

*multi-criteria optimization, Pareto efficiency,  
vehicle routing problem, artificial immune system*

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## **COMPARISON OF PARETO EFFICIENCY AND WEIGHTED OBJECTIVES METHOD TO SOLVE THE MULTI-CRITERIA VEHICLE ROUTING PROBLEM USING THE ARTIFICIAL IMMUNE SYSTEM**

### **Abstract**

*The solutions to the multi-criteria vehicle routing problem, dependent on route length and travelling time of delivery van, are presented in the paper. This type of problem is known as a traveling salesman problem. The artificial immune system is used to solve it in this article. Since there are two variables – route length and travelling time – two methods are employed. They are: Weighted Objectives Method and Pareto Efficiency Method. The results of calculation are compared.*

### **1. INTRODUCTION**

A major challenge in the distribution of goods is finding the optimal vehicle routes. Most often optimization seeks to minimize the number of kilometers, which primarily are translated into fuel and maintenance costs. Travelling time is also important because of drivers' wages, quick and timely delivery of goods to the customer, and consequently it affects customer satisfaction. The emissions, which recently have become more stringent, rely on such parameters as the age of vehicles, route length and individual driving styles. Optimization of vehicle routes is dependent on many variables. This paper presents how to optimize delivery schedule from one warehouse to multiple vendors.

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The controlled variables are: distribution of goods to a specified number of locations using one delivery van in one day. For such a scenario calculations were made using both methods.

## 2. TASK FORMULATION

From a mathematical standpoint, the travelling salesman problem is to find the shortest route that passes through each of a set of points.

To solve the traveling salesman problem, one of the known methods can be used. The solution offered by the construction of the Hamilton cycles in the graph representing the network of roads and finding the best one is possible only for small graphs. Therefore, the other methods are used. The branch and bound method is very popular and recently supported by artificial intelligence methods (Karaoglana et al., 2011). The genetic or evolutionary algorithms (Król, 2017), k-means methods (Ambroziak & Jachimowski, 2012), neighborhood search (Kytöjokia et al., 2007) and others heuristic methods are applied successfully. However, this approach uses artificial immune system. Optimization, which simultaneously takes into account the roadway and time is important for route delivery planning with varying traffic patterns. In this case, the travel time is a nonlinear function of the road and it should be taken into account as the independent variable.

There are many methods of multi-criteria optimization (Kovács & Bóna, 2009). Presented calculations use two different approaches to find the solution to the problem of multiple-criteria: weighted objectives method and Pareto efficiency.

### 2.1. Weighted Objectives Method

Weighted objectives method is used often to solve multi-criteria problems. In the case of minimizing road and travel times optimization criteria can be expressed in the following objective function:

$$f: \mathcal{D} \subset \mathbb{N}^m \rightarrow \mathbb{R} \quad (1)$$

where:  $\mathcal{D} = \{x=(n_1, n_2, \dots, n_m): \forall i \in \mathbb{N}, i \leq m \ n_i \in \mathbb{N}\}$  – an acceptable set of all possible routes being held by  $m$  points of receipt of goods,  
 $\mathbb{R}$  – a set of real values of objective function.

In the method of the weighted objectives the functions expressing the criteria are linked using weighted coefficients as follows:

$$f: (n_1, n_2, \dots, n_m) \rightarrow w_1 f_1 + w_2 f_2 + \dots + w_n f_n \quad (2)$$

where:  $f_i$  – criteria,  $i=1, \dots, n$ ,  
 $n$  – number of criteria,  
 $w_i$  – weighted coefficients;  $i=1, \dots, n$ ; often it assumes that

$$\sum_{i=0}^n w_i = 1 \quad (3)$$

As a result, multi-criteria task is reduced to one-criteria, which can be solved by the methods used to optimize one objective function.

In the case where the minimization of the road is the first criterion, and a travel time is a second, the objective function has the following equation:

$$f = w_1 \frac{d_{oi_1} + \sum_{k=1}^{m-1} d_{i_k i_{k+1}} + d_{i_m 0}}{d_{\max}} + w_2 \frac{t_{oi_1} + \sum_{k=1}^{m-1} t_{i_k i_{k+1}} + t_{i_m 0}}{t_{\max}} \quad (4)$$

where:  $m$  – number of delivery points,  
 $0$  – warehouse,  
 $\{i_1, i_2, \dots, i_m\}$  – a sequence of points in the order of service,  
 $d_{ij}$  – the shortest distance form  $i$ -th point to  $j$ -th point,  
 $t_{ij}$  – the shortest travel time form  $i$ -th point to  $j$ -th point,  
 $w_1, w_2$  – weight coefficients,  
 $d_{\max}$  – the maximum route length estimated in advance,  
 $t_{\max}$  – the maximum travel time estimated in advance.

In the weighted objectives method, the selection of appropriate weight coefficients may be of a problem since manipulating them is subjective. It calls for assigning a dominant criteria in a situation where all of them are equal.

## 2.2. Pareto efficiency

Pareto efficiency is the other method of solving the problem. It is frequently used in applications of genetic and evolutionary algorithms. The models described in (Goldberg, 1989) were followed to perform calculations presented in this paper.

Pareto optimality is a state where the improvement of one criterion has a negative impact on another one. In the case of minimization, a solution not dominated is defined as:

$$\mathbf{f}^*=(\mathbf{f}_1, \dots, \mathbf{f}_m): \quad \forall_{i \in \mathbb{N}} \mathbf{f}_i^* \leq \mathbf{f}_i \wedge \exists_{j \in \mathbb{N}} \mathbf{f}_j^* < \mathbf{f}_j \quad (5)$$

which means that the function  $\mathbf{f}$  is dominated by  $\mathbf{f}^*$  ( $\mathbf{f}^* < \mathbf{f}$ ).

A set of non-dominated solutions is called the Pareto front. The best solution is selected from the Pareto front. In the case of minimizing road and travel time optimization criteria can be expressed in the following objective function:

$$\mathbf{f}: \mathcal{D} \subset \mathbb{N}^m \rightarrow \mathbb{R}^2 \quad (6)$$

where the meanings are as in formula (1). To put it differently an objective function is the following mapping of:

$$\mathbf{f}: (n_1, n_2, \dots, n_m) \rightarrow (s, t) \quad (7)$$

where:  $s$  – the distance between the pickup points in the order given,  
 $t$  – the time needed to cover the selected option.

In terms of numerical solutions, pairs are searched  $(s^*, t^*)$  such as

$$\forall (s, t) \in \mathbb{R}^2 \exists s^* \in \mathbb{R}: s^* \leq s \vee \exists t^* \in \mathbb{R}: t^* \leq t \wedge (s, t) \neq (s^*, t^*) \quad (8)$$

### 3. NUMERICAL MODEL

#### 3.1. Clonal Selection

In this model artificial immune system has been utilised to designate routes for deliveries.

The artificial immune system is one of the methods of artificial intelligence which has been inspired by the human immune system (Wierzchoń, 2001). This network is activated when foreign antigens invading an organism have overcome the body's mechanical barriers such as the skin, mucus membranes, and the cornea.

When antigens get into the bloodstream they are captured by antibodies. In order to remove antigens, first they must be physically immobilized. This involves the reshaping of the antibody to bind to the antigen. Then antibodies are mutated. These antibodies, which are best matched to the antigen, are abundantly cloned and with the blood penetrate throughout the body in search of the enemy.

Captured antigens are destroyed, but the way in which this takes place is no longer essential for the numerical model. When antigens do not threaten the body any longer, the antibody population is suppressed. The patterns to diagnose the next attack are stored in the body. This simplified model of recognition of antigen presented here is called a clonal selection and it is a paradigm of clonal selection of artificial immune system.

### 3.2. Artificial Immune System

Numerical algorithm imitating the clonal selection can be described by first defining:

antigen – the optimum solution for the task,

antibody – an approximate solution,

affinity – measure of fitting an antibody to the antigen – the value of the objective function for a given solution,

population of antibodies – a finite set of different solutions,

cloning – copying existing solutions,

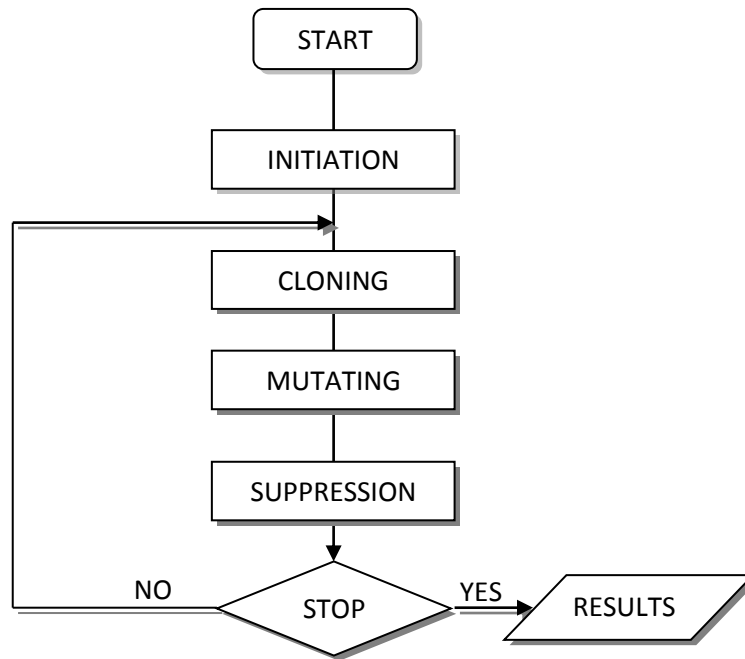
mutating – mapping  $M: (n_1, n_2, \dots, n_m) \rightarrow (n_{1j}, n_{2j}, \dots, n_{mj})$ , which changes the order of selected points  $n_j$ .

The steps of the algorithm are as follows: at first, the random population of antibodies is generated and an affinity of antibodies is evaluated.

Next the best fitted antibodies are cloned and mutated. Each mutated antibody is evaluated. The best antibodies pass to the next generation. The rest is eliminated in the process of suppression. Everything is repeated until the condition to stop the calculation. A flowchart of clonal selection is shown in Figure 1.

The evaluation of the affinity, important in this algorithm, depends on the accepted criterion. The route length and the travel time are determined for each solution in every method of optimization. These parameters are substituted into the formula (3) in weighted objectives method for calculating the affinity.

The affinity rating in the case of Pareto optimal solutions requires consideration of some steps of the algorithm (Figure 1) of the substeps as described in Figure 2.



**Fig. 1. Clonal selection's algorithm (own study)**

1. In the entire population of antibodies all individuals not dominated are searched (Pareto front) and the highest rank is assigned to them.
2. In a subpopulation consisting of the remaining antibodies another Pareto front is determined and antibodies from it receive a lower rank.
3. The substep 2 is repeated until each antibody from the population receives a rank.
4. Antibodies are subjected to cloning. Number of clones is directly proportional to the rank of the antibodies.
5. In the process of the suppression, the antibodies are eliminated sequentially starting from the lowest ranks.

**Fig. 2. Clonal selection algorithm – Pareto optimality (own study)**

#### 4. CALCULATION

The calculations of Pareto optimality was made using komi\_pareto01i.exe (Figure 3).

```

int i,j;
printf("\n front ini");
for(i=0;i<popu
{
if(antycbo
{
antycb
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for(j=
{
if(
{
20 17 16 32 33 31 30 29 28 14 13 15 10 2 3 4 34 38 5 6 11
12 26 27 24 25 40 41 35 36 39 37 45 42 48 49 47 46 43
9 1 -1 128.000001 13736.000000 1000 1000 * 44 7 1 8 9 21 23 22 19 18
20 17 16 32 33 31 30 29 28 14 13 15 10 2 3 4 34 38 5 6 11
12 26 27 24 25 40 41 35 36 39 37 45 42 48 49 47 46 43
10 1 -1 129.100000 13729.000000 1000 1000 * 44 1 7 8 9 21 23 22 19 20
18 17 16 32 33 31 30 29 28 14 13 15 10 2 3 4 34 38 5 6 11
12 24 27 26 25 40 41 35 36 39 37 45 42 48 49 47 46 43
11 1 -1 127.700000 13771.000000 1000 1000 * 44 7 1 8 21 9 23 22 20 18
19 17 16 32 33 31 30 29 28 14 13 15 10 2 3 4 34 38 5 6 11
12 24 27 26 25 40 41 35 36 39 37 45 42 48 49 47 46 43
12 1 -1 129.400000 13729.000000 1000 1000 * 44 1 7 8 9 21 23 22 19 18
20 17 16 32 33 31 30 29 28 14 13 15 10 2 3 4 34 38 5 6 11
12 24 26 27 25 40 41 35 36 39 37 45 42 48 49 47 46 43
13 1 -1 129.500000 14004.000000 1000 1000 * 44 1 7 8 9 21 23 22 19 18
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for(j=
{
18 20 17 16 32 33 31 30 29 28 14 13 15 10 2 3 4 34 38 5 6
11 12 24 26 27 25 40 41 35 36 39 37 45 42 48 49 46 43
***** K O N I E C *****
Press any key to continue
    
```

Fig. 3. The window of komi\_pareto01i.exe with the end results (own study)

Komiwojazer08i.exe was used for calculations by the weighted objectives method (Figure 4).

```

44 2 0.000000
45 0 0.000000
45 1 0.000000
45 2 0.000000
46 0 0.000000
46 1 0.000000
46 2 0.000000
47 0 0.000000
47 1 0.000000
47 2 0.000000
48 0 0.000000
48 1 0.000000
48 2 0.000000
49 0 0.000000
49 1 0.000000
49 2 0.000000
Obliczenia trwaj
300 49 404 0 0 0.586166 0.586166 0.000000 170.599999 18397.000000 0.
000000
***** K O N I E C *****
Press any key to continue
    
```

Fig. 4. The window of Komiwojazer08i.exe with the end results (own study)

Komi\_pareto01i.exe and komiwojazer08i.exe are own implementations of Artificial Immune System in C++.

As already mentioned, the purpose of the calculation is to obtain the shortest route and the quickest time of delivery from a warehouse to customers. On a selected day the warehouse is required to deliver goods to 49 recipients using one van. There is no time limit. The shortest distance and travel time between collection points are available. Table 1 shows the distance and transit times.

**Tab. 1. The distance and transit times for each edge of the graph (own study)**

No. edge	the beginning of the edge	the end of the edge	distance [km]	Travel time [s]	No. edge	the beginning of the edge	the end of the edge	distance [km]	Travel time [s]	No. edge	the beginning of the edge	the end of the edge	distance [km]	Travel time [s]
1	1	3	0,8	88	34	14	32	4,3	345	66	48	45	4,8	713
2	1	2	1	99	35	32	33	1,7	139	67	47	49	2	323
3	2	10	3	304	36	31	33	3,6	261	68	46	47	2,8	325
4	1	9	2,1	216	37	31	32	4,4	344	69	43	46	2,6	372
5	1	7	2,2	254	38	29	32	2,7	178	70	46	48	4,1	535
6	1	44	3,1	435	39	27	30	5,6	380	71	43	44	2	254
7	1	43	4,6	625	40	26	27	2,1	162	72	44	50	1,2	182
8	1	42	3,1	381	41	24	26	2,9	220	73	22	50	3,8	532
9	2	5	2,1	272	42	24	29	4,3	308	74	8	17	2,9	345
10	3	5	2,2	285	43	12	28	2,9	227	75	17	19	1,3	180
11	3	4	0,6	80	44	12	24	2,5	151	76	19	22	2,2	256
12	4	5	2	236	45	24	25	2,2	136	77	21	22	1,9	247
13	2	9	2,6	351	46	25	39	6,9	444	78	17	18	2	300
14	2	8	2,8	349	47	10	11	3,4	192	79	8	10	2,1	250
15	8	9	1,2	139	48	11	12	1	63	80	43	50	2	238
16	8	21	1,8	251	49	6	11	3,9	323	81	23	44	2,3	331
17	9	21	0,7	100	50	5	10	4,1	503	82	43	48	6,2	481
18	7	9	2	247	51	5	6	2	254	83	47	48	5,7	716
19	9	23	1,1	146	52	6	38	3,8	433	84	49	48	4,8	581
20	21	23	1,5	211	53	5	38	2,5	333	85	42	48	1,7	205
21	22	23	1,2	198	54	4	38	3	352	86	42	45	2,4	288
22	20	22	3,2	471	55	4	34	2,1	235	87	34	45	4,5	575
23	19	21	3,2	407	56	34	38	1,5	143	88	38	41	7,5	572
24	19	20	1,5	202	57	36	39	3,8	479	89	35	41	5,1	473
25	18	20	1,6	178	58	35	36	2	252	90	40	41	6,1	766
26	18	19	1	108	59	35	39	3,2	478	91	36	40	2,5	313
27	16	18	1,6	183	60	37	39	1,8	236	92	16	32	4,1	335
28	16	17	1,2	139	61	35	37	2	247	93	13	32	3,1	255
29	15	17	3,8	307	62	37	38	2	329	94	11	28	2,6	195
30	10	15	1,1	76	63	37	6	5,9	683	95	28	29	2,4	160
31	13	28	3,2	230	64	38	45	3,3	411	96	29	30	1	70
32	13	15	1	106	65	45	49	1,5	202	97	12	25	6,2	393
33	13	14	1,6	178										

The calculation results for the method of weighted criteria, and for Pareto optimality are listed in Table 2. Despite using coefficients in a function (3) which according to the experience of the author were well matched, the results of Pareto optimal solution are much better. As you can see the both solutions of weighted objectives method are dominated by solutions of Pareto efficiency method. Both solutions of Pareto optimality do not dominate each other. In the first solution, the road is shorter than in the other one, and in a second travel time is less than in the first solution.



**Tab. 2. The best results of the calculation (own study)**

	route length	travel time [s]	The sequence of points in the order of service																																																		
Weighted Objectives Method	150,2	16305	0	43	44	7	9	8	21	22	23	1	2	5	6	11	12	28	29	30	31	33	32	16	18	20	19	17	15	13	14	24	25	26	27	10	3	4	34	38	45	49	47	46	48	42	35	41	40	36	39	37	0
			0	43	44	23	9	21	22	19	20	18	16	17	8	2	1	3	4	34	38	45	49	47	46	48	42	10	13	14	15	5	6	37	39	25	24	12	11	28	29	32	33	31	30	27	26	40	36	35	41	7	0
			0	44	1	7	9	8	21	23	22	19	20	18	17	16	32	33	31	30	29	28	13	14	15	10	2	3	4	34	38	5	6	11	12	24	27	26	25	40	41	35	36	39	37	45	42	48	49	47	46	43	0
	0	44	7	1	8	9	21	23	22	20	18	19	17	16	32	33	31	30	29	28	13	14	15	10	2	3	4	34	38	5	6	11	12	26	27	24	25	40	41	35	36	39	37	45	42	48	49	47	46	43	0		
	129	13729	0	44	1	7	9	8	21	23	22	19	20	18	17	16	32	33	31	30	29	28	13	14	15	10	2	3	4	34	38	5	6	11	12	24	27	26	25	40	41	35	36	39	37	45	42	48	49	47	46	43	0
	127,5	13736	0	44	7	1	8	9	21	23	22	20	18	19	17	16	32	33	31	30	29	28	13	14	15	10	2	3	4	34	38	5	6	11	12	26	27	24	25	40	41	35	36	39	37	45	42	48	49	47	46	43	0

The solution to the problem using each of the methods has been achieved in a similar, very short computational time. It can therefore be expected that the use of Pareto optimality in further calculations will give good results too.

## 5. CONCLUSIONS

The Pareto optimality method used to plan the shortest and fastest route of the delivery van is effective and as fast as the method of weighted factors. Pareto optimal solutions are better than those obtained with the weighted objectives method, despite using the coefficients that have been tested in this type of calculation.

The chosen example is a simplified fragment of a larger, more complex, with restrictions on the delivery time and takes into account the varying intensity of movement. Results of calculations encourage the author to use the Pareto optimality method to solve this bigger example and in further studies.

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