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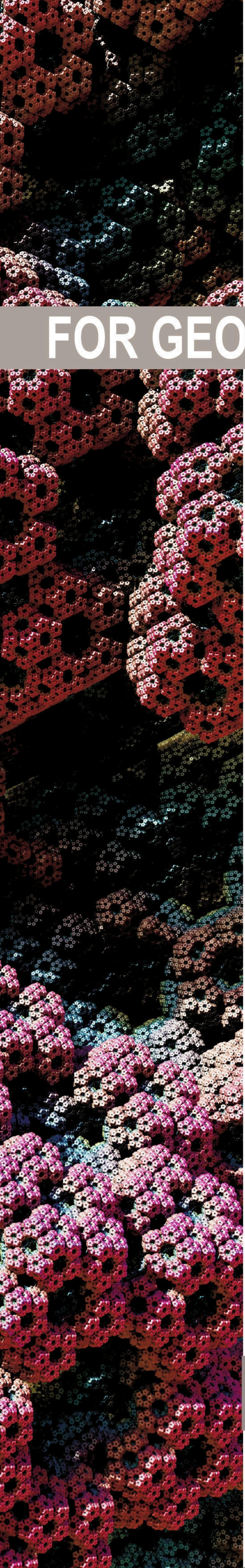

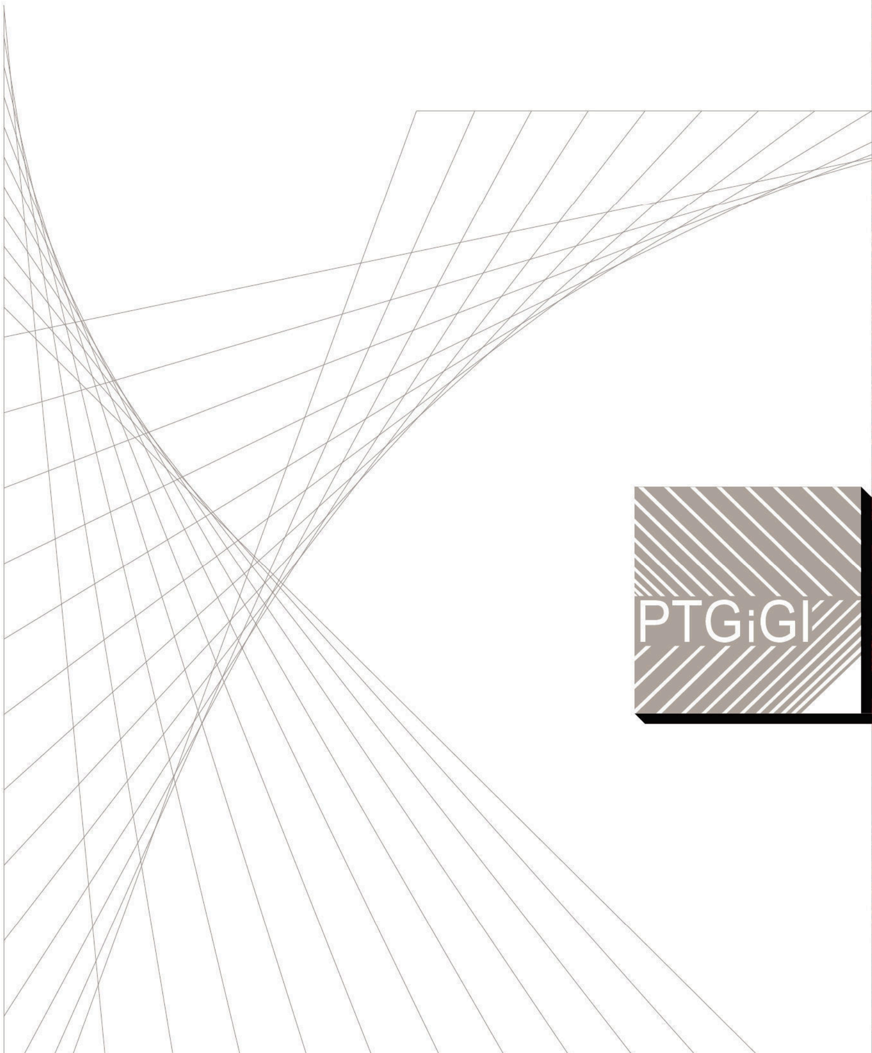

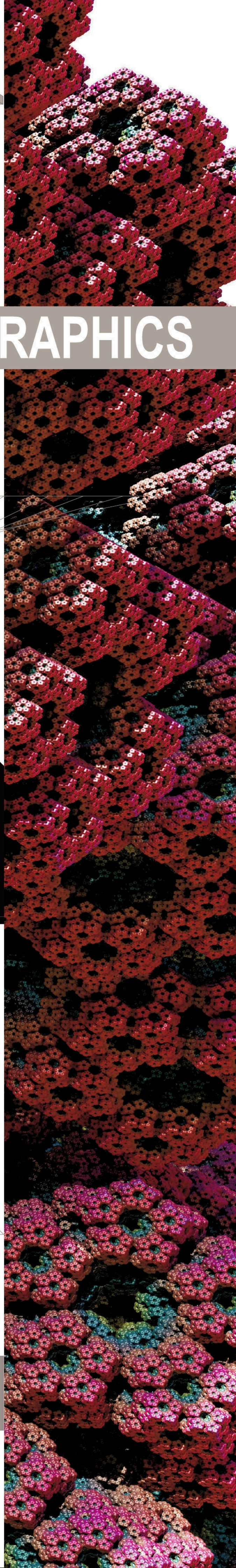

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CONTENTS

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PRE-SERVICE TEACHERS' (MIS)-UNDERSTANDING OF THE CONSTRUCTIONS OF DYNAMIC GEOMETRY OBJECTS

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Abstract: Future secondary school math teachers learned the basics of GeoGebra and were then tasked with the construction of a specific quadrilateral according to a given description. Qualitative analysis of their solutions discloses typical student misconceptions and errors.

Keywords: dynamic geometry tools, prospective math teachers, students' misconceptions, GeoGebra

1 1 Introduction

In order to assure wide and appropriate use of computer technologies in teaching and learning mathematics in school, pre-service teachers, in the framework of their academic training, must learn about and experiment with the wide range of applications, hardware, software, and dynamic environments. For example, in Gordon Academic College, all pre-service mathematics teachers take a number of 'general' courses whose goal is to boost the use of technology in teaching and learning the discipline [1].

As for GeoGebra, pre-service secondary school mathematics teachers meet this dynamic environment tool only occasionally during mathematical and didactic lessons, usually when they use it as a tool to plot graphs of functions, explore properties of functions, or visualize 2D or 3D geometrical objects. For many pre-service teachers, these are their first experiences with dynamic geometry.

2 Learning the basics of GeoGebra

In Gordon Academic College, students are introduced to the basic features and possibilities of the dynamic geometry environment for school geometry during the 'Teaching Workshop', a course offered in their third year of academic studies. The main goal of this workshop is to prepare the prospective teacher for practical work in the classroom, and thus, the course focuses on the mathematical and didactic aspects of important issues in secondary school mathematics. Exploring the use of the GeoGebra tool is integrated into the subject matter and didactic issues.

The teaching strategy for working with GeoGebra was to use the 'flipped classroom' method [2]. In this method, students are meant to acquire – on their own, using online sources of information – basic knowledge content of the subject to be studied prior to their classroom encounter. Work in the classroom is then devoted to interactive research activities to develop cognitive and collaborative processes. In practice, however, students required support for their self-learning, leading to two-stage classroom sessions: first the instructor demonstrated a feature of the tool appropriate for a specific geometric construction, and then the pre-service teachers worked in small groups (3-4 students) to solve a problem that involved implementation and further exploration of the feature.

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In the framework of the course, we chose to examine two topics of school geometry – 'classification of triangles' and 'family of quadrilaterals' – the exploration of which emphasizes logical relations between different geometrical objects [3]. This is an invaluable exercise for pre-service teachers who will often have to deal with pupil errors in this topic [4].

Students first discussed the subject matter with respect to the properties and definitions of triangles and quadrilaterals, with emphasis on the interplay of necessity and sufficiency. Following this, the students were asked to use different methods to construct a number of families of quadrilaterals in the GeoGebra environment, each time using a different characteristic property to define the object. Below, we present some findings of typical students errors when dealing with these problems.

3 The problem: the construction of quadrilaterals

Constructing geometrical objects with given properties is a classic motif in elementary mathematics. Mathematical educators consider this skill a powerful tool for the comprehension of the deductive structure of geometry and for the development of mathematical 'habits of mind' [5].

Over twenty years ago, problems involving 'ruler-and-compass' constructions were removed from the Israeli school curricula without the introduction of any suitable replacement. As a result, students not only did not learn how to construct the simplest geometrical objects, they were also deprived of activities that invite analysis of a wide range of properties of geometrical objects and the interrelation between different objects. This led to a significant degradation in understanding the logical structure of geometry – even for advanced-level high school graduates [6] and pre-service math teachers [7]. It has only been recently that construction problems using classic tools and/or a dynamic geometric environment – typically to determine a single geometric object – have been reintroduced into the curricula and textbooks.

Because pre-service teachers have an almost total lack of experience in solving construction problems, it is crucial to integrate into the course curriculum some simple step-by-step constructions to introduce GeoGebra shortcuts to them. Such basic activities include constructing rectangles with a given or variable perimeter and exploring their areas, constructing quadrilaterals with orthogonal diagonals and with various additional restrictions, or inscribing a rectangle within a circle. These exercises can demonstrate the dynamic nature of the environment with emphasis on construction of families of geometric objects rather than figures with given dimensions.

After the preparatory demonstration and trial, students were asked to construct a specific quadrilateral (their choice of rhombus, square, parallelogram, etc.) using the GeoGebra tool. For this, students needed to decide on an appropriate definition of the figure in question and then construct the entire family of figures that met this definition. For almost any specific quadrilateral, the set of requirements consists of more than one feature, meaning that the construction requires a multi-stage procedure.

From the logical point of view, the first step appears to be non-problematic: to choose the characteristic properties of the figure. Nevertheless, there are didactic dilemmas here. Students may encounter cases of different wording to express the same conditions in different textbooks (or even different instances in the same textbook), or the use of different but equivalent conditions to characterize the same object. Moreover, in some cases, redundant characteristics are customarily used so that the description is given in a 'common language' [8] and/or to help students deduce the properties of the object [4] (for example, 'a rectangle is a parallelogram with four right angles').

4 The analysis of solutions

In the framework of the activity, students chose one (correct) definition of some quadrilateral and tried to construct the figure accordingly. The need to construe the description of the object as a construction procedure has an immediate advantage in understanding that description, as it forces the reconceptualization of the problem with almost immediate elimination of superfluous components (Student: 'If we have constructed three right angles in a quadrilateral, the fourth angle is certainly also a right angle').

For the next step, each group of students received an assignment to construct the figure with GeoGebra according to the formulated description and to present their results. Students were able to successfully formulate the relevant description, but several groups had difficulty with the construction process.

It is important to emphasize that even if the final result of the construction seems to be correct, manipulations with the figure and analysis of the construction protocol may detect a defect in the procedure. There were three main types of errors.

1) Students used only part of the requirements in the construction. As a result, the set of figures was too extensive and included possibilities that did not meet the definition. While their figure appeared correct, it could undergo changes that contradict one or more of the necessary conditions. For instance, in order to construct a rhombus, students chose an arbitrary segment as a side of a quadrilateral and provided the same length for only two more sides, not three. The built-in GeoGebra option 'construct segment of given length' (where the results are horizontally oriented segments) supports the illusion of a proper construction: after drawing the fourth segment, the resulting figure was, indeed, a rhombus (Figure 1a). However, the constructed figure may be easily corrupted and transformed into a quadrilateral with only three sides equal in length (Figure 1b).

a b

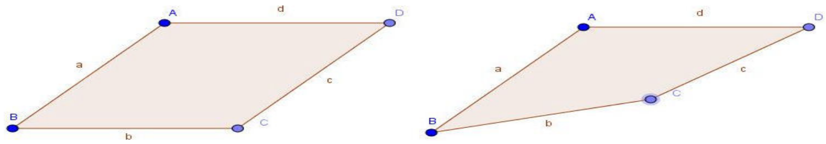

Figure 1. Result of construction of rhombus by the group of students: a) The figure constructed is a rhombus; b) The problem discovered: only three sides have been defined as necessary equal in length

2) Students started the construction properly, but at some stage added an additional constraint. Typically, they assigned some specific value to two geometric objects that need to be congruent, for example, the two opposite angles of a parallelogram were given a fixed measure, or a pair of opposite sides were set to a fixed length. This meant that only a partial set of possible figures were derived by this construction (see Figure 2). Because they obtained a family of geometric objects that fit the definition, it was hard for the students to understand the nature of this error.

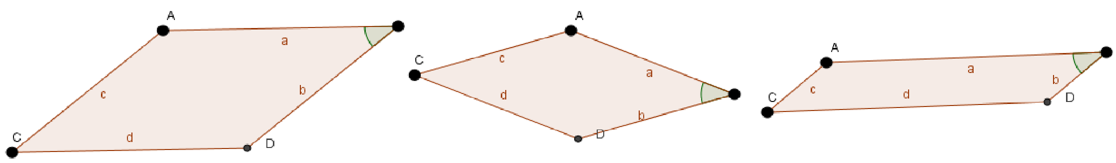

Figure 2. Construction of a parallelogram as a quadrilateral with two pairs of adjacent supplementary angles: in each parallelogram, the acute angles were set to 40°, thus restricting the answer to only a subset of parallelograms

3) Students began the construction according to the given set of requirements, but unconsciously replaced one requirement by another that could be realized easier in the construction process. For example, instead of constructing a rhombus as a four-sides-equal-in-length quadrilateral, one group after construction of two equal adjacent sides has completed the construction by drawing sides that are parallel to the constructed segments. While the final result is, indeed, a whole family of rhombi, only analysis of the protocol [8] can detect the erroneous substitution.

After these misunderstandings were discussed with the group, we asked the participants to evaluate the role of computer technology in the activity. Most of them agreed that the dynamic environment does provide a deeper comprehension of the issue and that they believe they will try to incorporate dynamic software in their teaching.

5 Summary

Constructing geometrical objects in a dynamic geometry environment is an activity that has both mathematical and didactic value. Through these activities, students can 'visualize the definition' of an object and understand the advantages of GeoGebra as a tool to construct entire families of objects. By examining how pre-service teachers use GeoGebra, teacher educators can obtain significant information about the students' comprehension of how mathematical objects are defined and their ability to construct the object according to a specific set of definitions. Following a discussion of how the solutions were obtained, pre-service secondary school math teachers seemed convinced that GeoGebra (or similar tool) can help illuminate typical difficulties and misconceptions that students might have in the logic behind the definition of geometric shapes. They also seemed amenable to the use of dynamic geometry in their teaching in school.

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NIE(ROZUMIENIE) PRZEZ KANDYDATÓW NA NAUCZYCIELI KONSTRUKCJI DYNAMICZNYCH OBIEKTÓW GEOMETRII

Przyszli nauczyciele matematyki w szkole średniej uczyli się podstaw programu GeoGebra, a następnie mieli za zadanie skonstruowanie określonego czworoboku zgodnie z danym opisem. Jakościowa analiza ich rozwiązań ujawnia typowe błędne wyobrażenia studentów i popełnione przez nich błędy.