AGH DRILLING, OIL, GAS • Vol. 35 • No. 2 • 2018

https://doi.org/10.7494/drill.2018.35.2.377

Michał Figiel*, Joanna Lewandowska-Śmierzchalska**

FRACTAL ANALYSIS OF SANDSTONE PORE SPACE GEOMETRY

Abstract: Fractal analysis is currently one of the fastest evolving branches of science. Numerous objects in nature exhibit a fractal structure. Additionally, the vast majority of rocks – especially reservoir rocks – take the form of a fractal.

Computer image analysis based on thin-section images has been used for examining the fractal structure of pore spaces, directly applying the definition of the fractal box-counting dimension. For the examined sandstone sample, thin sections were made and photographed, and the corresponding values of the fractal dimension and lacunarity were calculated. Each of the photos was encompassed by porosity that was calculated based on the number of pixels. Furthermore, the volatility of the fractal dimension and lacunarity were studied as well as their relationships with the porosity. A correlation analysis between the fractal parameters and the porosity was carried out. The results were compared with the data obtained from a mercury porosimetry of the same sample of sandstone.

Keywords: pore space, fractal structure, fractal dimension, lacunarity, thin section, computer image analysis

---

* AGH University of Science and Technology, Faculty of Drilling, Oil, and Gas, Krakow, Poland – MSc student
** AGH University of Science and Technology, Faculty of Drilling, Oil, and Gas, Krakow, Poland

1. INTRODUCTION

Next to permeability, porosity is one of the most important petrophysical and reservoir parameters that characterize rocks. The geometry of pore space is an issue of great importance. However, its classical Euclidean description is not sufficient enough due to the considerably high level of complexity and apparent randomness. The used approximations do not fully resemble the physical aspects of rocks appearing in nature. These reasons lead researchers to look for new ways of describing the rocks pore space. One of them is fractal analysis – a rapidly evolving branch of mathematics whose origins can be found in the second half of the 20th century. Because of the fact that fractal shapes very often occur in nature, fractal analysis has many applications in earth sciences, especially in geophysical and geological surveys (which includes the study of the shape and distribution of pores in reservoir rocks).

This article undertakes the problem of measuring the change of the fractal dimension and lacunarity of a rock pore space structure and examines their relationships with porosity. The research is based on a computer image analysis of reservoir rock pores visualized in thin sections.

2. BASIC DEFINITIONS

The term **fractal** comes from the Latin word *fractus* (broken, shattered) and was introduced by Bendoit Mandelbrot in the 1970s [1]. For a long time, it had been considered as an interesting geometric issue (although one without any greater practical applications). It wasn't until years later when it was proven that many naturally occurring phenomena can be described with the use of fractals [2]. Additionally, the pore system of rocks as well as their grain distributions possess fractal or multifractal properties [3–6]. What is more, introducing elements of fractal analysis allowed for attempts to parametrize the percolation model [7].

Generally, fractals are characterized by the following:

- construction from a simple and recursive definition instead of an equation,
- exact or statistical self-similarity,
- non-integer dimension.

A geometric object is called self-similar when, after its division, each part is an exact replica of the whole [8, 9]. Such patterns are mainly mathematical structures. The most famous examples are the Mandelberot set, Koch curve, Julia set, or Menger sponge. In reality, it is often impossible to determine the scaling factor or its differences in the x- and y-directions (self-affine fractals) and then the self-similarity occurs in a stochastic sense.

Due to the certain freedom for interpretation left by Mandelbrot, the fractal dimension has a whole range of definitions that can be used interchangeably by scientists [10]. It can be interpreted as the factor of irregularity and roughness of a given structure. Basic definitions of a fractal dimension have been gathered and listed below. Additionally, it is important to note that a fractal dimension calculated using different methods does not necessarily have to be equal; thus, how it was determined should always be mentioned.

A self-similarity dimension describes the relationship between the scaling factor and the number of pieces that the structure was divided into. It can be written that

$$n = \frac{1}{s^D},$$

where:

n – the number of pieces after division,
s – the scaling factor,
D – the fractal dimension.

Taking the logarithm of both sides of the equation and considering the smallest possible scaling factor,

$$D = \lim_{s \to 0} \frac{\log(n)}{\log\left(\frac{1}{s}\right)}.$$

This definition of a fractal dimension is usually used for fractals, which allows us to determine the necessary parameters – s and n. Usually, these are mathematical fractals. For example, for the Menger sponge,

$$D = \frac{\log(20)}{\log(3)} = 2.723.$$

A box-counting dimension (Minkowski-Bouligand dimension) is based on the box-counting method. An examined area is divided with a regularly spaced grid. To calculate the fractal dimension of the considered object, all of the squares containing its portion are counted. Next, the density of the grid is increased, and the counting process repeats. The straight line on the graph of the scaling factor against the grid density (created from the acquired data) is required for the structure to be a fractal; its dimension is calculated from the slope of the line. For the two following steps where the scale changes from *a* to *b,* the box-counting dimension can be estimated from the following equation:

$$D = \frac{\log(n_a) - \log(n_b)}{\log\left(\frac{1}{s_a}\right) - \log\left(\frac{1}{sb}\right)},$$

where:

n_a, n_b – numbers of squares containing the examined structure,
s_a, s_b – scaling factors.

The box-counting algorithm is the most popular way of determining a fractal dimension thanks to its simplicity and ease in writing adequate scripts and computer algorithms. It can successfully be applied to two- as well as three-dimensional objects. In the second scenario, cubes are counted instead of squares. Experiments show that the box-counting dimension of self-similar fractals such as the Menger sponge or Koch curve is satisfactorily similar to their self-similarity dimension.

It is possible for two different objects to have the same fractal dimension; thus, it alone does not fully describe a fractal. An additional parameter (lacunarity, describing pore distribution and heterogeneity) has been introduced. It was observed that, if a fractal has large "holes", it also has high lacunarity. Furthermore, lacunarity characterizes fractal invariance to its rotation, which shows the greater homogeneity of the studied object. A lack of rotation influences the appearance of a fractal and implies lower lacunarity.

3. RESEARCH METHODOLOGY

To determine the fractal dimension and lacunarity, it was necessary to make thin sections from the selected rock sample and then apply computer image analysis techniques. A sample of quartzitic sandstone from the southeastern part of the Polish lowlands was subjected to study. The sample was chosen based on its porosity (which was the only criteria of the selection). The porosity was measured using a vacuum pump and estimated at approx. 15%. It was a coarse-grained sandstone with a psammite structure. It was uneven, with a medium to well-sorted grainy skeleton. It consisted mostly of quartz with admixtures of iron oxide and mica. Carbonate cementation on the various development levels was observed. The adhesive was a various formed carbonate cement.

From the sandstone core, five thin sections were created (Fig. 1). The distance between each cut was chosen in such a way that it covered the longest possible distance throughout the greater side of the core (additionally keeping a relatively small distance between the following cuts). The thin sections were based on a blue-colored resin for better pore space observation. To eliminate the problems in distinguishing the pores from the grains, it is important to use the correct amount of pigment [11].

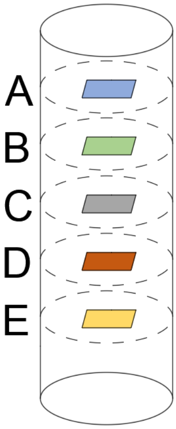


Fig. 1. Distribution of thin section throughout core of diameter 9.5 cm; distance between each cut is close to 2 cm

A binocular magnifier (×4) was used to examine the samples. Due to the high porosity, a greater zoom didn't clearly show the pore space. After examining the sample, the thin sections were photographed in random selected spots, which resulted in a set of 20 photos (3264 × 2449 pixels) for each of them.

Further image analysis was done using the free ImageJ software with the FracLac plugin, which allowed to estimate the fractal dimension and lacunarity based on the box-counting algorithm. The process of converting the image is described below. The photograph (Fig. 2a) shows a magnified view of the studied sandstone. Using the color threshold function built in to the program, it was possible to mark the pore space filled with blue resin (Fig. 2b). Each time, it required the adjustment of three different parameters: hue, saturation, and brightness. The hue was set to a wide range of blues to capture all of its tints and show the diversity of the pore space. It was noticed that the appropriate adjustment of the saturation allows the program to limit the marking pixels on the grains. On top of that, modifying the brightness parameter helps distinguish between the darker minerals and the resin. Finally, a binary image was created (Fig. 2c). It consists only of black pixels (cement, grains, minerals, etc.) and white pixels (pores).

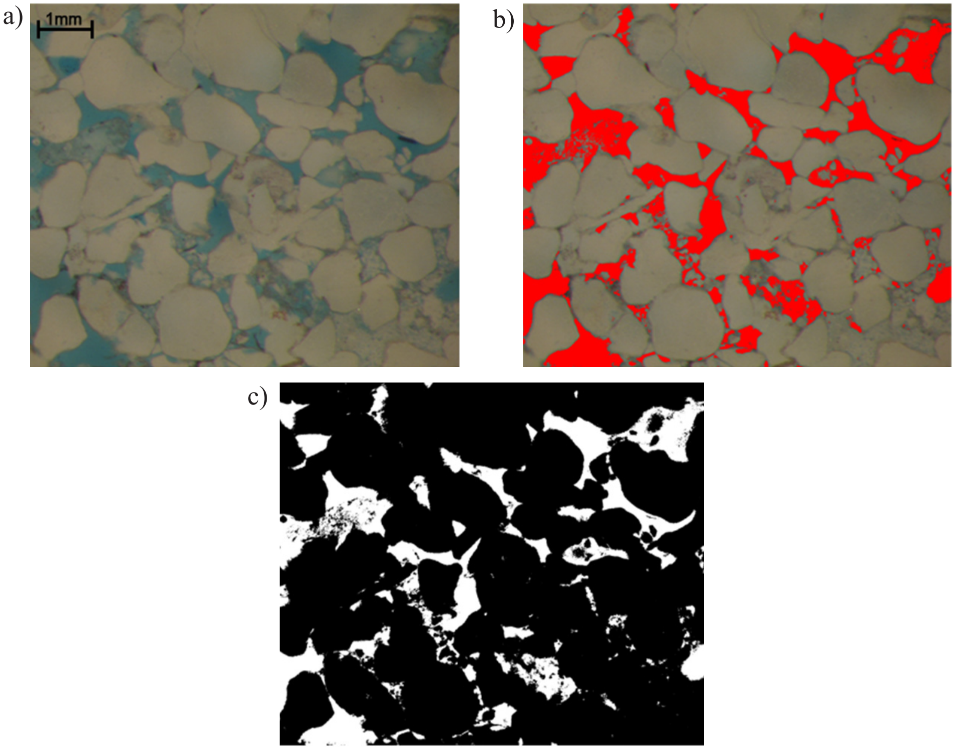


Fig. 2. Process of converting image on example of representative sample of thin section E; scale shown in figure corresponds to 1mm

The FracLac plugin was used to calculate the fractal dimension and lacunarity.

Knowing the resolution of a photo (and thus the total numbers of white and black pixels as well as their sum), it was possible to calculate the porosity for each photo using the following equation:

$$\varphi = \frac{W}{T},$$

where:

φ – total porosity,
W – number of white pixels,
T – sum of pixels of image.

The above procedure was carried out for all of the thin section photos of the studied sandstones A–E (Fig. 1).

4. RESULTS AND DISCUSSION

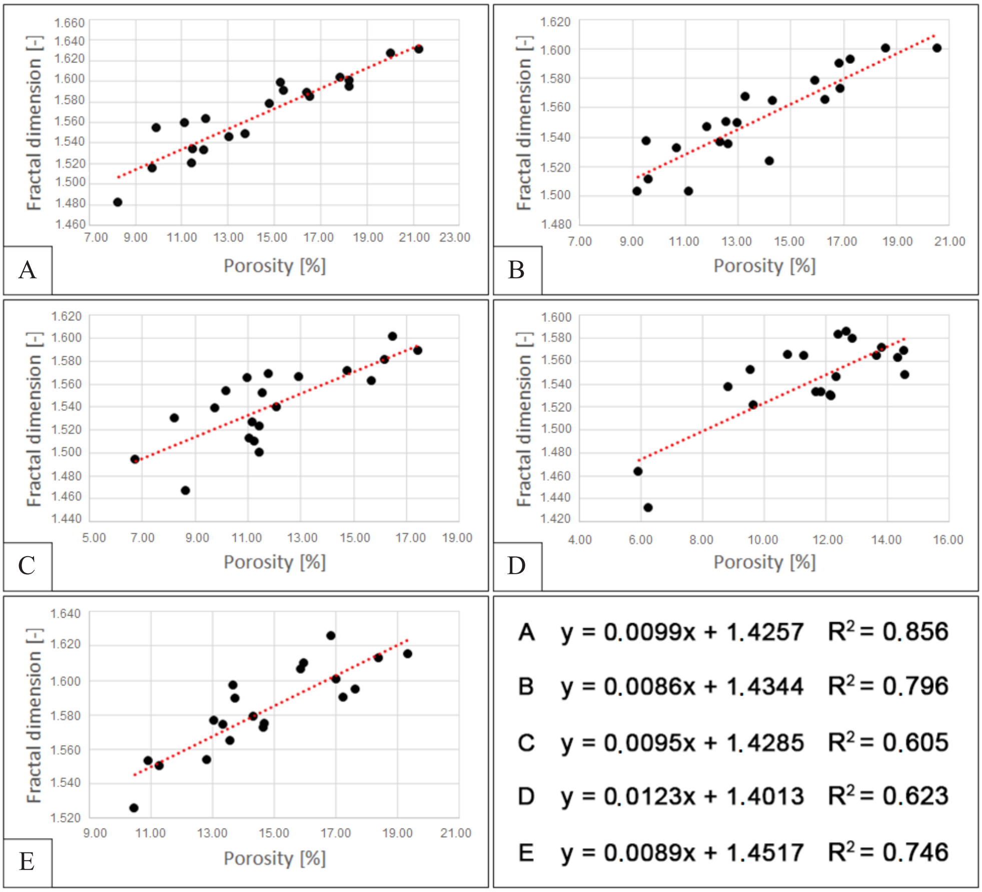
The values of the fractal dimension range from 1.432 to 1.631 but the vast majority of records remain between 1.511 and 1.617. The lacunarity values reach a much greater spread (from 0.607 to 1.820); however, the larger past is less than 1. The average porosity is at 13.32% with the min-max values of 5.93% and 21.28%, respectively. A detailed summary of calculated results for each thin section has been included below (Tab. 1).

Table 1

Minimum, maximum, and average values of porosity, fractal dimension, and lacunarity for sandstone

Thin section	Porosity [%] min–max/average	Fractal dimension min–max/average	Lacunarity min–max/average
A	8.290–21.284/14.365	1.482–1.631/1.567	0.624–1.596/1.567
B	9.200–20.583/13.851	1.503–1.600/1.553	0.607–1.061/0.833
C	6.782–17.470/12.017	1.466–1.601/1.542	0.700–1.589/1.023
D	5.934–14.589/11.583	1.432–1.585/1.543	0.730–1.820/0.986
E	10.494–19.356/14.759	1.525–1.626/1.583	0.665–1.440/0.864

The relationship between the fractal dimension and the porosity is shown on the graph (Fig. 3). Each data series is encompassed by its own trend line. Correlation coefficients R^2 were calculated for each of the curves; these differ between 0.605 and 0.856. For all of the thin sections, the correlation curve was a straight line. The directional coefficients are as follows (respectively): for thin section A – 0.010; B – 0.009; C – 0.010; D – 0.012; and E – 0.009. For the same types of rocks (sandstones in particular), the correlation curve is always a straight line with a directional coefficient (which is included within a narrow range of numerical values). A graph demonstrating the relationship between lacunarity and porosity was also made (Fig. 4).



A $y = 0.0099x + 1.4257$ $R^2 = 0.856$

B $y = 0.0086x + 1.4344$ $R^2 = 0.796$

C $y = 0.0095x + 1.4285$ $R^2 = 0.605$

D $y = 0.0123x + 1.4013$ $R^2 = 0.623$

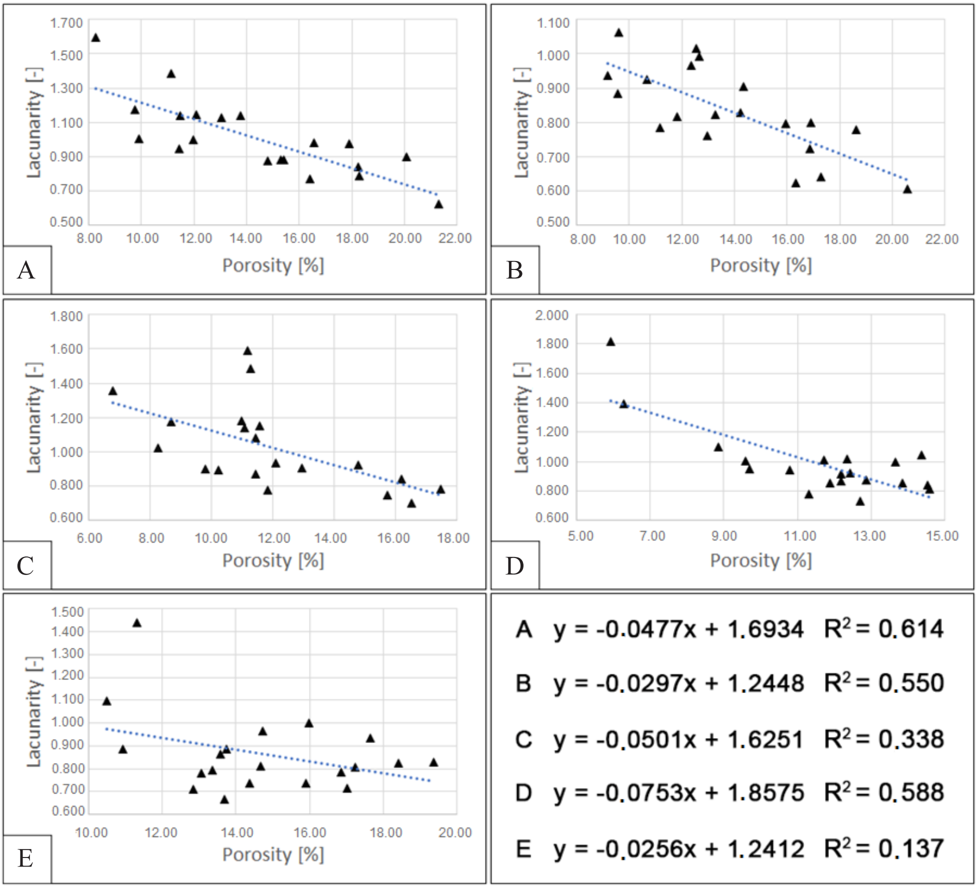
E $y = 0.0089x + 1.4517$ $R^2 = 0.746$

Fig. 3. Relationship between fractal dimension and porosity (with linear regression for each sample of sandstone)

A statistical analysis was carried out for the better visualization of the distribution of the values for the calculated fractal parameters and porosity – the basic mathematical statistics of the data sets (Tab. 2) and histograms (Figs. 5–7). The calculated arithmetic means of the fractal dimension, lacunarity, and porosity vary slightly across the studied sample of the sandstone. However, the median for these parameters oscillate around the same value.

The porosity of the analyzed thin sections from the sandstone sample varies considerably; on average, this is 13.315% with a relatively small standard deviation that is equal to 3.175% (Tab. 2). The minimum porosity value is about 5%, and the maximum – about 22%. This parameter has low variability. The distribution of the porosity is asymmetrical on the right side (Fig. 5). Most of the porosities are lower than the average. There is a concentration of porosity around the average.

The fractal dimension is relatively small varied – the average is 1.558; with a very small standard deviation equal to 0.026 (Tab. 2). The minimum fractal dimension is 1.432; and the maximum – 1.630. This parameter has very low variability. The distribution of the fractal dimension is asymmetrical on the left side (Fig. 6). Most of the values of the fractal dimension are higher than the average. There is a large values concentration around the average.



A $y = -0.0477x + 1.6934$ $R^2 = 0.614$

B $y = -0.0297x + 1.2448$ $R^2 = 0.550$

C $y = -0.0501x + 1.6251$ $R^2 = 0.338$

D $y = -0.0753x + 1.8575$ $R^2 = 0.588$

E $y = -0.0256x + 1.2412$ $R^2 = 0.137$

Fig. 4. Relationship between lacunarity and porosity (with corresponding regression lines)

Table 2

Mean, standard deviation, and median values for porosity, fractal dimension, and lacunarity

Thin section	Mean	Standard deviation	Median
Porosity [%]			
A	14.365	3.643	14.291
B	13.851	3.200	13.133
C	12.017	2.851	11.447
D	11.583	2.468	12.176
E	14.759	2.493	14.510
Average	13.315	3.175	12.907

Table 2. cont.

Fractal dimension [–]			
A	1.567	0.039	1.570
B	1.553	0.031	1.550
C	1.542	0.035	1.545
D	1.543	0.038	1.550
E	1.583	0.026	1.584
Average	1.558	0.026	1.563
Lacunarity [–]			
A	1.008	0.222	0.978
B	0.833	0.128	0.819
C	1.023	0.246	0.929
D	0.986	0.242	0.932
E	0.864	0.173	0.818
Average	0.943	0.218	0.896

The lacunarity is an attribute with very varied values – the average is 0.943; with a very small standard deviation equal to 0.218 (Tab. 2). The minimum lacunarity is 0.607, and the maximum – 1.820. This parameter is highly variable. The distribution of the lacunarity is asymmetrical on the right side (Fig. 7). The great part of the lacunarity values are lower than the average. There is a weak concentration of values around the average.

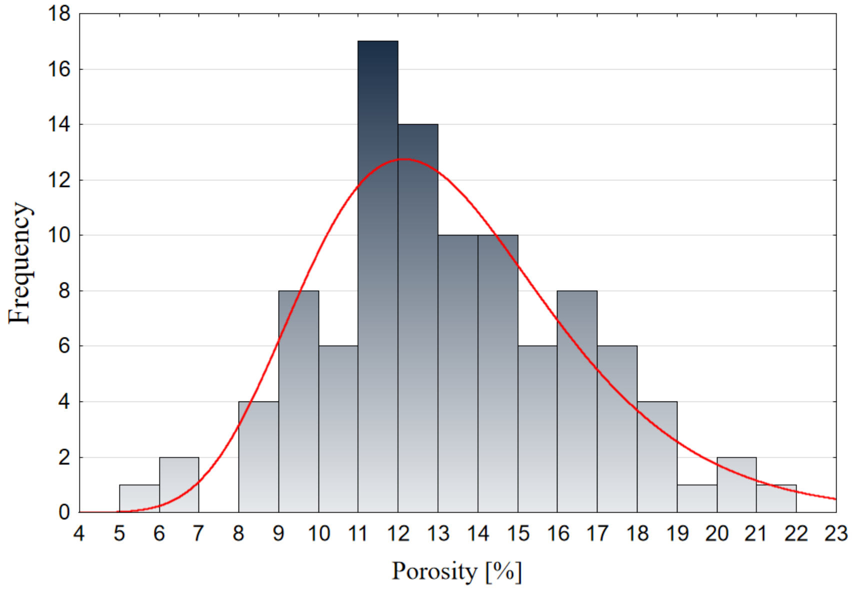


Fig. 5. Distribution of sandstone porosity

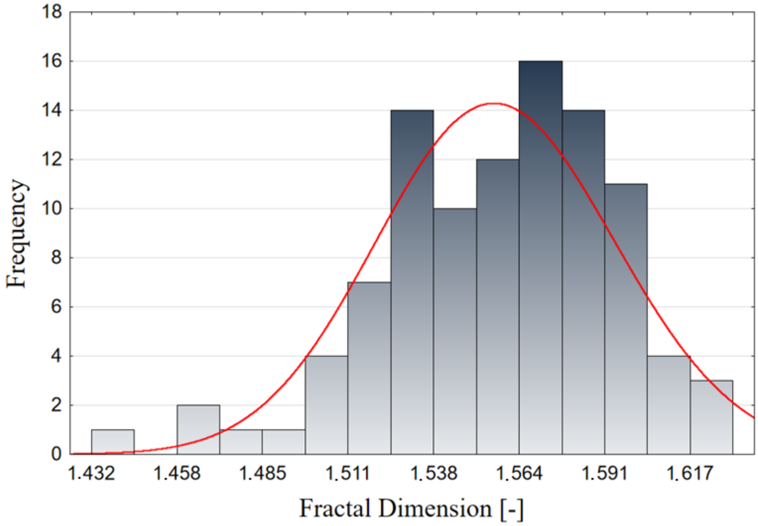


Fig. 6. Distribution of sandstone fractal dimension

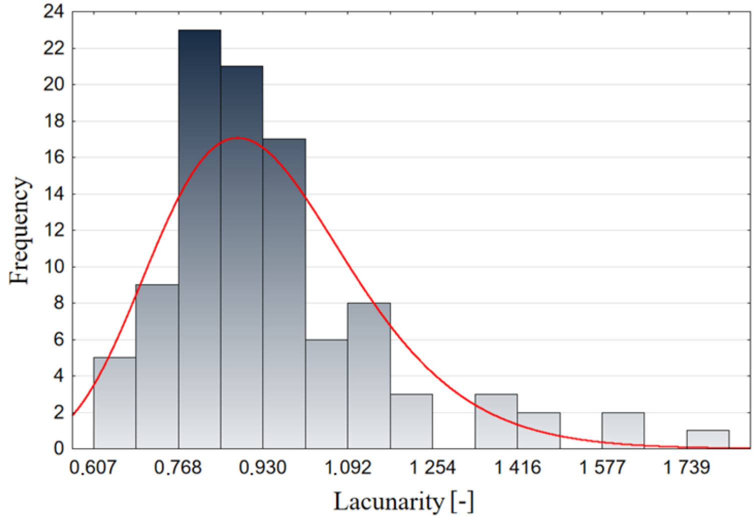


Fig. 7. Distribution of sandstone lacunarity

The research was supplemented by mercury porosimetry to compare the results with the computer image analysis of the thin sections. Using the Micromeritics AutoPore IV 9500 porosimeter located at the Department of Geology and Geochemistry of the Oil and Gas Institute – National Research Institute, it was possible to acquire additional data based on six samples of sandstone. The total porosity varies from 12.259% to 17.588% (Tab. 3). Additionally, a graph showing the relationship between the dynamic porosimeter porosity and total porosity of the studied samples was made (Fig. 8).

Table 3

Porosimetry data of six samples of analysis sandstone

Sample	Weight [g]	Volume [cm^3]	Bulk density [g/cm^3]	Dynamic porosity [%]	Total porosity [%]
1	4.177	1.525	2.739	10.139	12.964
2	2.326	0.838	2.778	12.356	15.547
3	3.035	1.105	2.746	10.191	12.441
4	2.541	0.916	2.773	11.556	14.994
5	4.363	1.609	2.713	16.489	17.588
6	4.473	1.644	2.721	10.175	12.259
		Average	2.745	11.818	14.299

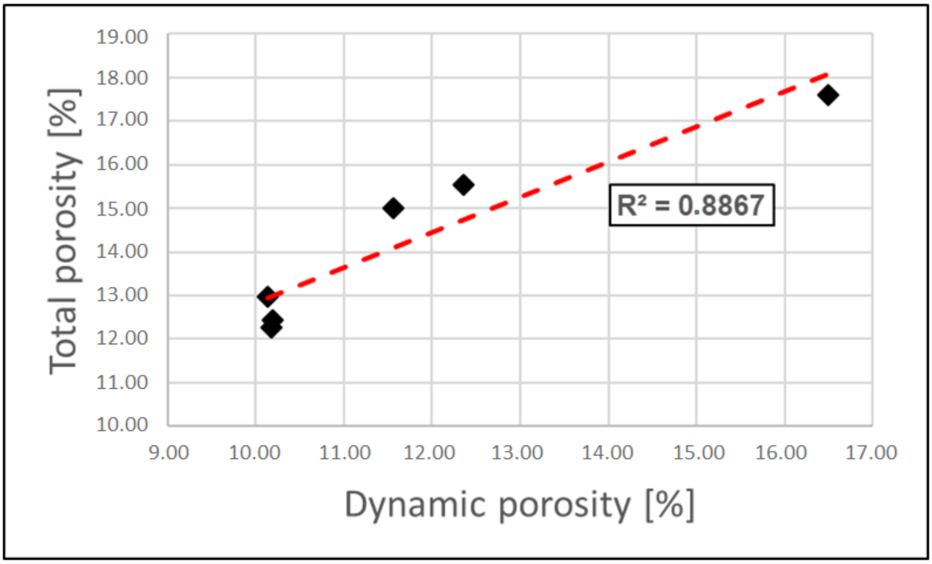


Fig. 8. Relationship between total porosity and dynamic porosity of sandstone

The computer image analysis of the thin sections confirmed that both the fractal dimension and lacunarity change throughout the whole studied core sample. The differences in the fractal parameters were noticed within a single thin section and between each of the thin sections. The obtained data shows a strong correlation between the fractal dimension and the porosity (an increase in one causes an increase in the other) and a relatively moderate (or no) correlation between the lacunarity and the porosity.

The thresholding method appeared to be the most important factor (as well as correctly adjusting the hue, saturation, and brightness parameters). This was based on human judgement, which is always relative and conditioned by the knowledge and experience of the researcher. However, the large number of pictures allowed us to minimalize the influence of this defect. With the increasing number of photos, the analysis became less subjective.

The research done on the studied thin section of the sandstone allowed to assess the pore space structure of the greater pores (macropores) and the change of the fractal parameters (fractal dimension and lacunarity). Such an analysis can be useful when mercury porosimetry doesn't fully describe the distribution of pores. However, this is a time-consuming and arduous study whose accuracy greatly depends on the photo resolution. It turns out that a good way of estimating the change in fractal parameters is an analysis done on an image that shows the largest possible area of the sample. However, a high number of pixels is associated with larger file sizes as well as conversion difficulties.

The change of a fractal dimension and lacunarity can manifest itself in very small sections and fragments of a sample (range of micrometers). This conclusion is very important in modeling the heterogeneity of the pore structure. The obtained results should be interpreted illustratively in terms of a studied core sample as well as the entire deposit.

The porosity from the mercury porosimetry measurement slightly differs from the one acquired from the image analysis. This suggests that some small portion (micropores) of it was not captured on the images.

5. CONCLUSION

The thin-section analysis confirms the fractal nature of the pore space of reservoir rocks like sandstone. A computer analysis showed a change in both the fractal dimension and the lacunarity for the studied sample (throughout the whole core as well as in a single thin section).

The graphs of the calculated parameters point to a strong linear correlation between porosity and the fractal dimension, which justifies the straight regression line. The data shows a weak linear correlation between porosity and lacunarity. Additionally, a fractal dimension that is a defined number characterizing a pore space as a whole is a very good parameter used for correlation with other analysis methods such as porosimetry.

Although time consuming, a computer analysis of thin sections is a direct means of observing and studying the pore structures of macropores. Additionally, there is a real problem of the propagation of two-dimensional data into a 3D space and the connection of the structure of the large pores with rest of the pore space. This is the reason why obtained numerical data should be interpreted illustratively (instead of precisely), which is caused by the physical structure of the rocks.

Acknowledgment

This work was supported by the Ministry of Science and Higher Education; statutory works of AGH University of Science and Technology, Faculty of Drilling, Oil, and Gas (No 11.11.190.555).

The authors would like to great thanks to Professor Piotr Such of The Department of Geology and Geochemistry of Oil and Gas Institute – National Research Institute in Cracow for the possibility of carry out the petrophysical measurements in his laboratory by Michał Figiel.

REFERENCES

[1] Mandelbrot B.: *The fractal geometry of nature*. W.H. Freeman and Company, New York 1977.

[2] Turcotte D.L.: *Fractals and chaos in geology and geophysics*. Cambridge University Press, Cambridge 1992.

[3] Garrison Jr. J.R., Pean W.C., von Rosenberg D.U.: *The Fractal Nature of Geological Data Sets: Power Law Processes Everywhere*. SPE Paper 22842, 1991.

[4] Katz A.J., Thompson A.H.: *Fractal Sandstone Pores: Implications and Conductivity and Pore Formation*. Physical Review Letters, vol. 54, no. 12, 1985, pp. 1325–1328.

[5] Such P.: *Studium badań przestrzeni porowej skał dla potrzeb geologii naftowej*. Prace Instytutu Górnictwa Naftowego i Gazownictwa, no. 104, IGNiG, Kraków 2000.

[6] Turcotte D.L.: *Fractals and Fragmentation*. Journal of Geophysical Research, vol. 91, no. B2, 1986, pp. 1921–1926.

[7] Patzek T.W.: *Verification of a Complete Pore Network Simulator of Drainage and Imbibition*. SPE Paper 59312, Society of Petroleum Engineers, 2000.

[8] Peitgen H.O., Jurgens H., Saupe D.: *Fraktale: granice chaosu. Cz. 1*. Wydawnictwo Naukowe PWN, Warszawa 1995.

[9] Peitgen H.O., Jurgens H., Saupe D.: *Fraktale: granice chaosu. Cz. 2*. Wydawnictwo Naukowe PWN, Warszawa 1996.

[10] Angulo R.F., Alvarado V., Gonzalez H.: *Fractal Dimensions from Mercury Intrusion Capillary Tests*. SPE Papers 23695, Society of Petroleum Engineers 1992.

[11] Leśniak G.: *Zastosowanie komputerowej analizy obrazu w badaniach petrofizycznych*. Przegląd Geologiczny, vol. 47, no. 7, 1999, pp. 644–651.