8223)} \approx \mathbf{0.0849}$$

For normal logarithmic distribution of the reliability function R(t), the expected duration of failure-free operation of the thermal and oil oven unit is 21.82%. The probability of proper operation of the unit in this reliability function E(t) is 8.49%.

The above reliability calculations were used to prepare a chart of damage severity for the thermal and oil oven unit in the years 2006-2018 and the expected distribution of the function of failure intensity in the years 2019-2026.

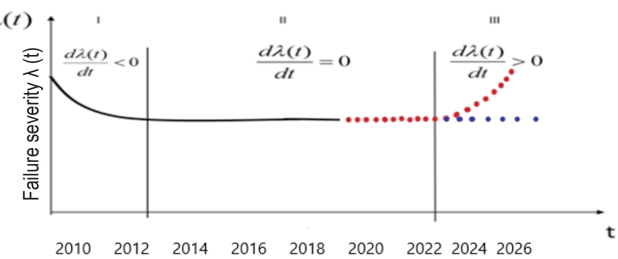

Figure 1 Diagram of failure intensity for the thermal and oil oven set (own tests).
Source: Own study based on tests performed in the bakery X

In the first interval, the failure intensity is a decreasing function. This is the so-called initial operating period, also referred to as adaptation and break-in period during which the components of low reliability are damaged. During this period, hidden defects of materials, design errors, assembly errors, control deficiencies, defects occurring during transport and storage, as well as other such defects of the object are revealed. Failures are also affected by the skills of the user who learns to operate the device (adaptation). In the

second interval, the failure intensity is constant. This is the so-called *normal operating period* during which failures caused by random factors prevail, e.g. sudden overload in harsh operating conditions. Such failures may be excluded by including the extreme harsh operating conditions in calculations and engineering design. However, this would cause deadweights of the machines to be excessive. The duration of this period depends on the operating and maintenance conditions of the equipment.
In the third interval, the failure intensity is an increasing function. This is mainly caused by equipment wear and ageing processes. Components of long-operated equipment are subject to irreversible physical and chemical changes which reduce their strength. As a result of wear of the top layer of the mating components, clearances increase and weakened components are deformed. Therefore, this is a period of increase in the failure probability.

4. CONCLUSIONS

In this article based on own tests (performed in the period from 02/01/2016 to 31/12/2018 in micro and small companies in the bakery industry), the author presented methods of measuring the reliability of the traditional production system based on the probabilistic concept, such as: exponential distribution, Weibull distribution, gamma distribution, normal standardised distribution, and normal logarithmic distribution.
A diagram of the bread production process as well as the machine facilities used on the production line are presented. On the basis of the above-mentioned probabilistic models, the author provided example calculations for the thermal and oil oven unit with a diagram of the failure intensity in the years 2006-2018 and the expected distribution of the failure intensity function in the years 2019-2026.

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