

Power transfer analysis in a single phase dual active bridge

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Abstract. This paper presents an analysis of the power transfer between two DC circuit by use a single phase galvanically isolated dual active bridge – DAB. The analytical description of instantaneous values of the currents in both DC and in AC circuits of the DAB is done. The influence of the dead time as well as voltage drops across the transistors and diodes of the bridges is examined. The different relations between voltages of the DC circuits coupled through DAB and various phase shift ratios are considered. The analytical relations describing the average values of the currents in DC circuits are derived. These currents can be used to predict the power in both DC circuits and power losses generated in semiconductor devices of the converter. It is assumed that the voltage drops across these devices in conduction states are constant. The calculation of the transferred power as well as power losses and energy efficiency for the DAB converter power rated 5600 VA which is used to energy transfer between DC circuits 280 V and 51 V \pm 20% is presented. The proposed relations and calculation results can be useful for preliminary evaluation of power losses generated in semiconductor devices and for design of the cooling system. Due to the high switching frequency of 100 kHz, the phase shift modulation for the control of DAB is used. To validate the theoretical investigations a few experimental results are presented.

Key words: bidirectional active bridge – DAB, power losses, shift phase modulation.

1. Introduction

The converters which enable bidirectional energy transfer mainly between DC circuits with different voltage levels and also assure galvanic separation are more and more widely used in power distribution systems (photovoltaic and wind energy, energy storage such as battery and supercapacitor) [1–7], automotive (hybrid and electrical vehicles) [8–11] as well as aerospace [12] applications. These converters have a topology consisting of two single- or three-phase bridges connected by a single- or three-phase medium- or high-frequency (e.g. 20–100 kHz) transformer providing galvanic separation between low-voltage and high-voltage sides. The three-phase topologies are generally appropriate for large power applications and require relative complicated control systems [4]. The DAB converters with single- or three-phase transformers can be made with the use of resonant topologies to limit the switching power losses [3–15]. However, the converters with resonant ZVS and quasi-ZCS scheme need additional passive elements. Thanks to the high power density (more than 10 kW/dm³) the DAB converters without resonant circuits are very interesting [5–12].

Achieving high power density is linked together with a necessity of precise determination of the power losses generated in all components of the converter, including particularly semiconductor devices and magnetic elements. Few papers have dealt with the determination of power losses and energy efficiency of the DAB converters [6, 16–21]. Most of them present complex mathematical relationships which are difficult to use in practice [10, 17]. In the literature there are presented also many experimental methods that enable global losses determination without the possibility of identifying the

losses in each elements of the converter [18, 20]. It is worth to notice the work [21], where the authors present an analytical description based on a simplified model of the DAB converter. In this work, the proposed analytical power transfer model captures the power flow characteristics by accounting the dead time and constant voltage drops on semiconductor devices, however the authors discussed only the positive phase shift between the both rectangular AC voltages applied to the transformer windings by an assumption that the DAB converter is fully symmetric and only a boost direction has been investigated. A new steady-state model for the DAB was presented in [12]. The model produced equations for device rms and average current, which can be useful in calculation the power losses occurred in devices. However, the equations do not take into account the impact of voltage drops on instantaneous values of AC voltage and currents in both DC circuits.

The subject of this paper is the DAB converter consisting two, realized using MOSFETs, single phase H bridges, which can serve as an interface between renewable energy sources, such an photovoltaic array or fuel cell stack and energy storage devices such as a supercapacitor bank or battery system (Fig. 1). The full bridge H1 on the left hand side of Fig. 1a is connected to the high-voltage DC port (E_1) and the full bridge H2 on the right hand side is connected to the low-voltage DC circuit (E_2). The AC sides of both H bridges are connected through a high – frequency planar transformer (100 kHz) and a coupling inductors $2 \times L_d/2$. The electrical connection between two bridges can be presented by the simplified scheme shown in Fig. 1b, where the AC voltages u_1 and u_2/n are referred to the HV side and L is the sum of the transformer leakage inductance L_σ and the auxiliary inductances $2 \times L_d/2$. The inductance L is a very important parameter, which has

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a strong influence on the power flow and the range of phase shift ratio. It is assumed that the DC high-voltage is constant ($E_1 = 280$ V, e.g. obtained by use DC/DC boost converter) and the DC voltage E_2 can fluctuate in the range $51 \text{ V} \pm 20\%$. Owing to a high switching frequency of semiconductor devices in both converters the dimensions of transformer power rated 5600 VA are small (ca $8 \text{ cm} \times 6 \text{ cm} \times 2.5 \text{ cm}$).

The operation range of the DAB can be divided into two regions: A and B, which are defined as illustrated in Fig. 1c. For the region A the DC voltage E_2 is in the range [40.8 V; 51 V] and for the region B this voltage can change in range [51 V; 61.2 V]. For the both regions following relations are valid

$$\text{A: } E_1 > \frac{E_2}{n} \text{ hence } k_u = \frac{E_2}{nE_1} < 1, \quad (1)$$

$$\text{B: } E_1 < \frac{E_2}{n} \text{ hence } k_u = \frac{E_2}{nE_1} > 1, \quad (2)$$

where k_u – DC voltage conversion ratio.

For transformer turns ratio $n = N_2/N_1 = 2/11$ and at constant value of $E_1 = 280$ V the balance of the DC voltages occurs for the nominal value of $E_{2Z} = 51$ V. For the

$40.8 \text{ V} \leq E_2 \leq 61.2 \text{ V}$ the voltage conversion ratio changes between $k_{u(\min)} = 0.80$ and $k_{u(\max)} = 1.20$. The DAB considered in this paper allows bidirectional power transfer by means of controlling the phase shift (expressed in time ore angle domain) between square – waves u_1 and u_2 generated by the bridges H1 and H2 [16, 17, 20, 21].

This paper presents the analytical description of the instantaneous (i_{E1}, i_{E2}) and average (I_{E1}, I_{E2}) currents in both DC circuits considering not only the so called major parameters, i.e. DC voltages and phase shift between the AC voltages, but also the voltage drops across the semiconductor devices and the dead time. As in [21], the formulas described the powers in both DC circuits and the power losses generated in all semiconductor are formulated as function of the average currents in DC circuits for different relations between the DC voltages (E_1, E_2) and various shift modulation ratio. It is worthwhile to point out, that this paper extends the analysis further and provides unique equations determining the output and input powers not only for positive but also for negative phase shift between the AC quasi-rectangular voltages u_1 and u_2 .

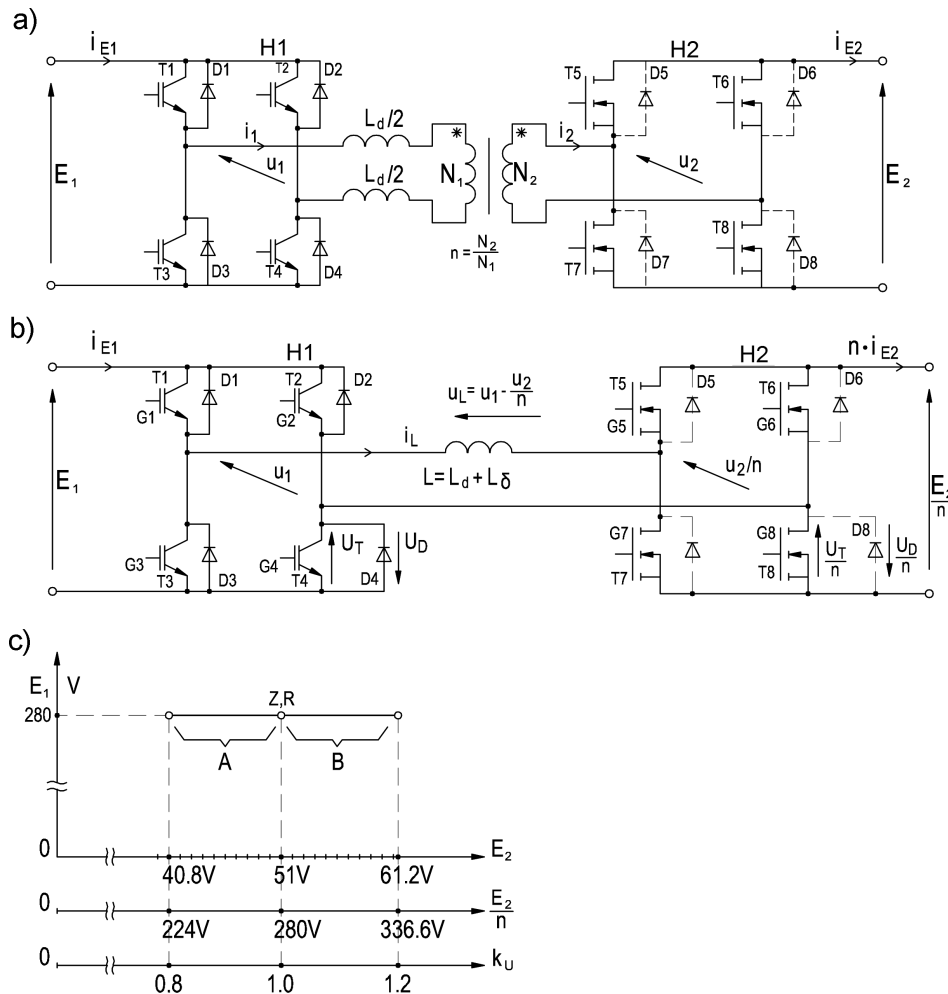


Fig. 1. Two single phase full bridges H1 (high – voltage bridge) and H2 (low – voltage bridge) coupled via high-frequency transformer and two inductors (DAB converter): a) complete scheme of the converter; b) equivalent circuit with parameters referred to high-voltage side: $L = L_\sigma + L_d = 21 \mu\text{H}$; c) graphic representation of the DC voltages both DC circuits. Z – point corresponds with the nominal value $E_2 = 51$ V for which the both DC voltages are balanced, i.e. $E_1 = E_2/n$, where $n = N_2/N_1 = 2/11 = 0.1818$ – transformer turns ratio

The rest of this paper is organized as follows. Section 2 is devoted to the presentation of the basic operation principle and the analysis of the power flow in DAB converter neglecting the voltage drop on the transistors ($U_T = 0$) and diodes ($U_D = 0$) and ignoring the dead time t_{dead} , i.e. the time interval between the gate impulses of the transistors constituting one branch in the bridges. Section 3 focuses on the analytical description of power flow in the DAB according to the voltage drop across semiconductor devices ($U_T \neq 0$, $U_D \neq 0$) at dead time t_{dead} and phase shift ratio D equals to zero. In Sec. 4 the power flow analysis with considering the voltage drop on the semiconductor devices ($U_T \neq 0$, $U_D \neq 0$) and the dead time ($t_{dead} \neq 0$) at phase shift D equal to zero is given. Section 5 presents the analytical relations suitable to determination the input and output powers and the power losses of the DAB by accounting the voltage drop on semiconductor devices and the dead time at positive value of the phase shift ratio ($D > 0$). Similarly to Sec. 5, in Sec. 6 the analysis of power transfer conditions but at negative value of the phase shift ($D < 0$) are presented. In Sec. 7 there are presented the calculated values of the powers in DC circuits as well as power losses and energy efficiency (excluding the power losses generated in transformer and inductors) for DAB converter with the following parameters: $E_1 = 280$ V; $E_2 = 51$ V \pm 20%; $L = 21$ μ H; $P_{max} = 5600$ VA. Several experimental results which validate the theoretical calculations are also given. In Sec. 8 the conclusions are drawn.

2. Simplified analysis of the power transfer in DAB ($U_T = 0$, $U_D = 0$, $t_{dead} = 0$)

In the used phase shift modulation method each bridge is controlled to generate a square wave (with 50% duty cycle) voltage at its AC terminals. Neglecting the dead time and the power losses generated in semiconductor devices, the value and flow direction of the power transferred between two DC circuits depend on the both DC voltages (E_1 and E_2), phase shift (expressed in angle ϕ or time delay t_{del} domain) between the two quasi-rectangular AC voltages u_1 and u_2 , transformer turns ratio n and the sum of inductances L in AC circuit. The power flows always from the bridge generating the leading square – wave. The phase shift (time delay t_{del}) is defined as positive if the power flows from the E_1 side to the E_2 side. The opposite direction of the energy flow occurs at the negative phase shift, i.e. if voltage u_2 leaves the voltage u_1 . In this paper the phase shift is expressed using the so called normalized phase shift ratio $D = t_{del}/(T/2)$, where T is the switching period.

Operating with small phase shift ($DT/2 = t_A$). Omitting the power losses generated in semiconductor devices ($U_T = 0$, $U_D = 0$) and in magnetic elements the following relations are accomplished:

$$P_{E1} = E_1 I_{E1} = E_1 I_{E2} n, \quad (3)$$

$$P_{E2} = E_2 I_{E2} = E_2 I_{E1} \frac{1}{n}, \quad (4)$$

$$P_{E1} = P_{E2}, \quad (5)$$

where P_{E1} , P_{E2} , I_{E1} , I_{E2} – powers and average currents in E_1 and E_2 side, respectively.

For the small phase shift ratio D and at $k_u \neq 1$, in the time interval $DT/2$ there is no change in direction of the current i_L in AC link and currents i_{E1} , i_{E2} in both DC circuits. In this case the relation is satisfied: $DT/2 = t_A$ (Fig. 2). By assumption that the DC voltages are constant, determination of the powers P_{E1} and P_{E2} is reduced to the determination of the average currents I_{E1} and I_{E2} . On the basis of Fig. 2, for the conditions occurring in both ranges A and B (i.e. $k_u < 1$ and $k_u > 1$) at $DT/2 = t_A$ can be written

$$I_1 = -\frac{nu_1 - u_2}{nL} t_B = \frac{nu_1 - u_2}{nL} t_A + I_0, \quad (6)$$

$$I_3 = \frac{nu_1 - u_2}{nL} t_C = -I_0. \quad (7)$$

By assumption that $U_T = 0$ and $U_D = 0$ it can be derived

$$I_1 = -\frac{nE_1 - E_2}{nL} t_B = \frac{nE_1 + E_2}{nL} t_A + I_0, \quad (8)$$

$$I_3 = \frac{nE_1 - E_2}{nL} t_C = -I_0. \quad (9)$$

From equations (8) and (9) indicate that

$$I_0 = -\frac{nE_1 - E_2}{nL} t_B - \frac{nE_1 + E_2}{nL} t_A, \quad (10)$$

$$I_0 = -\frac{nE_1 - E_2}{nL} t_C \quad (11)$$

and hence

$$(nE_1 + E_2) t_A + (nE_1 - E_2) t_B - (nE_1 - E_2) t_C = 0. \quad (12)$$

Taking into account that $t_A + t_B + t_C = T/2$ and $t_A = DT/2$, from Eq. (12) the following equations can be derived

$$t_B = \frac{T}{2} \frac{nE_1 - E_2 - 2DnE_1}{2(nE_1 - E_2)} = \frac{T}{4} \frac{nE_1(1 - 2D) - E_2}{nE_1 - E_2}, \quad (13)$$

$$t_C = \frac{T}{2} - \frac{T}{2} \frac{nE_1 - E_2 - 2DnE_1}{2(nE_1 - E_2)} - D \frac{T}{2} = \frac{T}{4} \frac{nE_1 + E_2(2D - 1)}{nE_1 - E_2}. \quad (14)$$

The average current I_{E1} can be determined by a simple manner through calculation the area between the instantaneous waveform i_{E1} and the time axis (triangles and rectangles) dividing the result by the half of switching period $T/2$. According to Fig. 2a for the region A (i.e. for $E_1 > E_2/n$) the following equation can be obtained

$$I_{E1} = \frac{2}{T} \left[\frac{1}{2} (I_0 - I_1) t_A + I_1 t_A + \frac{1}{2} I_1 t_B + \frac{1}{2} I_3 t_C \right] = \frac{1}{T} [I_1 (t_A + t_B) + I_3 (t_C - t_A)]. \quad (15)$$

For the operating condition in the region B (i.e. for $E_1 < E_2/n$), on the basis of Fig. 2b, the average current in the circuit E_2 can be expressed as

$$I_{E1} = \frac{2}{T} \left[\frac{1}{2} (I_1 - I_0) t_A + I_0 t_A + \frac{1}{2} I_1 t_B + \frac{1}{2} I_3 t_C \right] = \frac{1}{T} [I_1 (t_A + t_B) + I_3 (t_C - t_A)]. \quad (16)$$

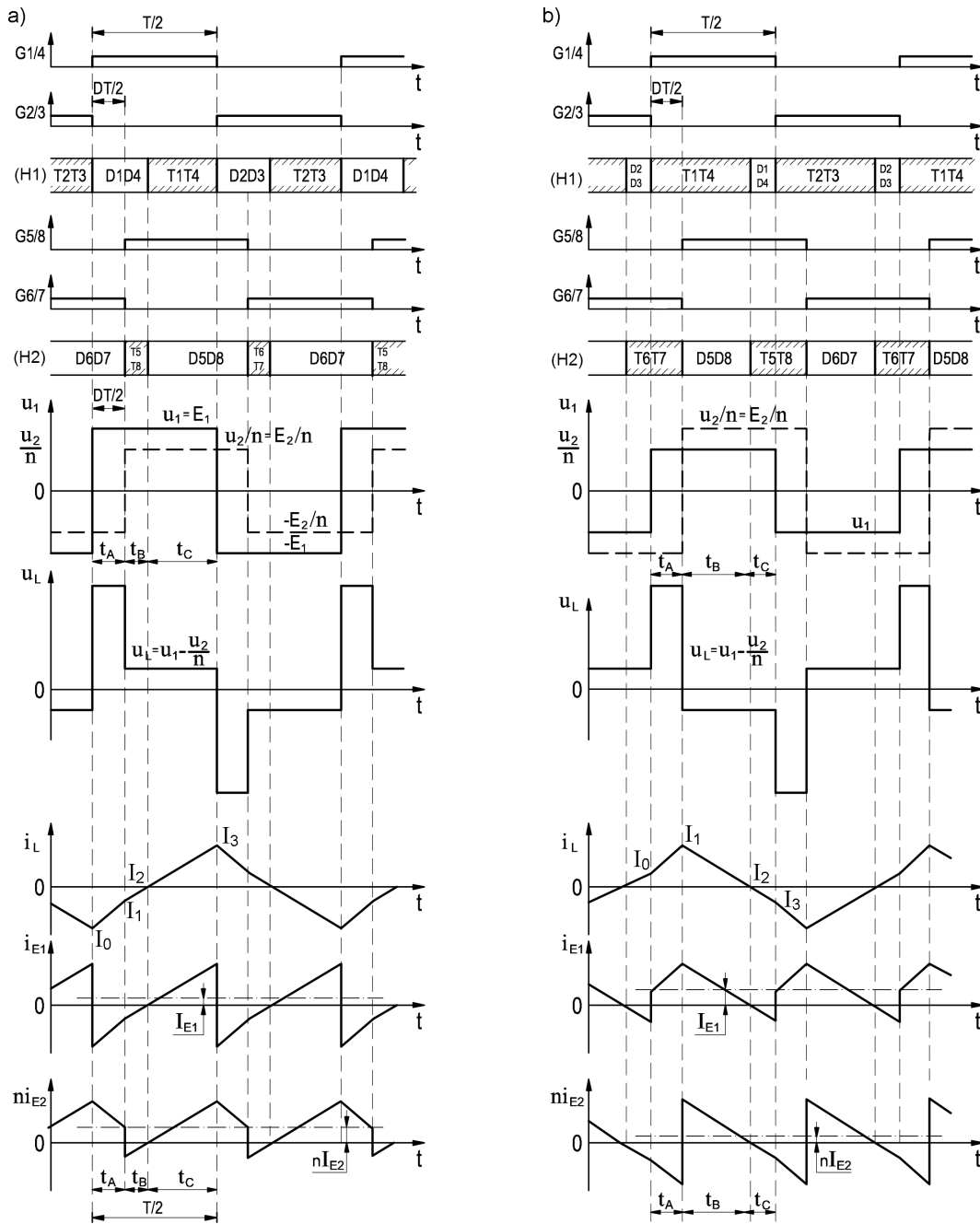


Fig. 2. Simplified waveforms of the transistors control signals ($G1 \div G8$) as well as voltage and currents without consideration of the voltage drop across semiconductor devices ($U_D = 0; U_T = 0$) and dead time ($t_{dead} = 0$) at small phase ratio $DT/2 = t_A$: a) $E_1 > E_2/n$; b) $E_1 < E_2/n$; positive instantaneous value of the voltages and currents correspond to the directions marked on scheme in Fig. 1

Using the formulas (3) and (4) as well as (15) and (16) the power in both DC circuits may be determined

$$P_{E1} = P_{E2} = \frac{1}{T} E_1 [I_1 (t_A + t_B) + I_3 (t_C - t_A)]. \quad (17)$$

Operating with large phase shift ratio ($DT/2 = t_A + t_B$). In the case of the phase shift ratio D , large enough that, regardless of the voltage conversion ratio k_u , during the time interval $DT/2$ there is a change in direction of currents i_L, i_{E1} and i_{E2} , from Fig. 3, for the regions A and B the following can be written

– in the time interval t_A :

$$u_{LtA} = E_1 + \frac{E_2}{n}, \quad (18)$$

$$I_0 = -\frac{nE_1 + E_2}{nL} t_A = -I_3, \quad (19)$$

– in the time interval t_B :

$$u_{LtB} = E_1 + \frac{E_2}{n}, \quad (20)$$

$$I_2 = \frac{nE_1 + E_2}{nL} t_B, \quad (21)$$

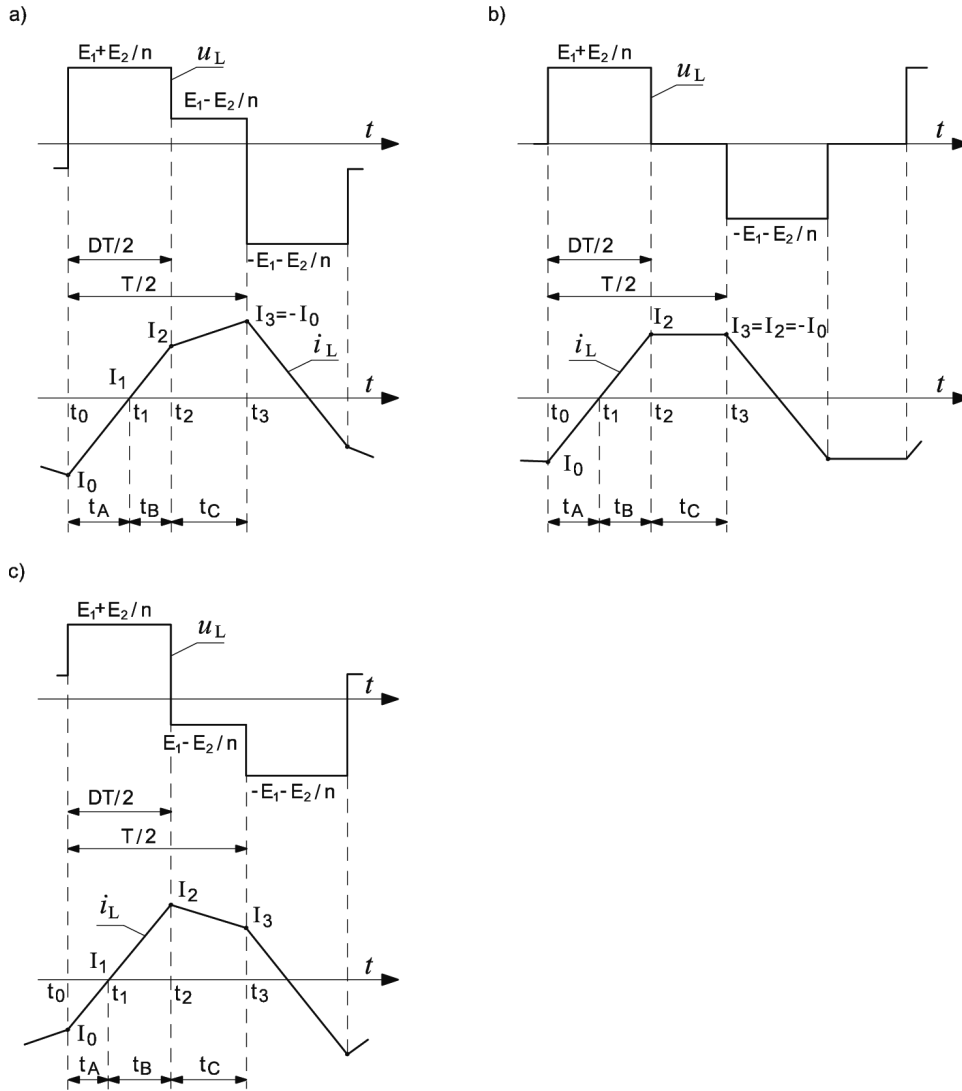


Fig. 3. The simplified voltage and current waveforms in DAB converter operating with large phase shift ratio D , which satisfied the relation $DT/2 = (t_A + t_B)$: a) $E_1 > E_2/n$; b) $E_1 = E_2/n$; c) $E_1 < E_2/n$

– in the time interval t_C :

$$u_{L t_C} = E_1 - \frac{E_2}{n}, \quad (22)$$

$$I_3 = I_2 + \frac{E_1 - \frac{E_2}{n}}{L} t_C = \frac{nE_1 + E_2}{nL} t_B + \frac{nE_1 - E_2}{nL} t_C. \quad (23)$$

From (19) and (23) the following relation can be formulated

$$(nE_1 + E_2) t_A - (nE_1 + E_2) t_B - (nE_1 - E_2) t_C = 0. \quad (24)$$

Taking into consideration $DT/2 = t_A + t_B$ and $t_C = T(1-D)/2$ from (24) the following equation can be obtained

$$t_A = \frac{T n E_1 + E_2 (2D - 1)}{4 n E_1 - E_2}. \quad (25)$$

For the given t_A , t_B and t_C , in accordance with (21) and (22) the characteristic values I_2 and I_3 of the current i_L can

be calculated. The average currents I_{E1} and I_{E2} in both DC circuits are described by

$$I_{E1} = n I_{E2} = \frac{2}{T} \left[-I_3 \frac{1}{2} t_A + \frac{1}{2} I_2 t_B + \frac{1}{2} (I_2 + I_3) t_C \right] = \frac{1}{T} [I_2 (t_B + t_C) + I_3 (t_C - t_A)]. \quad (26)$$

When $DT/2 = t_A + t_B$, then the power in both DC circuits is expressed as

$$P_{E1} = P_{E2} = \frac{1}{T} E_1 [I_1 (t_A + t_B) + I_3 (t_C - t_A)]. \quad (27)$$

For $k_u = 1$, i.e. $E_1 = E_2/n$, the equation describing the time intervals t_A , t_B , t_C and the characteristics value of the AC current I_2 and I_3 (Fig. 3b) take the simple form as follows

$$t_A = t_B = D \frac{T}{4}, \quad (28)$$

$$I_2 = I_3 = -I_0 = \frac{nE_1 + E_2}{2nL} D \frac{T}{2}. \quad (29)$$

In this case the average current in high-voltage DC side is given by

$$I_{E1} = nI_{E2} = I_2(1 - D) = \frac{T}{4} \frac{nE_1 + E_2}{nL} D(1 - D) \quad (30)$$

hence

$$P_{E1} = P_{E2} = E_1 I_{E1} = E_1 \frac{T}{4} \frac{nE_1 + E_2}{nL} D(1 - D). \quad (31)$$

Equations (17) and (27) can be simplified. Substituting (13) and (25) in (17) and (27) well as considering relation $t_A + t_B + t_C = T/2$, for lossless DAB converter at dead time equal to zero, the known relation can be obtained

$$P_{E1} = P_{E2} = \frac{T}{2} E_1 \frac{E_2}{Ln} D(1 - D). \quad (32)$$

It is worth noting that the sum $nE_1 + E_2$ in (31) for $k_u = 1$, can be presented as $2E_2$, which gives the relation (32).

Using the phase shift ϕ in angular domain, described by

$$\varphi = \pi D. \quad (33)$$

Equation (32) can be presented in the following form

$$P_{E1} = P_{E2} = \frac{E_1 E_2 \varphi (\pi - |\varphi|)}{2\pi^2 L n f} = \frac{E_1 E_2 \varphi (\pi - |\varphi|)}{\pi n \omega L} = \frac{E_2^2 \varphi (\pi - |\varphi|)}{k_u \pi^2 n \omega L}. \quad (34)$$

Neglecting the power losses, for $D = 0$ and for DC voltage conversion ratio $k_u \neq 1$, the instantaneous current i_L has a triangular form ($t_A = 0, t_B = t_C$). The average values I_{E1} and I_{E2} and the active power transferred between the DC circuits are equal to zero.

3. Power transfer analysis including the voltage drop across the semiconductor devices at $D = 0$ and $t_{dead} = 0$

In the presented analysis, it has been assumed that the transistors in the two bridges have the same voltage drop (U_T) and the diodes in the two bridges have the same voltage drop (U_D) in the states of conduction. The analysis was carried out for the region A and B at $D = 0$ (Fig. 4). It is also assumed, that the voltage drops across the semiconductor device are constant and independent of current [21].

The voltage drop on transistor and diodes cause power losses so that $P_{E1} \neq P_{E2}$. In order to determination the power P_{E1} it is necessary to find the relation describing the average current I_{E1} in high-voltage circuit E_1 . According to Fig. 4a for the condition $E_1 > E_2/n$ the following equations can be written

$$I_0 = -\frac{E_1 + 2U_D - \frac{E_2}{n} + \frac{2U_T}{n}}{L} t_B = -\frac{nE_1 + 2nU_D - E_2 + 2U_T}{nL} t_B, \quad (35)$$

$$I_2 = -\frac{E_1 - 2U_T - \frac{E_2}{n} - \frac{2U_D}{n}}{L} t_C = \frac{nE_1 - 2nU_T - E_2 - 2U_D}{nL} t_C. \quad (36)$$

Having regard that $I_0 = -I_2$, by using the Eqs. (35) and (36) the following relations can be obtained

$$(nE_1 + 2nU_D - E_2 + 2U_T) t_B = (nE_1 - 2nU_T - E_2 - 2U_D) t_C, \quad (37)$$

hence

$$\frac{t_B}{t_C} = \frac{nE_1 - E_2 - 2(nU_T + U_D)}{nE_1 - E_2 + 2(nU_D + U_T)}. \quad (38)$$

From (37) it follows that for $E_1 > E_2/n$ the steady state operation of the DAB is possible, if $nE_1 - E_2 > 2(nU_T + U_D)$ and the following relation is satisfied: $t_B < t_C$.

Given that $t_B + t_C = T/2$, based on (37) it can be derived

$$t_B = \frac{nE_1 - E_2 - 2(nU_T + U_D)}{nE_1 - E_2 + (n-1)(U_D - U_T)} \frac{T}{4}, \quad (39)$$

$$t_C = \frac{nE_1 - E_2 + 2(nU_D + U_T)}{nE_1 - E_2 + (n-1)(U_D - U_T)} \frac{T}{4}. \quad (40)$$

According to Eq. (40), the following relations can be formulated: $2(nU_D + U_T) > (n-1)(U_D - U_T)$ and $t_C > T/4$.

Using the formulas (35), (36), (39) and (40) the average current I_{E1} can be determined. Referring to the current waveform i_{E1} plotted in Fig. 4a it can be written

$$I_{E1} = \frac{2}{T} \int_0^{T/2} i_{E1} dt = \frac{2}{T} \left[\frac{1}{2} I_0 t_B + \frac{1}{2} I_2 t_C \right] = \frac{1}{T} I_2 (t_C - t_B), \quad (41)$$

where I_0 and I_2 – currents described by Eqs. (35) and (36); t_B, t_C – time intervals occurred in half cycle $T/2$, resulting from (39) and (40).

For considered conditions ($E_1 > E_2/n; D = 0; U_T \neq 0; U_D \neq 0; t_{dead} = 0$) the power drawing from the DC circuit E_1 can be expressed as

$$P_{E1} = \frac{2}{T} \int_0^{T/2} E_1 i_{E1} dt = E_1 I_{E1} = \frac{1}{T} E_1 I_2 (t_C - t_B) > 0. \quad (42)$$

The power P_{E1} is positive. The power delivered to the DC circuit with the voltage E_2 is described by the following relation

$$P_{E2} = \frac{2}{T} \int_0^{T/2} E_2 i_{E2} dt = E_2 I_{E2}. \quad (43)$$

Equation (43) can be presented in the form [21]

$$P_{E2} = \frac{E_2}{E_1} \frac{2}{T} \int_0^{T/2} E_1 \frac{i_{E1}}{n} dt = \frac{E_2}{E_1} \frac{1}{n} P_{E1} = k_u P_{E1}. \quad (44)$$

Because $t_C \neq t_B$, the powers P_{E1} and P_{E2} , though $D = 0$, are different from zero. For the considered conditions the power P_{E2} is less than P_{E1} .

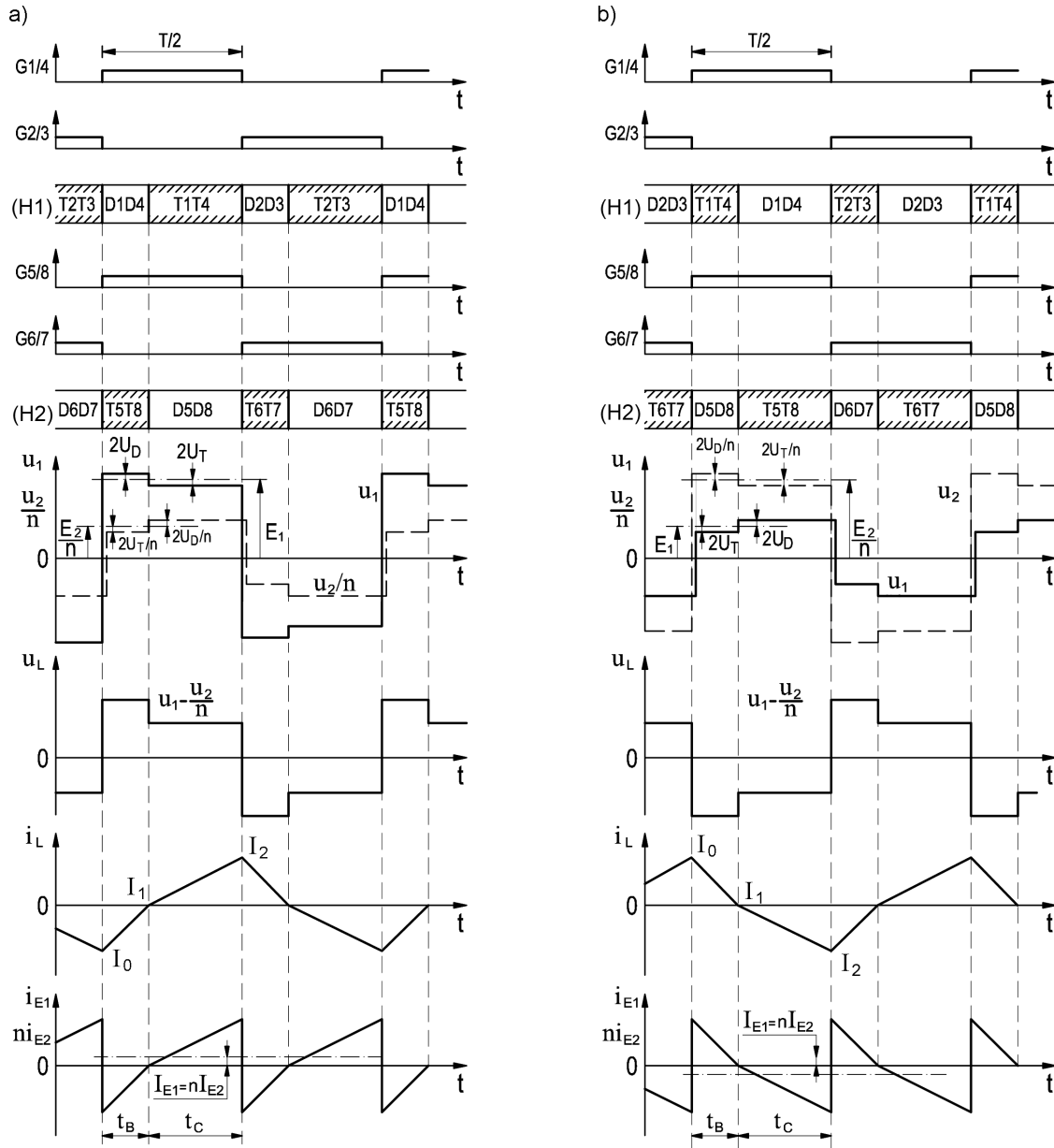


Fig. 4. Simplified waveforms of the transistors control signals ($G1 \div G8$) as well as voltage and currents with consideration of the voltage drop across semiconductor devices ($U_D \neq 0; U_T \neq 0$) for the phase shift ratio $D = 0$ and the dead time $t_{dead} = 0$: a) $E_1 > E_2/n$; b) $E_1 < E_2/n$

Similar investigation can be done for DAB operating in range B, in which the inequality $E_1 < E_2/n$ is validity. On the basis of Fig. 4b the following formulas can be determined

$$I_0 = -\frac{nE_1 - 2nU_T - E_2 - 2U_D}{nL}t_B, \quad (45)$$

$$I_2 = -\frac{E_1 + 2U_D - \frac{E_2}{n} + \frac{2U_T}{n}}{L}t_C = \frac{nE_1 + 2nU_D - E_2 + 2U_T}{nL}t_C. \quad (46)$$

For $I_0 = -I_2$, from (45) and (46) it can be derived

$$(nE_1 - 2nU_T - E_2 - 2U_D)t_B = (nE_1 + 2nU_D - E_2 + U_T)t_C \quad (47)$$

and

$$\frac{t_B}{t_C} = \frac{nE_1 - E_2 + 2(nU_D + U_T)}{nE_1 - E_2 - 2(nU_T + U_D)}. \quad (48)$$

Note, that for the relation $nE_1 - E_2 < 0$ the quotient $t_B/t_C < 1$, and hence $t_B < t_C$.

Taking into account that $t_B + t_C = T/2$, using (48), the time intervals can be calculated as [21]

$$t_B = \frac{nE_1 - E_2 + 2(nU_D + U_T)}{nE_1 - E_2 - (n-1)(U_T - U_D)} \frac{T}{4}, \quad (49)$$

$$t_C = \frac{nE_1 - E_2 - 2(nU_T + U_D)}{nE_1 - E_2 - (n-1)(U_T - U_D)} \frac{T}{4}. \quad (50)$$

From formula (49) it can be concluded that $t_B < T/4$.

Formulas (45), (46), (49) and (50) can be used for calculation the average current in DC circuit E_1 . According to Fig. 4b it can be written

$$I_{E1} = \frac{2}{T} \int_0^{T/2} i_{E1} dt = \frac{2}{T} \left[\frac{1}{2} I_0 t_B + \frac{1}{2} I_2 t_C \right] \quad (51)$$

$$= \frac{1}{T} I_2 (t_C - t_B).$$

Because $I_2 < 0$, the average current is negative and flows into positive pole of the DC circuit E_1 . This current has opposite direction in respect to the direction marked on Fig. 1. This means, that the high voltage circuit receives the power determined by following equation

$$P_{E1} = \frac{2}{T} \int_0^{T/2} E_1 i_{E1} dt = E_1 I_{E1} \quad (52)$$

$$= \frac{1}{T} E_1 I_2 (t_C - t_B) < 0.$$

For condition $E_1 < E_2/n$ at $D = 0$ the power is transferred from low-voltage circuit (E_2) to the high-voltage circuit (E_1) and the absolute value of the power P_{E2} is greater than P_{E1} . The difference between the powers is equal to the power losses generated in semiconductor devices.

4. Power transfer analysis including the voltage drop across the semiconductor devices and the dead time ($t_{dead} \neq 0$) at $D = 0$

In order to avoid a short circuit, in real DAB converter between gate signals of two transistors in each leg of the H-bridges must exist time delay t_{dead} – so called “dead time”. This time, although is very short (split of microsecond), especially at high switching frequency, causes the undesired phase shift between the AC voltages u_1 and u_2 . This additional phase shift, defined as phase drift [21] affect the power transfer.

Discontinuous AC current. If the difference between E_1 and E_2/n and the phase shift ratio are small, then the current i_L is also small. Under such condition the situation in which the current i_L becomes zero may occur (Fig. 5).

For $E_1 > E_2/n$ (i.e. $k_u < 1$), after the pair of transistors (e.g. T_2 and T_3) turned off, there is a time interval $t_A < t_{dead}$, in which the current i_L passes through the diodes of the high – voltages bridge D_1 and D_4 and the diodes of the low – voltage bridge D_6 and D_7 (Fig. 5a). During the time interval, from the moment when the current becomes zero to the moment when the transistors T1, T4 and T5, T8 control impulses occur, i.e. during the time $t_{dead} - t_A = t_B$, the current i_L is equal to zero. In this time all semiconductor devices are in off state and there is no power flow.

At discontinuous current i_L , for $k_u < 1$, there is a positive phase drift D_{dead} , which can vary in the range between 0 to $t_{dead}/(T/2)$, depending on the current value and its time derivative. According to Fig. 5a, the following formulas can be derived:

– in time interval t_A

$$u_{L(tA)} = u_1 - \frac{u_2}{n} = E_1 + 2U_D + \frac{E_2}{n} + \frac{2U_D}{n}, \quad (53)$$

$$I_0 = -\frac{E_1 + 2U_D + \frac{E_2}{n} + \frac{2U_D}{n}}{L} t_A \quad (54)$$

$$= -\frac{nE_1 + 2nU_D + E_2 + 2U_D}{nL} t_A,$$

– in time interval t_C

$$u_{L(tC)} = u_1 - \frac{u_2}{n} = E_1 - 2U_T - \frac{E_2}{n} - \frac{2U_D}{n}, \quad (55)$$

$$I_3 = \frac{E_1 - 2U_T - \frac{E_2}{n} - \frac{2U_D}{n}}{L} t_C \quad (56)$$

$$= \frac{nE_1 - 2nU_T - E_2 - 2U_D}{nL} t_C.$$

Since $I_0 = -I_3$, on the basis of (54) and (56) the following formulas can be written:

$$-(nE_1 + 2nU_D + E_2 + 2U_D) t_A \quad (57)$$

$$= (nE_1 - 2nU_T - E_2 - 2U_D) t_C,$$

$$\frac{t_A}{t_C} = \frac{nE_1 - E_2 - 2(nU_T + U_D)}{nE_1 + E_2 + 2(n+1)U_D}, \quad (58)$$

$$t_A = \frac{nE_1 - E_2 - 2(nU_T + U_D)}{nE_1 + E_2 + 2(n+1)U_D} t_C \quad (59)$$

$$= \frac{nE_1 - E_2 - 2(nU_T + U_D)}{nE_1 + E_2 + 2(n+1)U_D} \left(\frac{T}{2} - t_{dead} \right).$$

Knowing the time t_A , determined by the formula (59), and the time $t_C = T/2 - t_{dead}$ currents I_0 and I_3 can be established.

In accordance with Fig. 5a, the average current I_{E1} can be expressed as follows

$$I_{E1} = \frac{2}{T} \int_0^{T/2} i_{E1} dt = \frac{2}{T} \left[\frac{1}{2} I_0 t_A + \frac{1}{2} I_3 t_C \right] \quad (60)$$

$$= \frac{1}{T} I_3 (t_C - t_A).$$

The power in circuit E_1 is defined by the formula

$$P_{E1} = \frac{2}{T} \int_0^{T/2} E_1 i_{E1} dt = E_1 I_{E1} = \frac{1}{T} E_1 I_3 (t_C - t_A) \quad (61)$$

$$= \frac{1}{T} E_1 I_3 \left(\frac{T}{2} - t_{dead} - t_A \right) > 0.$$

Due to the fact that the average current $I_{E1} > 0$, the power P_{E1} is also positive. This means that it is drawn from the circuit with the value E_1 . As it can be seen from Fig. 5a, the voltage waveform u_1 leads the waveform u_2/n . The delay time between this waveforms corresponds to a phase drift, which is in this case positive.

Taking into account the current waveform i_{E2} (Fig. 5a) its average value can be determined.

$$I_{E2} = \frac{1}{n} \frac{2}{T} \left[-\frac{1}{2} I_0 t_A + \frac{1}{2} I_3 t_C \right] \quad (62)$$

$$= \frac{1}{T} \frac{I_3}{n} (t_A + t_C) > 0.$$

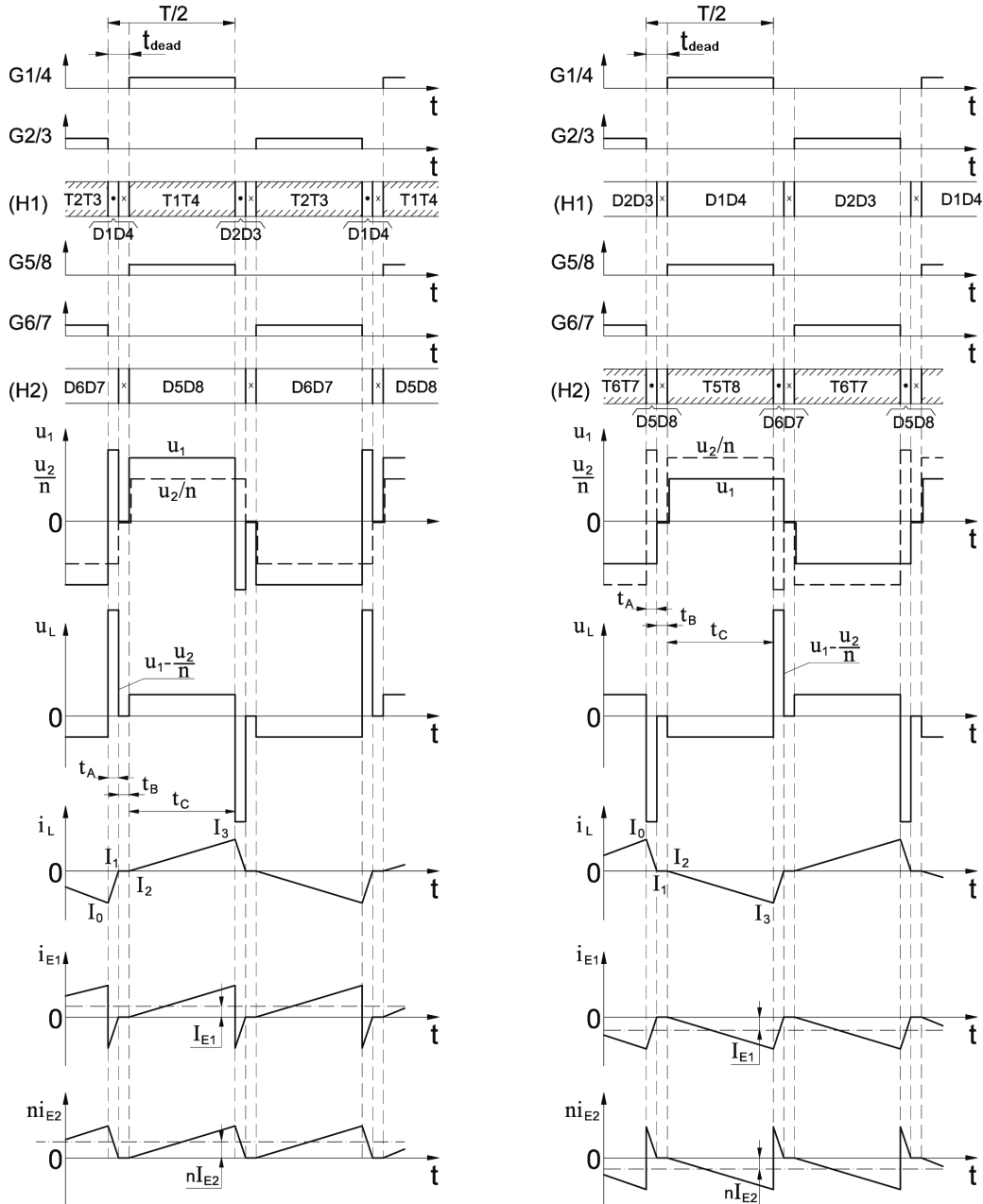


Fig. 5. The influence of the dead time t_{dead} (phase drift) between transistor gate impulses on the currents and voltages in DAB at small current i_L for $D = 0$, $U_T \neq 0$ and $U_D \neq 0$: a) $E_1 > E_2/n$; b) $E_1 < E_2/n$

The power delivered to the E_2 side can be described as

$$P_{E2} = \frac{2}{T} \int_0^{T/2} E_2 i_{E2} dt = E_2 I_{E2} = \frac{1}{T} E_2 \frac{I_3}{n} (t_A + t_C) \quad (63)$$

$$= \frac{1}{T} E_2 \frac{I_3}{n} \left(t_A + \frac{T}{2} - t_{dead} \right) > 0.$$

Since $E_2 = k_u n E_1$, the power P_{E2} is expressed by

$$P_{E2} = \frac{1}{T} k_u n E_1 \frac{I_3}{n} (t_A + t_C) \quad (64)$$

$$= \frac{1}{T} k_u E_1 I_3 (t_A + t_C) \frac{t_C - t_A}{t_C - t_A} = k_u P_{E1} \frac{t_A + t_C}{t_C - t_A}.$$

The power will be transmitted to the circuit E_2 when $P_{E1} > P_{E2}$. This means, that the following condition must be satisfied

$$k_u < \frac{t_C - t_A}{t_C + t_A}. \quad (65)$$

Due to the fully symmetrical topology of the converter, also in the in the region B (i.e. for $k_u > 1$), for small phase shift ratio D the current i_L can be discontinuous. In this case, after turning off the transistor pair of low-voltage bridge (e.g. T_6 and T_7), the state (continuing for the time $t_A < t_{dead}$) wherein the current flows through the diodes in low-voltage bridge (D_5 and D_8) and diodes in high-voltage bridge (D_2 and D_3) occurs. The state without current occurs during time

$t_B = t_{dead} - t_A$. Basing on Fig. 5b the following equations can be written

– in time interval t_A :

$$u_{L(tA)} = u_1 - \frac{u_2}{n} = -E_1 - 2U_D - \frac{E_2}{n} - \frac{2U_D}{n}, \quad (66)$$

$$I_0 = -\frac{nE_1 + 2nU_D + E_2 + 2U_D}{nL}t_A, \quad (67)$$

– in time interval t_C :

$$u_{L(tC)} = u_1 - \frac{u_2}{n} = E_1 + 2U_D - \frac{E_2}{n} + \frac{2U_T}{n}, \quad (68)$$

$$I_3 = \frac{nE_1 + 2nU_D - E_2 + 2U_T}{nL}t_C$$

$$= \frac{nE_1 + 2nU_D - E_2 + 2U_T}{nL} \left(\frac{T}{2} - t_{dead} \right). \quad (69)$$

Considering that $I_0 = I_3$, on the basis of (67) and (68) it can be obtained

$$t_A = -\frac{nE_1 - E_2 + 2(nU_D + U_T)}{nE_1 + E_2 + 2(n+1)U_D}t_C$$

$$= -\frac{nE_1 - E_2 + 2(nU_D + U_T)}{nE_1 + E_2 + 2(n+1)U_D} \left(\frac{T}{2} - t_{dead} \right). \quad (70)$$

Knowing the time interval t_A and t_C the values of the currents I and I_3 as well as the average current IE_1 can be determined which is described by the following relation

$$I_{E1} = \frac{2}{T} \int_0^{T/2} i_{E1} dt = \frac{2}{T} \left[\frac{1}{2}I_0t_A + \frac{1}{2}I_3t_C \right]$$

$$= \frac{1}{T}I_3(t_A + t_C). \quad (71)$$

The power in circuit E_1 is described as follows

$$P_{E1} = \frac{2}{T} \int_0^{T/2} E_1 i_{E1} dt = E_1 I_{E1} = \frac{1}{T} E_1 I_3 (t_A + t_C)$$

$$= \frac{1}{T} E_1 I_3 \left(t_A + \frac{T}{2} - t_{dead} \right) < 0. \quad (72)$$

Due to the fact that $I_3 < 0$ the power P_{E1} is negative, which means that it is delivered to the E_1 side (phase drift $D_{dead} < 0$).

In accordance with Fig. 5b formulas for the average current I_{E2} can be determined. Replacing the integration by calculating the areas contained between the instantaneous current i_{E2} and time axis it can be obtained

$$nI_{E2} = \frac{2}{T} \left[-\frac{1}{2}I_0t_A + \frac{1}{2}I_3t_C \right] = \frac{1}{T}I_3(t_C - t_A). \quad (73)$$

The power received from the circuit E_2 is given by

$$P_{E2} = \frac{2}{T} \int_0^{T/2} E_2 i_{E2} dt = E_2 I_{E2} = \frac{1}{T} E_2 \frac{I_3}{n} (t_C - t_A)$$

$$= \frac{1}{T} E_2 \frac{I_3}{n} \left(\frac{T}{2} - t_{dead} - t_A \right) < 0. \quad (74)$$

The relationship describing the power P_{E2} can be transferred thus ensuring express this value as a function of the power P_{E1} . Given that $E_2 = E_1 n k_u$, on the basis of (74) it can be written

$$P_{E2} = \frac{1}{T} E_2 \frac{I_3}{n} (t_C - t_A)$$

$$= \frac{1}{T} E_1 I_3 k_u (t_C - t_A) \frac{t_C + t_A}{t_C + t_A} = P_{E1} k_u \frac{t_C - t_A}{t_C + t_A}. \quad (75)$$

The power is transferred from the circuit E_2 to the circuit E_1 when

$$k_u > \frac{t_C + t_A}{t_C - t_A} > 1. \quad (76)$$

Continuous AC current. If the current i_L is so large that after switching off the transistor pair, the current does not reach a value of zero in the time interval t_{dead} , the phase drift D_{dead} between the AC voltages u_1 and u_2 occurs. In fulfillment of the condition $k_u < 1$, phase drift is equal to $2t_{dead}/T$, while for $k_u > 1$ it is negative and equal to $-2t_{dead}/T$ (Fig. 6).

In the case of continuous current i_L , in respect to $k_u < 1$ and $k_u > 1$ the following can be written

$$I_3 = \frac{u_{L(tC)}}{L} t_C = -I_0, \quad (77)$$

$$I_1 = -\frac{u_{L(tB)}}{L} t_B = I_0 + \frac{u_{L(tA)}}{L} t_A, \quad (78)$$

where $u_{L(tA)}$, $u_{L(tB)}$, $u_{L(tC)}$ – the instantaneous values of the voltage on replacement choke with inductance L in the AC circuit at intervals $t_A = t_{dead}$, t_B and t_C , respectively.

Comparing the current value I_0 determined from formulas (77) and (78) the following relation can be obtained

$$u_{L(tA)} t_A + u_{L(tB)} t_B + u_{L(tC)} t_C = 0. \quad (79)$$

Since $t_A + t_B + t_C = T/2$, based on equation (79) the general formulas, defining time intervals t_B and t_C can be determined

$$t_B = \frac{T}{2} \frac{u_{L(tC)}}{u_{L(tB)} + u_{L(tC)}} - \frac{u_{L(tA)} + u_{L(tC)}}{u_{L(tB)} + u_{L(tC)}} t_A$$

$$= \frac{u_{L(tC)} \frac{T}{2} - (u_{L(tA)} + u_{L(tC)}) t_A}{u_{L(tB)} + u_{L(tC)}}, \quad (80)$$

$$t_C = \frac{u_{L(tB)} \frac{T}{2} + (u_{L(tA)} - u_{L(tB)}) t_A}{u_{L(tB)} + u_{L(tC)}}. \quad (81)$$

In the region A, i.e. for $E_1 > E_2/n$ (Fig. 6a) the following relations are satisfied

$$u_{L(tA)} = u_1 - \frac{u_2}{n} = E_1 + 2U_D - \left(-\frac{E_2}{n} - \frac{2U_D}{n} \right), \quad (82)$$

$$u_{L(tB)} = u_1 - \frac{u_2}{n} = E_1 + 2U_D - \left(\frac{E_2}{n} - \frac{2U_T}{n} \right), \quad (83)$$

$$u_{L(tC)} = u_1 - \frac{u_2}{n} = E_1 - 2U_T - \left(\frac{E_2}{n} + \frac{2U_D}{n} \right). \quad (84)$$

From Eqs. (80)–(84) it is obtained [21]

$$t_B = \frac{(nE_1 - E_2 - 2nU_T - 2U_D) \frac{T}{4} - n(E_1 + U_D - U_T) t_{dead}}{nE_1 - E_2 + (n-1)(U_D - U_T)}, \quad (85)$$

$$t_C = \frac{(nE_1 - E_2 + 2U_T + 2nU_D) \frac{T}{4} + (E_2 + U_D - U_T) t_{dead}}{nE_1 - E_2 + (n-1)(U_D - U_T)}. \quad (86)$$

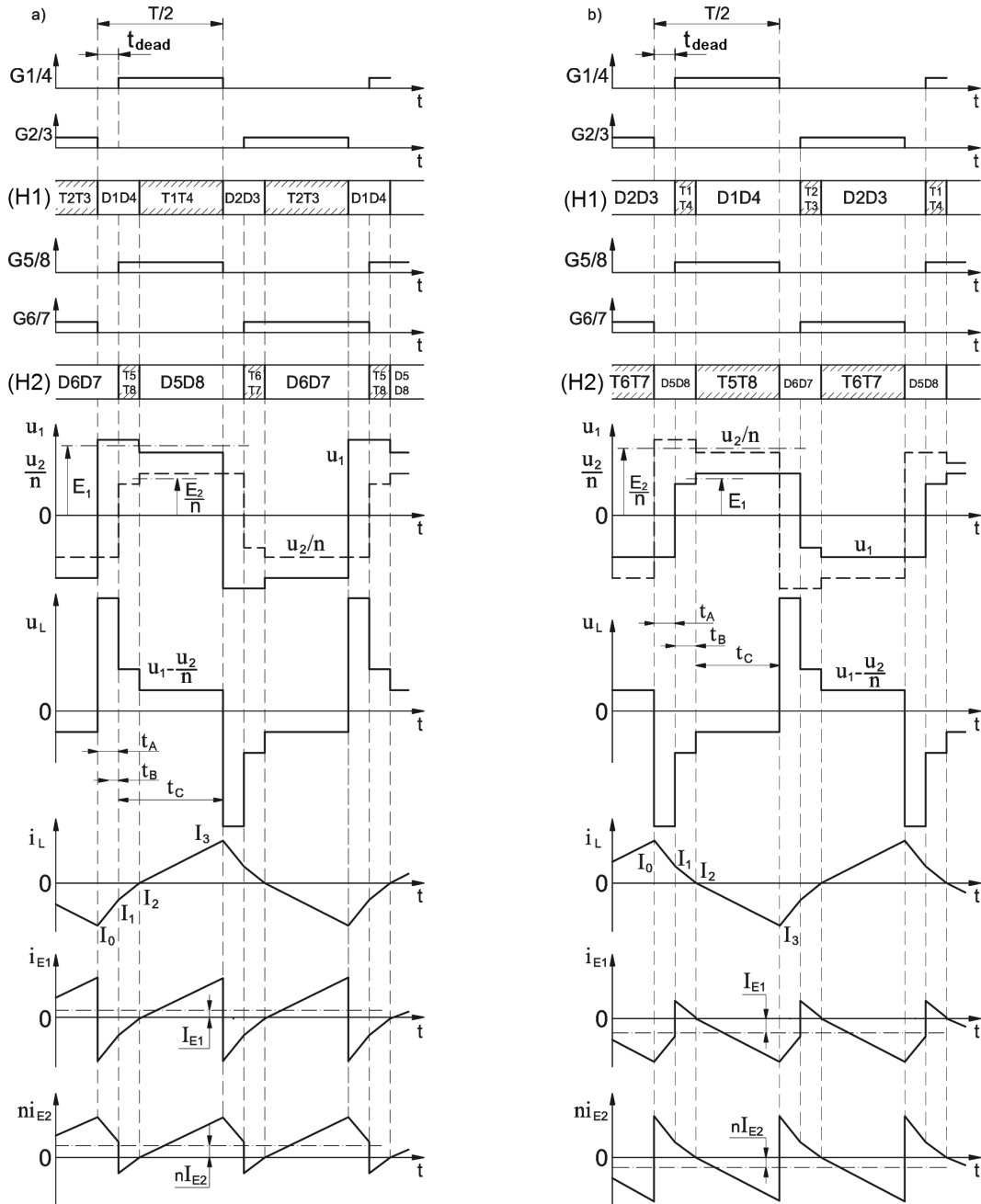


Fig. 6. Effect of the time delay t_{dead} between the transistor control signals on the voltage and current waveforms for continuous AC current at $D = 0$: a) $E_1 > E_2/n$; b) $E_1 < E_2/n$

Based on (77) and (78) and (82)–(84) characteristic values of the current i_L , may be determined

$$I_3 = \frac{nE_1 - 2nU_T - E_2 - 2U_D}{nL} t_C = -I_0, \quad (87)$$

$$I_1 = -\frac{nE_1 + 2nU_D - E_2 + 2U_T}{nL} t_B. \quad (88)$$

Knowing the time intervals $t_A = t_{del}$, t_B and t_C and the currents I_1 and $I_3 = -I_0$, the formulas describing the aver-

age values I_{E1} and I_{E2} can be determined. On the basis of Fig. 6a it can be written

$$\begin{aligned}
 I_{E1} &= \frac{2}{T} \int_0^{T/2} i_{E1} dt \\
 &= \frac{2}{T} \left[I_1 t_A + \frac{1}{2} (I_0 - I_1) t_A + \frac{1}{2} I_1 t_B + \frac{1}{2} I_3 t_C \right] \\
 &= \frac{1}{T} [I_1 (t_A + t_B) + I_3 (t_C - t_A)], \\
 I_{E2} &= \frac{1}{n} \frac{2}{T} \left[\frac{1}{2} (-I_0 + I_1) t_A - I_1 t_A + \frac{1}{2} I_1 t_B + \frac{1}{2} I_3 t_C \right] \\
 &= \frac{1}{n} \frac{1}{T} [I_3 (t_A + t_C) + I_1 (t_B - t_A)].
 \end{aligned} \tag{89}$$

The average values of currents in DC circuits I_{E1} and I_{E2} are positive, which means that they have a direction such as shown in Fig. 1. The powers P_{E1} and P_{E2} in these circuits are also positive and are determined by the formulas

$$\begin{aligned}
 P_{E1} &= \frac{2}{T} \int_0^{T/2} E_1 i_{E1} dt = E_1 I_{E1} \\
 &= \frac{1}{T} E_1 [I_1 (t_A + t_B) + I_3 (t_C - t_A)] > 0,
 \end{aligned} \tag{91}$$

$$\begin{aligned}
 P_{E2} &= \frac{2}{T} \int_0^{T/2} E_2 i_{E2} dt = E_2 I_{E2} \\
 &= \frac{1}{n} \frac{1}{T} E_2 [I_1 (t_B - t_A) + I_3 (t_A + t_C)] > 0,
 \end{aligned} \tag{92}$$

wherein $P_{E1} > P_{E2}$.

Given that $E_2 = k_u n E_1$, on the base (91) and (92) following relations can be obtained

$$\begin{aligned}
 P_{E2} &= \frac{1}{T} k_u E_1 [I_1 (t_B - t_A) + I_3 (t_A + t_C)] \\
 &\quad \frac{I_1 (t_A + t_B) + I_3 (t_C - t_A)}{I_1 (t_A + t_B) + I_3 (t_C - t_A)} \\
 &= k_u P_{E1} \frac{I_1 (t_B - t_A) + I_3 (t_A + t_C)}{I_1 (t_A + t_B) + I_3 (t_C - t_A)}
 \end{aligned} \tag{93}$$

and

$$k_u \frac{I_1 (t_B - t_A) + I_3 (t_A + t_C)}{I_1 (t_A + t_B) + I_3 (t_C - t_A)} < 1. \tag{94}$$

In the operation region B, i.e. with $E_1 < E_2/n$ (Fig. 6b), the instantaneous voltage of the replacement inductor L is described as follows

$$\begin{aligned}
 u_{L(tA)} &= u_1 - \frac{u_2}{n} = -E_1 - 2U_D - \left(\frac{E_2}{n} + \frac{2U_D}{n} \right) \\
 &= -E_1 - 2U_D - \frac{E_2}{n} - \frac{2U_D}{n},
 \end{aligned} \tag{95}$$

$$\begin{aligned}
 u_{L(tB)} &= u_1 - \frac{u_2}{n} = E_1 - 2U_T - \left(\frac{E_2}{n} + \frac{2U_D}{n} \right) \\
 &= E_1 - 2U_T - \frac{E_2}{n} - \frac{2U_D}{n},
 \end{aligned} \tag{96}$$

$$\begin{aligned}
 u_{L(tC)} &= u_1 - \frac{u_2}{n} = E_1 + 2U_D - \left(\frac{E_2}{n} - \frac{2U_T}{n} \right) \\
 &= E_1 + 2U_D - \frac{E_2}{n} + \frac{2U_T}{n}.
 \end{aligned} \tag{97}$$

Using formulas (80) and (81) as well as (95)–(97) it can be written

$$t_B = \frac{(nE_1 - E_2 + 2nU_D + 2U_T) \frac{T}{4} + (E_2 + U_D - U_T) t_{dead}}{nE_1 - E_2 + (n-1)(U_D - U_T)}, \tag{98}$$

$$t_C = \frac{(nE_1 - E_2 - 2U_D - 2nU_T) \frac{T}{4} - n(E_1 + U_D - U_T) t_{dead}}{nE_1 - E_2 + (n-1)(U_D - U_T)}. \tag{99}$$

Based on (77) and (78) as well as (95)–(97) characteristic values of the current i_L , can be determined

$$I_3 = \frac{nE_1 + 2nU_D - E_2 + 2U_T}{nL} t_C = -I_0, \tag{100}$$

$$I_1 = -\frac{nE_1 - 2nU_T - E_2 - 2U_D}{nL} t_B. \tag{101}$$

From Fig. 6b, using formulas (98)–(101), it can be obtained

$$\begin{aligned}
 I_{E1} &= \frac{2}{T} \int_0^{T/2} i_{E1} dt \\
 &= \frac{2}{T} \left[-I_1 t_A - \frac{1}{2} (I_0 - I_1) t_A + \frac{1}{2} I_1 t_B + \frac{1}{2} I_3 t_C \right] \\
 &= \frac{1}{T} [I_1 (t_B - t_A) + I_3 (t_A + t_C)], \\
 I_{E2} &= \frac{1}{n} \frac{2}{T} \left[\frac{1}{2} (I_0 - I_1) t_A + I_1 t_A + \frac{1}{2} I_1 t_B + \frac{1}{2} I_3 t_C \right] \\
 &= \frac{1}{n} \frac{1}{T} [I_3 (t_C - t_A) + I_1 (t_A + t_B)].
 \end{aligned} \tag{102}$$

The average values of currents I_{E1} and I_{E2} are negative, which means that the power P_{E1} and P_{E2} are also negative. These powers are expressed by formulas

$$\begin{aligned}
 P_{E1} &= \frac{2}{T} \int_0^{T/2} E_1 i_{E1} dt = E_1 I_{E1} \\
 &= \frac{1}{T} E_1 [I_1 (t_B - t_A) + I_3 (t_A + t_C)] < 0,
 \end{aligned} \tag{104}$$

$$\begin{aligned}
 P_{E2} &= \frac{2}{T} \int_0^{T/2} E_2 i_{E2} dt = E_2 I_{E2} \\
 &= \frac{1}{n} \frac{1}{T} E_2 [I_1 (t_A + t_B) + I_3 (t_C - t_A)] < 0,
 \end{aligned} \tag{105}$$

wherein $|P_{E2}| > |P_{E1}|$.

Considering that $E_2 = k_u n E_1$, it can be written

$$\begin{aligned}
 P_{E2} &= \frac{1}{T} k_u E_1 [I_1 (t_A + t_B) + I_3 (t_C - t_A)] \\
 &\quad \frac{I_1 (t_B - t_A) + I_3 (t_A + t_C)}{I_1 (t_B - t_A) + I_3 (t_A + t_C)} \\
 &= k_u P_{E1} \frac{I_1 (t_A + t_B) + I_3 (t_C - t_A)}{I_1 (t_B - t_A) + I_3 (t_A + t_C)},
 \end{aligned} \tag{106}$$

and

$$k_u \frac{I_1(t_A + t_B) + I_3(t_C - t_A)}{I_1(t_B - t_A) + I_3(t_A + t_C)} > 1. \quad (107)$$

5. Power transfer analysis including the voltage drop cross the semiconductor devices and the dead time at $D > 0$

When the phase shift D is positive, for both conditions $E_1 > E_2/n$ and $E_1 < E_2/n$, the relations describ-

ing the characteristic voltages, currents and powers are the same.

On the basis of Fig. 7a it can be obtained – for time interval t_A :

$$u_1 = E_1 + 2U_D, \quad (108)$$

$$\frac{u_2}{n} = -\frac{E_2}{n} - \frac{2U_D}{n}, \quad (109)$$

$$u_{L(t_A)} = u_1 - \frac{u_2}{n} = E_1 + 2U_D + \frac{E_2}{n} + \frac{2U_D}{n}, \quad (110)$$

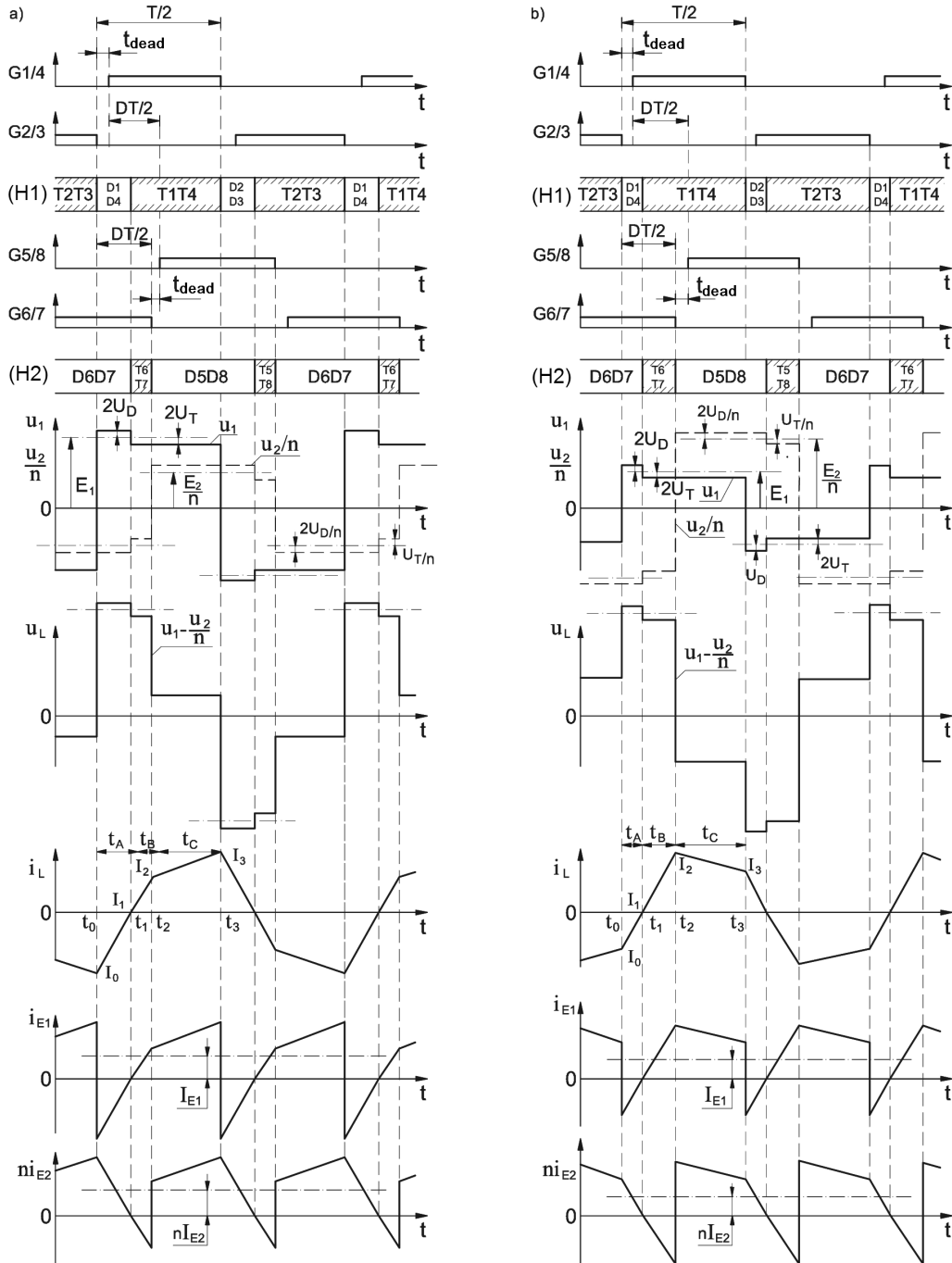


Fig. 7. Control signals, voltage and current waveforms in the DAB operating with continuous AC current at $D > 0$: a) $E_1 > E_2/n$; b) $E_1 < E_2/n$

– for the time interval t_B :

$$u_1 = E_1 - 2U_T, \quad (111)$$

$$\frac{u_2}{n} = -\frac{E_2}{n} + \frac{2U_T}{n}, \quad (112)$$

$$u_{L(tB)} = u_1 - \frac{u_2}{n} = E_1 - 2U_T + \frac{E_2}{n} - \frac{2U_T}{n}, \quad (113)$$

– for the time interval t_C :

$$u_1 = E_1 - 2U_T, \quad (114)$$

$$\frac{u_2}{n} = \frac{E_2}{n} + \frac{2U_D}{n}, \quad (115)$$

$$u_{L(tC)} = u_1 - \frac{u_2}{n} = E_1 - 2U_T - \frac{E_2}{n} - \frac{2U_D}{n}. \quad (116)$$

Based on Fig. 7 it can be written

$$\begin{aligned} I_0 &= -\frac{u_{L(tA)}}{L}t_A \\ &= -\frac{nE_1 + 2nU_D + E_2 + 2U_D}{nL}t_A = -I_3, \end{aligned} \quad (117)$$

$$I_2 = \frac{u_{L(tB)}}{L}t_B = \frac{nE_1 - 2nU_T + E_2 - 2U_T}{nL}t_B, \quad (118)$$

$$I_3 - I_2 = \frac{u_{L(tC)}}{L}t_C, \quad (119)$$

$$\begin{aligned} I_2 &= -I_0 - \frac{u_{L(tC)}}{L}t_C = \frac{nE_1 + 2nU_D + E_2 + 2U_D}{nL}t_A \\ &\quad - \frac{nE_1 - 2nU_T - E_2 - 2U_D}{nL}t_C. \end{aligned} \quad (120)$$

Comparison of expression (118) and (120) gives

$$\begin{aligned} &(nE_1 + 2nU_D + E_2 + 2U_D)t_A \\ &- (nE_1 - 2nU_T + E_2 - 2U_T)t_B \\ &- (nE_1 - 2nU_T - E_2 - 2U_D)t_C = 0. \end{aligned} \quad (121)$$

Taking into account that

$$t_A + t_B + t_C = \frac{T}{2}, \quad (122)$$

$$t_B = D\frac{T}{2} - t_A, \quad (123)$$

$$t_C = \frac{T}{2} - D\frac{T}{2} = \frac{T}{2}(1 - D) \quad (124)$$

on the basis of Eqs. (121) and (122)–(124), after the transformation it can be obtained

$$t_A = \frac{T}{4} \frac{2D(E_2 + U_D - U_T) + nE_1 - E_2 - 2nU_T - 2U_D}{nE_1 + E_2 + (n+1)(U_D - U_T)}. \quad (125)$$

From Eqs. (123) and (125) it follows that

$$\begin{aligned} t_B &= D\frac{T}{2} - t_A \\ &= \frac{T}{4} \frac{(1+2D)nE_1 - E_2 - 2U_D(1-Dn) - 2nU_T(1+D)}{nE_1 + E_2 + (n+1)(U_D - U_T)}. \end{aligned} \quad (126)$$

Knowing the time intervals t_A , t_B and t_C (formulas (125), (123) and (124)) and the characteristics current values I_0 , I_2

and I_3 (formulas (117) and (118)) the average current I_{E1} can be determined. Based on Fig. 7a the following equation can be formulated

$$\begin{aligned} I_{E1} &= \frac{2}{T} \int_0^{T/2} i_{E1} dt \\ &= \frac{2}{T} \left[-\frac{1}{2}I_3t_A + \frac{1}{2}I_2t_B + \frac{1}{2}(I_2 + I_3)t_C \right] \\ &= \frac{1}{T} [I_2(t_B + t_C) + I_3(t_C - t_A)]. \end{aligned} \quad (127)$$

The average value of the current in the high – voltage DC circuit $I_{E1} > 0$. The power in this circuits is also positive and is defined by the following formula:

$$\begin{aligned} P_{E1} &= \frac{2}{T} \int_0^{T/2} E_1 i_{E1} dt = E_1 I_{E1} \\ &= \frac{1}{T} E_1 [I_2(t_B + t_C) + I_3(t_C - t_A)] > 0. \end{aligned} \quad (128)$$

The power P_{E1} reduced by the power losses generated in semiconductor devices and magnetic elements flows to the low – voltage side. The power can be determined on the basis of DC voltage E_2 and the average current which is given by

$$\begin{aligned} I_{E2} &= \frac{2}{T} \int_0^{T/2} i_{E2} dt \\ &= \frac{1}{nT} \left[\frac{1}{2}I_3t_A - \frac{1}{2}I_2t_B + \frac{1}{2}(I_2 + I_3)t_C \right] \\ &= \frac{1}{nT} [I_2(t_C - t_B) + I_3(t_A + t_C)] > 0. \end{aligned} \quad (129)$$

The average value of the current I_{E2} is positive, which means that the power P_{E2} is delivered to the low-voltage circuit E_2 . This power is described by

$$\begin{aligned} P_{E2} &= \frac{2}{T} \int_0^{T/2} E_2 i_{E2} dt = E_2 I_{E2} \\ &= \frac{1}{nT} E_2 [I_2(t_C - t_B) + I_3(t_A + t_C)] > 0. \end{aligned} \quad (130)$$

Energy efficiency of the two bridges, taking into account the power losses in eight semiconductor switches, under conditions where energy is transferred from the high - voltage side E_1 to low-voltage side E_2 defines the relationship

$$\eta_{H12} = \frac{P_{E2}}{P_{E1}} = k_u \frac{I_2(t_C - t_B) + I_3(t_A + t_C)}{I_2(t_B + t_C) + I_3(t_C - t_A)}. \quad (131)$$

In order to determine the energy efficiency of the DAB converter η_{H12} of both bridges must be multiplied by the efficiency of transformer and inductors.

The relationships presented in this section can be used if

$$0 \leq t_A \leq D\frac{T}{2}. \quad (132)$$

According to Eq. (125), the condition (132) is satisfied for the phase shift ratio D arising from the following relation

– for $k_u \leq 0$

$$D > \frac{nE_1 - E_2 - 2(nU_T + U_D)}{2n(E_1 + U_D - U_T)}, \quad (133)$$

– for $k_u \geq 0$

$$D > \frac{E_2 - nE_1 + 2(nU_T + U_D)}{2(E_2 + U_D - U_T)}. \quad (134)$$

6. Power transfer analysis including the voltage drop across the semiconductor devices and the dead time at $D < 0$

If the voltage waveform u_2 leads the waveform u_1 , i.e. when $D < 0$, for the both conditions: $E_1 > E_2/n$ and $E_1 < E_2/n$,

the characteristic voltages and currents in the DAB are described by the same relations. Using Fig. 8 it is possible to write following equations

– for the time interval t_A :

$$u_1 = -E_1 - 2U_D, \quad (135)$$

$$\frac{u_2}{n} = \frac{E_2}{n} + \frac{2U_D}{n}, \quad (136)$$

$$u_{L(tA)} = u_1 - \frac{u_2}{n} = -E_1 - 2U_D - \frac{E_2}{n} - \frac{2U_D}{n}, \quad (137)$$

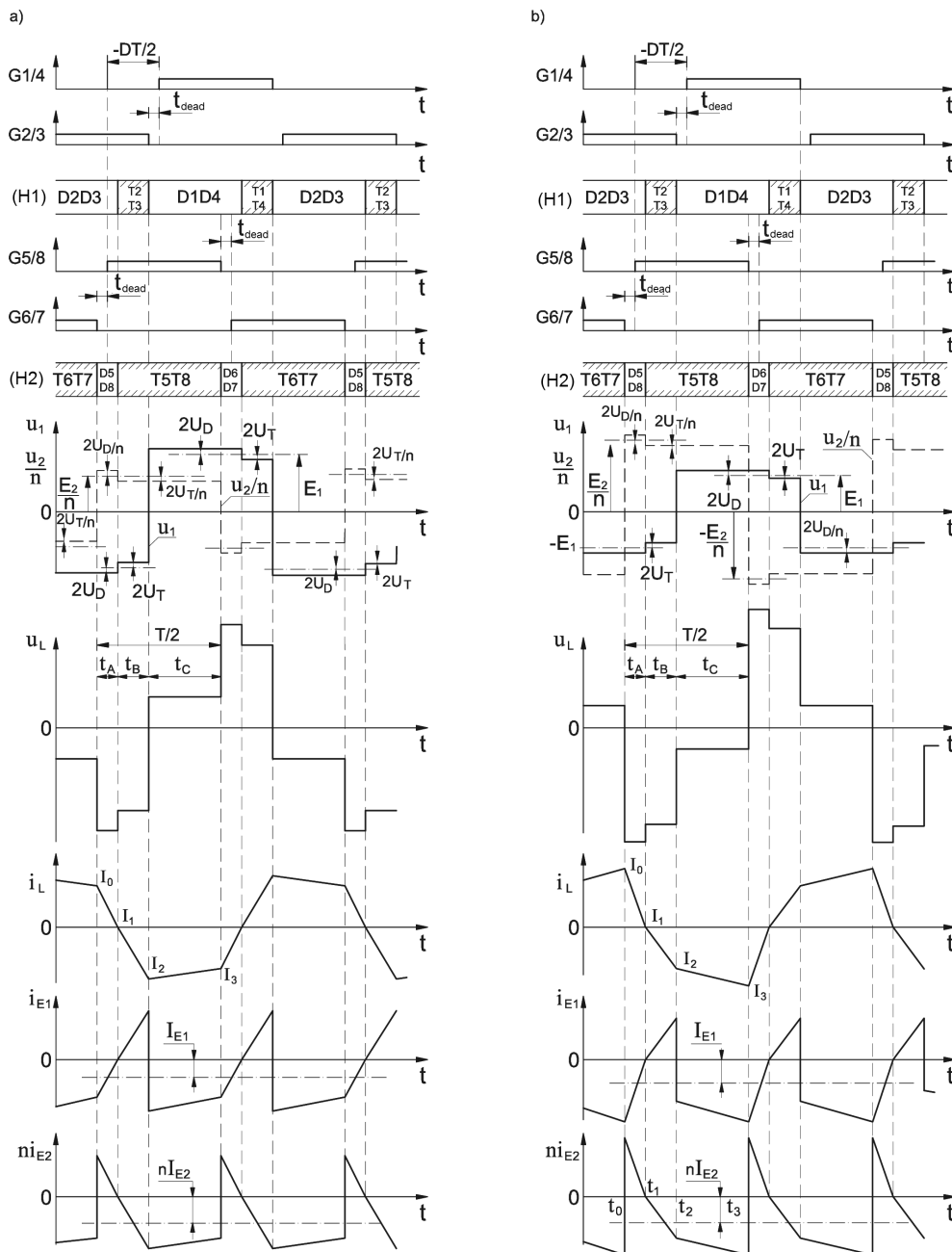


Fig. 8. Control signals and voltage and current waveforms in the DAB operating with continuous AC current at $D > 0$: a) $E_1 > E_2/n$; b) $E_1 < E_2/n$

– for the time interval t_B :

$$u_1 = -E_1 + 2U_T, \quad (138)$$

$$\frac{u_2}{n} = \frac{E_2}{n} - \frac{2U_T}{n}, \quad (139)$$

$$u_{L(tB)} = u_1 - \frac{u_2}{n} = -E_1 + 2U_T - \frac{E_2}{n} + \frac{2U_T}{n} \quad (140)$$

– for the time interval t_C :

$$u_1 = E_1 + 2U_D, \quad (141)$$

$$\frac{u_2}{n} = \frac{E_2}{n} - \frac{2U_T}{n}, \quad (142)$$

$$u_{L(tC)} = u_1 - \frac{u_2}{n} = E_1 + 2U_D - \frac{E_2}{n} + \frac{2U_T}{n}. \quad (143)$$

Using formulas (137), (140) and (143), on the basis of Fig. 8 following relationships can be written:

$$I_0 = -\frac{u_{L(tA)}}{L} t_A = \frac{nE_1 + 2nU_D + E_2 + 2U_D}{nL} t_A = -I_3, \quad (144)$$

$$I_2 = \frac{u_{L(tB)}}{L} t_B = \frac{-nE_1 + 2nU_T - E_2 + 2U_T}{nL} t_B \quad (145)$$

and

$$I_2 = -I_0 - \frac{u_{L(tC)}}{L} t_C = \frac{-nE_1 - 2nU_D - E_2 - 2U_D}{nL} t_A - \frac{nE_1 + 2nU_D - E_2 + 2U_T}{nL} t_C. \quad (146)$$

A comparison of (145) and (146) yields the following expression

$$\begin{aligned} & (nE_1 + 2nU_D + E_2 + 2U_D) t_A \\ & + (-nE_1 + 2nU_T - E_2 + 2U_T) t_B \\ & + (nE_1 + 2nU_D - E_2 + 2U_T) t_C = 0. \end{aligned} \quad (147)$$

After considering that

$$t_A + t_B = D \frac{T}{2} + t_{dead}, \quad (148)$$

$$t_C = \frac{T}{2} - D \frac{T}{2} - t_{dead} \quad (149)$$

on the basis of (147) it can be obtained

$$t_A = \frac{T}{4} \left[\frac{(D+a)(nE_1 - 2nU_T + E_2 - 2U_T) + (D+a-1)(nE_1 + 2nU_D - E_2 + 2U_T)}{nE_1 + E_2 + (n+1)(U_D - U_T)} \right], \quad (150)$$

$$t_B = D \frac{T}{2} + t_{dead} - t_A = \frac{T}{2} (D+a) - t_A, \quad (151)$$

$$t_C = \frac{T}{2} - D \frac{T}{2} - t_{dead} = \frac{T}{2} (1-D-a), \quad (152)$$

where $a = 2t_{dead}/T$ – maximum phase drift.

Determined on the basis of (150)–(152) the time intervals t_A , t_B and t_C can be used to calculate the characteristic values of the AC current I_0 , I_2 and I_3 (Fig. 8, Eqs. (144) and

(145)) and the average currents I_{E1} and I_{E2} . By using Fig. 8 it can be written

$$\begin{aligned} I_{E1} &= \frac{2}{T} \int_0^{T/2} i_{E1} dt \\ &= \frac{2}{T} \left[\frac{1}{2} I_3 t_A - \frac{1}{2} I_2 t_B + \frac{1}{2} (I_2 + I_3) t_C \right] \end{aligned} \quad (153)$$

$$= \frac{1}{T} [I_2 (t_C - t_B) + I_3 (t_A + t_C)] < 0,$$

$$\begin{aligned} I_{E2} &= \frac{2}{T} \int_0^{T/2} i_{E2} dt \\ &= \frac{2}{T} \left[-\frac{1}{2} \frac{I_3}{n} t_A + \frac{1}{2} \frac{I_2}{n} t_B + \frac{1}{2} \left(\frac{I_2}{n} + \frac{I_3}{n} \right) t_C \right] \\ &= \frac{1}{nT} [I_2 (t_B + t_C) + I_3 (t_C - t_A)] < 0. \end{aligned} \quad (154)$$

The power values in the two coupled DC circuits are described by the following formulas

$$P_{E1} = \frac{2}{T} \int_0^{T/2} E_1 i_{E1} dt = E_1 I_{E1} \quad (155)$$

$$= \frac{1}{T} E_1 [I_2 (t_C - t_B) + I_3 (t_A + t_C)] < 0,$$

$$P_{E2} = \frac{2}{T} \int_0^{T/2} E_2 i_{E2} dt = E_2 I_{E2} \quad (156)$$

$$= \frac{1}{nT} E_2 [I_2 (t_B + t_C) + I_3 (t_C - t_A)] < 0.$$

Negative values of the powers P_{E1} and P_{E2} indicates that the power is transferred from low – voltage circuit E_2 to the high – voltage side E_1 , i.e. in the opposite direction than assumed in Fig. 1.

At energy transfer from E_2 side to the E_1 at $E_1 > E_2/n$ and $E_1 < E_2/n$ the energy efficiency of the two bridges can be expressed as follows

$$\eta_{M21} = \frac{P_{E1}}{P_{E2}} = \frac{1}{k_u} \frac{I_2 (t_C - t_B) + I_3 (t_A + t_C)}{I_2 (t_B + t_C) + I_3 (t_C - t_A)}. \quad (157)$$

The equations contained in this section are valid, if the following condition is satisfied

$$0 \leq t_A \leq \frac{T}{2} (D+a). \quad (158)$$

From equation (150) show that the condition (158) is satisfied for the phase shift ratio D defined by the following relationships

– for $k_u \leq 0$

$$D > \frac{nE_1 - E_2 + 2nU_D + U_T}{2n(E_1 + U_D - U_T)}, \quad (159)$$

– for $k_u \geq 0$

$$D > \frac{E_2 (1-2a) - nE_1 - 2U_T (1-a) - 2U_D (n+a)}{2(E_2 + U_D - U_T)}. \quad (160)$$

7. Examples of calculation of power transferred by DAB

Relation given in the previous section were used to assess the power transfer and preliminary estimate power losses and energy efficiency of DAB converter at different phase shift ratio D and various voltage conversion ratio k_u .

In presented in this section calculations it will be assumed that the voltages of the coupled DC circuits are: $E_1 = 280$ V and $E_2 = 51$ V \pm 20%, which corresponds to a voltage conversion ratio k_u contained in the range of 0.8–1.2. The other parameters of the system under consideration are as follows:

- total transformer leakage inductance: $L_\sigma = L_{\sigma 1} + L'_{\sigma 2} = 1.52 \mu\text{H}$,
- inductance of each of two identical inductors in the circuit of the high – voltage transformer winding: $L_d/2 = 9.75 \mu\text{H}$,
- resultant AC circuit inductance: $L = L_\delta + L_d = 21 \mu\text{H}$,
- period of AC voltages of the bridges: $T = 10 \mu\text{s}$,
- dead time: $t_{dead} = 0.125 \mu\text{s}$,
- transformer magnetizing inductance: $L_m = 0.7$ mH,
- voltage drop across each conducting transistor: $U_T = 2$ V,
- voltage drop across each conducting diode: $U_D = 1$ V,
- transformer turns ratio: $n = N_2/N_1 = 0.18$,

It was assumed that the voltage drop on the semiconductor elements are constant and have do not depend on the current.

Figure 9 shows the transferred power as a function of the phase shift D for voltage conversion ratio as parameter. The curves were obtained neglecting of the voltage drops on the semiconductor devices and the dead time (Eqs. (47) and (48)).

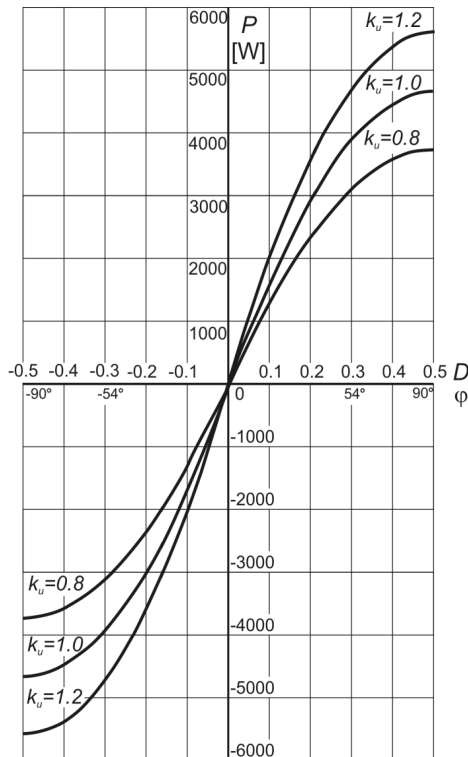


Fig. 9. The power transferred between the DC circuits calculated on the basis of (47) and (48)

If the semiconductor devices are ideal, the power transferred through the DAB is a parabolic function of the phase shift ratio. In this case, due to the symmetry of the converter, power values for $|D| > 0.5$ take the same values as for $|D| < 0.5$.

Another results are obtained if the voltage drops on the semiconductor devices and the dead time is taken into account. For such a condition the power transferred between two DC circuits P_{E1} and P_{E2} at phase shift ratio $D = 0$ have been calculated on the basis of formulas (87)–(94) and (99)–(106) respectively to $k_u = 0.8$ and $k_u = 1.2$. Due to the phase drift $a = 2t_{dead}/T = 0.025$ (i.e. $\phi = 4.5^\circ$), for $k_u = 0.8$ the powers in DC circuits are positive: $P_{E1} = 595$ W and $P_{E2} = 541$ W. For $k_u = 1.2$ these powers are negative: $P_{E1} = -705.6$ W and $P_{E2} = -773.2$ W. For $k_u = 1.0$ in the range $|D| < a$ there is no active power transfer between the both DC circuits.

The calculations taking into account the voltage drops $U_T = 2$ V and $U_D = 1$ V as well as dead time ($a = 0.025$) were accomplished for $D > 0$ and $D < 0$ using appropriate formulas (118)–(131) and (141)–(153). The limits of the phase shift ratio, for which it is possible to use these formulas are determined from the conditions (133), (134), (159) and (160).

The calculated powers P_{E1} and P_{E2} for positive and negative phase shift ratio D at $k_u = 0.8; 1.0$ and 1.2 are shown in Fig. 10. To highlight the impact of the dead time the calculations are limited to the scope of changes the phase shift $|D| \leq 0.3$, which also covers the acceptable energy efficiency of the DAB.

Figure 10 indicates that:

- the characteristics determined by the formulas taking into account the voltage drops on semiconductor devices and the dead time (continuous line) are different from the characteristics of the ideal (dotted line) for all considered voltage conversion ratio ($k_u = 0.8; = 1.0; = 1.2$),
- presented in a simplified manner the power flow characteristics at small phase shift D significantly differ from the ideal characteristics mainly due to the phase drift caused by the dead time,
- the power transfer characteristic for the positive and negative phase shift ratio D at a given voltage conversion ratio k_u are not symmetric, which is caused not only by the phase drift but also by difference in power losses generated in the semiconductor devices.

The graphs depicted in Fig. 11 allow the assessment of energetic properties of the DAB converters. The power losses (with respect to only the power losses generated in semiconductor devices) ΔP_E and the energy efficiency η are described as follows:

$$\Delta P_E = |P_{E1} - P_{E2}|, \tag{161}$$

$$\eta = \eta_{H12} = \frac{P_{E2}}{P_{E1}} \quad \text{for } D > 0, \tag{162}$$

$$\eta = \eta_{H21} = \frac{P_{E1}}{P_{E2}} \quad \text{for } D < 0. \tag{163}$$

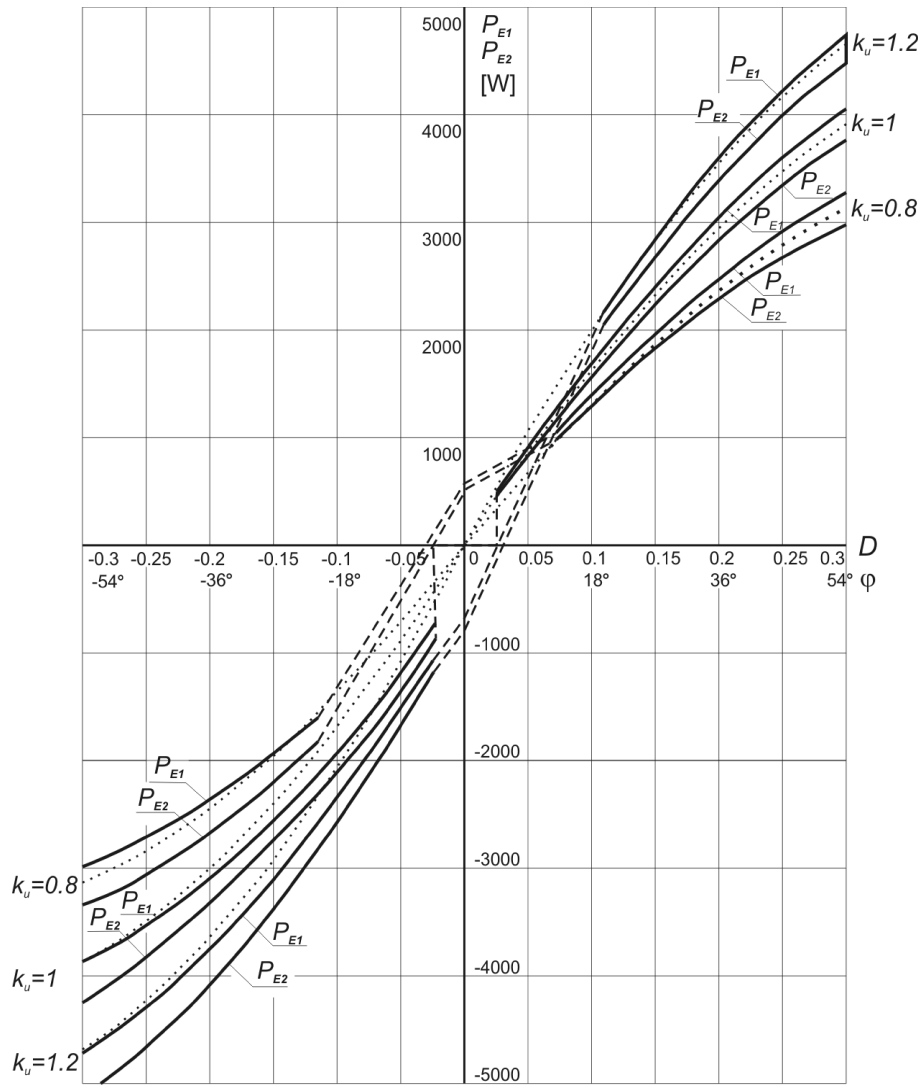


Fig. 10. The power transfer characteristics of the DAB made considering the voltage drops on the semiconductor devices ($U_T = 2$ V and $U_D = 1$ V) and the phase drift ($a = 0.025$) for $k_u = 0.8$; 1.0 and 1.2

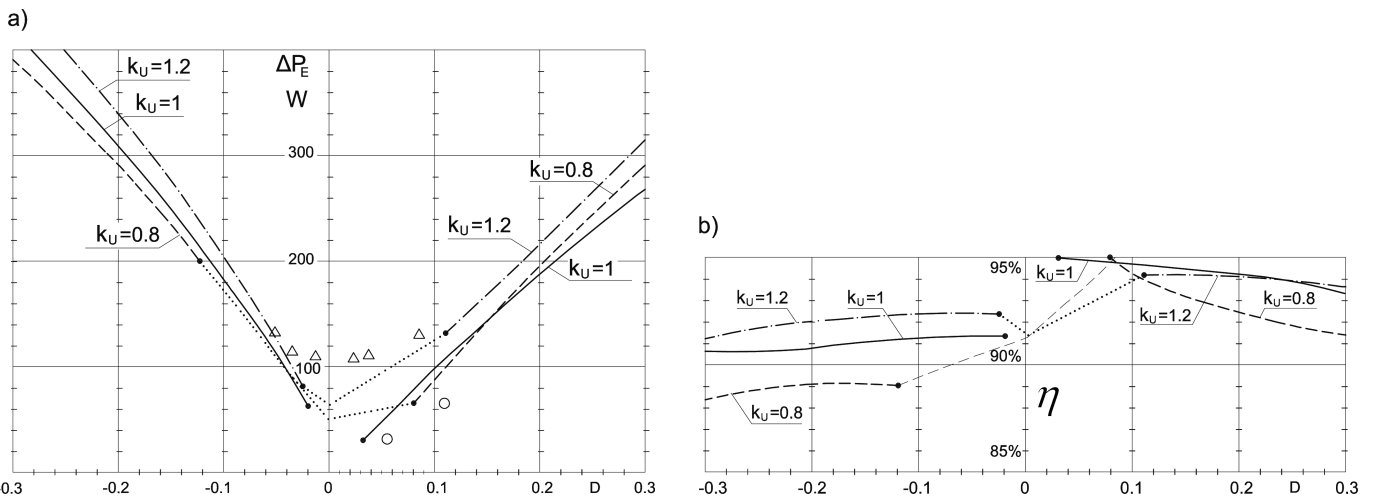


Fig. 11. Calculated power losses (a) and energy efficiency (b) of the two bridges in DAB converter for different voltage conversion ratio. A few experimental results are marked: \circ – measurements for $k_{u1} = 1.0$; Δ – measurements for $k_{u1} = 1.2$

Figure 11 shows that:

- in considered range of phase shift ratio D the energy efficiency of the converter for the positive values of D is higher than for negative,
- in the case of $D > 0$, in the high – voltage bridge (operating in inverter mode) dominate the power losses generated in transistors, which however, conduct a small current, and in the low-voltage bridge (operating as rectifier) dominate the power losses in diodes, which, although they conduct high current, have a small voltage drop, which makes the total power dissipation in all semiconductor devices relatively small,
- the most favorable conditions, in terms of minimizing the power losses in both bridges occur when the time-duration of the on-state of transistors in low-voltage bridge is as small as possible, which occurs at $k_u = 1.2$ and at low phase shift D .

It is worthy to notice, that the results of power losses calculations, although they are inaccurate, have far-reaching convergence with the results of measurements made by infrared camera [22]. Experimental investigations were carried out in a laboratory model power rated 5600 VA/100 kHz. A description of these tests and measurement methods is beyond the scope of this work.

8. Conclusions

The presented in this paper analytical description allows to determine the power transferred through the DAB converter between two DC circuits. The calculation of this power is based on average values of currents in DC circuits by taking into account the voltage drops on semiconductor devices and the dead time. The power losses generated in these devices, as differences of the powers in DC circuits, can be used for a pre-selection of the phase shift ratio ranges so that they correspond to the acceptable energy efficiency of the DAB converter. Due to very simple models of the semiconductor devices, power loss calculation results should be regarded as indicative, however, give information about the requirements for the cooling system of transistors and diodes.

Presented waveforms allow the evaluation of the conditions for which there is a soft switching of the transistors in two bridges.

In assessing the energy efficiency of the DAB converter, the power losses generated in transformer and inductors in AC circuit should be also taken into account. However, they are far (up to ten) smaller than the losses in semiconductor devices.

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