A Study on the Performance Comparison of Three Optimal Alpha-Beta-Gamma Filters and Alpha-Beta-Gamma-Eta Filter for a High Dynamic Target

T.G. Jeong
Korea Maritime and Ocean University, Busan, Korea

A.W. Njonjo
Jomo Kenyatta University of Agriculture and Technology, Juja, Kenya

B.F. Pan
Korea Maritime and Ocean University, Busan, Korea

ABSTRACT: The α-β-γ tracking filter is useful for tracking a constant acceleration target with zero lag error in the steady state. It, however, depicts a constant lag error for a maneuvering target. Various algorithms of the α-β-γ tracking filter exist in literature and each one of them presents its own unique challenges and advantages depending on the design requirement.

This study investigates the operation of three α-β-γ tracking filter design methods which include Benedict-Bordner also known as the Simpson filter, Gray-Murray filter and the fading memory constant acceleration filter. These filters are then compared based on the ability to reduce noise and follow a maneuvering target with minimum lag error, against the jerky model α-β-γ-η. Results obtained from simulations of the input model of the target dynamics under consideration indicate an improvement in performance of the jerky model in comparison with the constant acceleration models.

1 INTRODUCTION

The tracking radar system has a wide application in both the military and civilian fields. In the military, tracking is essential for fire control and missile guidance whereas in civilian application it is useful for controlling traffic of manned maneuverable vehicles such as ships, submarines and aircrafts which require accurate tracking. Tracking filters play the key role of target state estimation from which the tracking system is updated. One of the tracking filters in use today in many tracking applications is the α-β-γ filter which is a development of the α-β filter aimed at tracking an accelerating target since the α-β filter is only effective when the target model input is a constant velocity model.

Due to the essential role that tracking filters play in a tracking system, many researchers have taken quite an interest in understanding the theory and application leading to valuable insights into design developments and improvement. In the early work of Benedict and Bordner (1962), the authors based their analysis of the α-β filter on the frequency domain (Z-transform). They proposed a relationship between the α and β filtering coefficients derived from a pole matching technique in order to optimize the tracker's ability to reduce noise and achieve a good transient performance. This led to what is known today as the Benedict-Bordner relationship. Simpson (1963) further extended this study to the α-β-γ filter by including the acceleration term thus arriving at the optimization condition between the filter weight coefficients.

Kalata (1984) proposed the use of a tracking index that relates the filter coefficients and is a function of position uncertainty due to target maneuverability, radar measurement uncertainty and update time interval. He utilized the tracking index parameter to derive implicit closed form equations of the

smoothing coefficients which resulted in optimal performance. A more convenient way to determine the optimal filtering weights was investigated by Gray and Murray (1993) whereby a damping parameter that computes the smoothing coefficients directly was derived analytically.

Njonjo et al (2016) investigated the performance of the fading memory α-β-γ filter on a high dynamic target warship. The research concluded that the filter was capable of tracking the highly maneuvering vessel with a relatively good accuracy in terms of noise reduction. This research was further extended by Pan et al (2016) where the filter was optimized in order to improve its tracking ability by reducing the noise further. The optimization procedure involved varying the value of the discounting factor, ξ, with the residual error and determining the ξ that corresponds to the minimum error. The study concluded that the optimal filter uniquely varies with the initial speed and average speed of the target under consideration.

In this study, different algorithms of the steady state Kalman filter are investigated and compared based on capability to reduce noise of a highly maneuvering target and hence provide quality estimates. The study focusses on performance comparison of the Benedict-Bordner filter also known as the Simpson filter, Gray-Murray filter and the fading memory α-β-γ filter, also known as the critically damped filter. These filters are then compared with the optimal α-β-γ-η filter.

2 THEORY OF THE A-B-Γ FILTER

The α-β-γ filter is a steady state Kalman filter which assumes that the input model of the target's dynamics is a constant acceleration model. The model has a low computational load since only two steps are involved that is, estimation and updating the prediction estimates of position, velocity and acceleration as shown in Equations 1-6. In addition, the smoothing coefficients of the filter are constant for a given sensor which further contributes to its design simplicity. The selection of the weighting coefficient is an important design consideration as it directly affects the error reduction capability. The optimal filter of three different designs of the α-β-γ filter are investigated and compared based on their ability to reduce tracking error and improve the tracking response. The three designs differ in their selection of the smoothing coefficients α, β and γ.

Prediction;

$$P_p(n) = P_s(n-1) + tV_s(n-1) + \frac{t^2}{2}A_s(n-1), \qquad (1)$$

$$V_p(n) = V_s(n-1) + tA_s(n-1), \qquad (2)$$

$$A_p(n) = A_s(n-1). \qquad (3)$$

Smoothing;

$$P_s(n) = P_p(n) + \alpha(P_o(n) - P_p(n)), \qquad (4)$$

$$V_s(n) = V_p(n) + \frac{\beta}{t}(P_o(n) - P_p(n)), \qquad (5)$$

$$A_s(n) = A_p(n) + \frac{2\gamma}{t^2}(P_o(n) - P_p(n)). \qquad (6)$$

where o, p and s denote the observed, predicted and smoothed state parameters respectively; P, V and A are the target's position, velocity and acceleration respectively; t is the simulation time interval; and n is the sample number.

2.1 Benedict-Bordner model

The optimal filter is obtained when the condition in Equation 7 is satisfied.

$$\beta = \frac{\alpha^2}{2 - \alpha}. \qquad (7)$$

The design of this filter does not specify the optimal position smoothing coefficient, α, hence it is chosen based on the system application. It is proposed to vary α with observed high frequency power fluctuations of the tracking error residual or the innovation, $(P_o(n) - P_p(n))$.

The Benedict-Bordner filter coefficient relationship becomes an optimal third order tracking filter when the condition in Equation 8 is satisfied;

$$2\beta - \alpha(\alpha + \beta + \frac{\gamma}{2}) = 0. \qquad (8)$$

2.2 Gray-Murray Model

This filter is an extension of the Kalata filter coefficients relationship which employs the tracking index to compute a damping parameter which is consequently used to calculate the position smoothing coefficient, α. The tracking index is determined from the relationship given in Equation 9.

$$\Lambda = \frac{t^2\sigma_w}{\sigma_v}. \qquad (9)$$

where t = target tracking period; σ_w = maneuverability noise; and σ_v = measurement noise.

The damping parameter, r, is computed as shown in Equation 10. The position, velocity and acceleration gain parameters, α, β, and γ, are computed explicitly as shown in Equations 11-13.

$$(3)$$

$$r = \frac{(4 + \Lambda) - \sqrt{(8\Lambda + \Lambda^2)}}{4}, \qquad (10)$$

$$\alpha = 1 - r^2, \qquad (11)$$

$$\beta = 2(2 - \alpha) - 4\sqrt{1 - \alpha}, \qquad (12)$$

$$\gamma = \frac{\beta^2}{2\alpha}. \qquad (13)$$

2.3 *The fading memory filter model*

The fading memory filter has three real roots and represents the filter minimizing the discounted least squares error for a constantly accelerating target as discussed Brookner (1998). The position, velocity and acceleration gain coefficients are computed from the damping parameter, ξ, which is the discounting factor and whose value was investigated through an optimization process and found to depend on the initial and average speed of the target under consideration as was determined by Pan et al (2016). The smoothing coefficients are obtained as shown in Equations 14-16.

$$\alpha = 1 - \xi^3, \qquad (14)$$

$$\beta = 1.5(1 - \xi)^2(1 + \xi), \qquad (15)$$

$$\gamma = (1 - \xi)^3. \qquad (16)$$

3 THEORY OF α-β-γ-η FILTER

The α-β-γ-η filter is a constant gain, four-state tracking filter. The four state vector includes position, velocity, acceleration and jerk, a time derivative of acceleration. The jerk is modelled as a constant and includes zero mean white Gaussian noise. Equations 17- 20 are the prediction equations for position, velocity, acceleration and jerk respectively where they are updated from the estimated state thereby lowering the tracking error. Equations 21-24 are the smoothing equations which are computed by adding a weighted difference between the observed and the predicted position to the forecast state.

Prediction;

$$P_p(n) = P_s(n-1) + tV_s(n-1) + \frac{t^2}{2}A_s(n-1) + \frac{t3}{6}j_s(n-1), \qquad (17)$$

$$V_p(n) = V_s(n-1) + tA_s(n-1) + \frac{t^2}{2}j_s(n-1), \qquad (18)$$

$$A_p(n) = A_s(n-1) + tj_s(n-1), \qquad (19)$$

$$j_p(n) = j_s(n-1). \qquad (20)$$

Smoothing;

$$P_s(n) = P_p(n) + \alpha(P_o(n) - P_p(n)), \qquad (21)$$

$$V_s(n) = V_p(n) + \frac{\beta}{t}(P_o(n) - P_p(n)), \qquad (22)$$

$$A_s(n) = A_p(n) + \frac{\gamma}{2t^2}(P_o(n) - P_p(n)), \qquad (23)$$

$$j_s(n) = j_p(n) + \frac{\eta}{6t^3}(P_o(n) - P_p(n)). \qquad (24)$$

where the subscripts *o*, *p* and *s* denote the observed, predicted and smoothed state parameters respectively; *P*, *V* and *A* are the target's position, velocity and acceleration respectively; *t* is the simulation time interval; and *n* is the sample number.

The filter weight constants, α, β, γ and η, are computed using the fading memory filter model as shown in Equations 25-28 (Brookner, 1998). ξ is the discounting factor that minimizes the least squares error for a constant jerk model input. The smoothing constants are determined from the value of the discounting factor hence the optimization of the filter is applied on the ξ as illustrated by Pan et al (2016).

$$\alpha = 1 - \xi^4, \qquad (25)$$

$$\beta = \frac{1}{6}(1 - \xi)^2(11 + 14\xi + 11\xi^2), \qquad (26)$$

$$\gamma = 2(1 - \xi)^3(1 + \xi), \qquad (27)$$

$$\eta = \frac{1}{6}(1 - \xi)^4. \qquad (28)$$

4 SIMULATION

4.1 *Input model of target's dynamics*

The simulation tests were carried out on a high dynamic target moving at the initial speed of 50 m/s as observed from a stationary own ship. A sample signal of n=1000 data samples was investigated at sampling interval time of t=3 s which corresponds to the time of one aerial rotation of the radar antenna. The target's initial position as observed from the radar range measurements was (573, 1038.4) after scan-conversion to produce Cartesian coordinates. The input model employed to generate the target dynamics is as shown below in Equations 29-30.

$$X_i = a[10\sin(1.2wi) + 7\cos(0.99wi) + 8\sin(0.7wi) + 6\cos(2wi) + \tag{29}$$
$$9\sin(3wi) + 5\cos(3wi)] + 10i,$$

$$Y_i = b[20\cos(0.3wi) + 22\sin(2wi)]. \tag{30}$$

The resulting data was then sampled at intervals of three seconds to obtain the true trajectory of the target as shown in Figure 1.
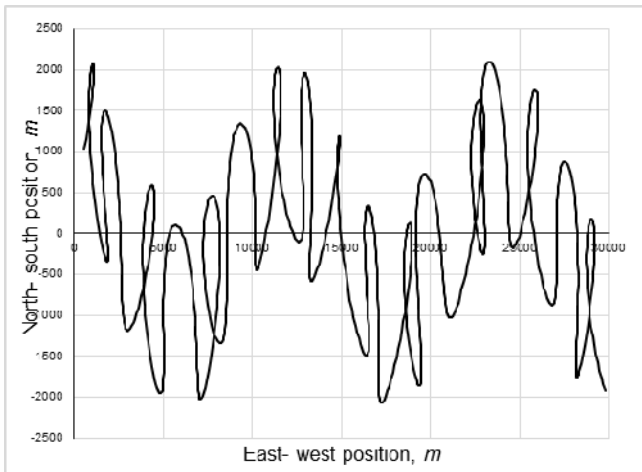
Figure 1. Target's true trajectory.

4.2 *Noise addition*

The observation measurement obtained from the radar sensor contains an error which was accounted for by corrupting the true positions with zero mean random white Gaussian noise with a standard deviation, σ, of 10 m. Figures 2a & 2b show the error distribution in the observation.
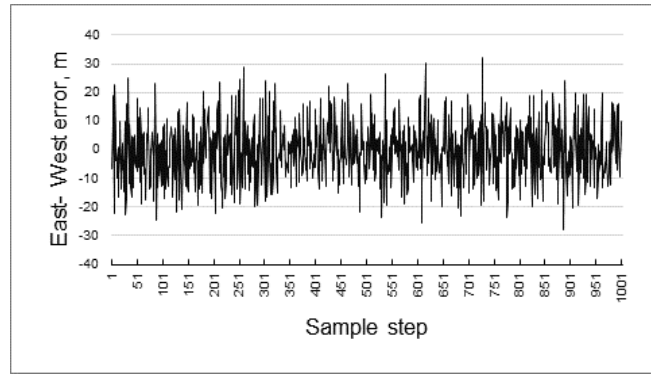

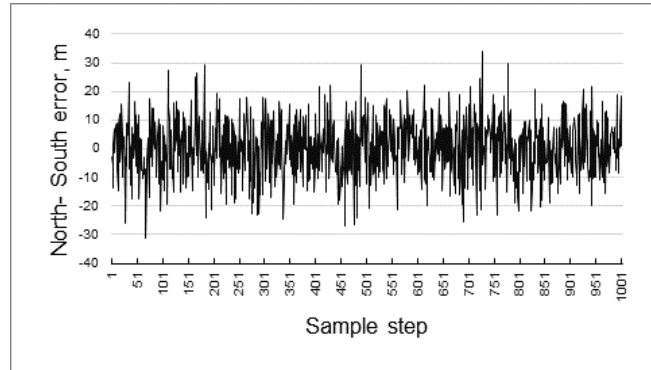Figure 2a. East-west error distribution in the observation.

Figure 2b. North-south error distribution in the observation.

4.3 *Filter gain coefficient selection and computation*

4.3.1 *Filter gain coefficients selection using the Benedict-Bordner model*

Since this design method does not provide an analytical solution for determining the position smoothing coefficient α, in this study, the position smoothing coefficient was determined experimentally through a trial and error method by plotting it against the corresponding innovation which is the total residual obtained from the difference between the observed position and predicted position trajectories as shown in Figure 3. The interval evaluated was selected based on the stability constraints provided for by Jury (1964) for the α-β-γ tracking filter. The value of α that best reduced the innovation was found to be α=0.86. Equations 7 & 8 were then used to compute the values of the velocity and acceleration smoothing coefficients as shown in Table 3.1.
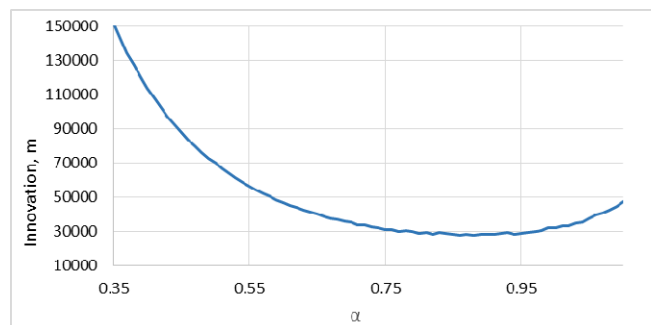

Figure 3. Total residual between observed and predicted positions against corresponding value of position smoothing coefficient, α.

Table 1. Smoothing coefficients obtained from Benedict-Bordner model.

α	β	γ
0.86	0.6488	4.4409×10^{-16}

4.3.2 Filter gain coefficients selection using the Gray-Murray model

The maneuverability and measurement noise variances were determined experimentally by an iterative trial and error method by changing the values of maneuverability and measurement error variances while simultaneously feeding the measurement data to the filter for each error variance. The output was then used to compute cumulative positional error which was then plotted against corresponding error variances. The purpose of this procedure was to determine the error variance coefficient corresponding to the least error. From the Figures 4-7, the values of the maneuverability and measurement error variance coefficients corresponding to the minimum residual error are 10^{-3} and 1 respectively. Consequently, the respective standard deviations are estimated to be $\sigma_w=0.03162$ and $\sigma_v=1$.

The tracking index was, therefore, computed as $\Lambda=0.2846$ and, consequently the damping parameter, $r=0.6873$. The smoothing coefficients are then computed using Equations 11-13 and are obtained as displayed in Table 2.

Table 2. Smoothing coefficients obtained from Gray-Murray model.

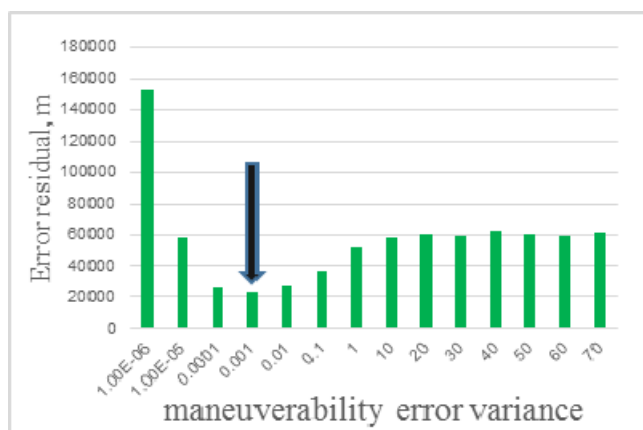| α	β	γ
0.5277	0.1956	0.0101

Figure 4. Cumulative error difference between observed and predicted positions against maneuverability error variance.
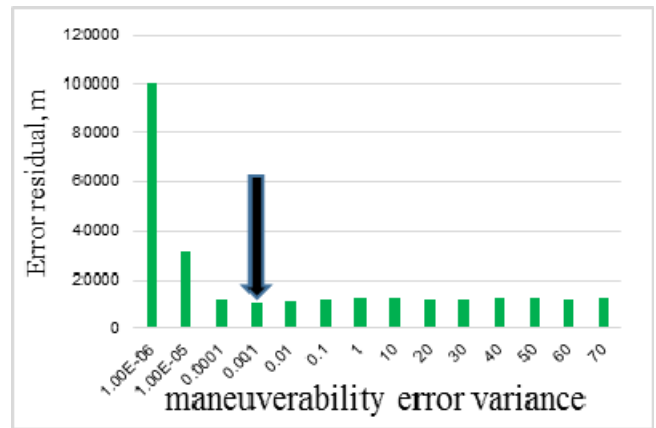

Figure 5. Cumulative error difference between true and smoothed positions against maneuverability error variance.
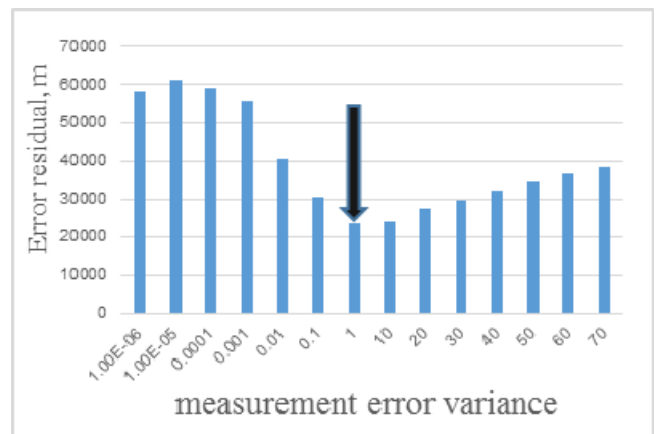

Figure 6. Cumulative error difference between observed and predicted positions against measurement error variance.

Figure 7. Cumulative error difference between true and smoothed positions against measurement error variance.

4.3.3 Filter gain coefficients selection using the fading memory model

The optimal value of the damping parameter ξ was experimentally found to be 0.62 for a maneuvering target with an initial speed of 50.4 m/s (Pan et al. 2016). The Equations 14-16 were then employed to compute the optimal filtering coefficients which were obtained as shown in Table 3.

Table 3. Smoothing coefficients obtained from fading memory filter model.

α	β	γ
0.7379	0.3188	0.0467

4.3.4 *Filter gain coefficients selection using the jerky model*

The optimal value of the damping parameter ξ was determined through an iterative trial and error method and found to be 0.74 for a maneuvering target with an initial speed of 50.4 *m/s* and sampled at intervals of 3 *seconds* (Pan et al. 2016). Equations 25-28 were then employed to compute the optimal filtering coefficients as shown on Table 4.

Table 4. Smoothing coefficients obtained from jerky filter model.

α	β	γ	η
0.7001	0.3085	0.0612	7.6163^{-04}

5 FILTER PERFORMANCE COMPARISON

In this study, the comparison of the filters was based on the following performance indices i.e. tracking and estimation error reduction, sensitivity of the filter to target maneuvers and output data stability.

5.1 *α-β-γ filter results and performance comparison*

Figures 8-10 show the true, observed, predicted and smoothed positions trajectories obtained from the tracking problem using the various *α-β-γ* filter models under consideration in this study. The figures represent the positional trajectories for the Benedict-Bordner filter, Gray-Murray model, the Fading memory filter model respectively. Of the three models under investigation, the Gray-Murray model appears to follow the target quite well with high sensitivity to changes in target maneuvers as indicated by the stability and steadiness of the trajectories as the target transitions from one point to the next. In addition, the output trajectories which include the predicted and smoothed position trajectories can be observed to transition very smoothly and closely to the true trajectory for the entire duration of the tracking period. The fading memory model performs nearly as well as the Gray-Murray model except for a few fluctuations of data samples at several points along the target's curves indicating a reduced sensitivity at these points on the targets' trajectories as it maneuvers. As for the Benedict-Bordner model, shown in Figure 8, the filter performs worst, based on sensitivity to target maneuvers and data stability, compared to the other two α-β-γ filters as indicated by the visibly clear jerky motion at the beginning of the tracking process. However, as tracking continues the trajectories stabilize and the tracking accuracy can be seen to also improve.
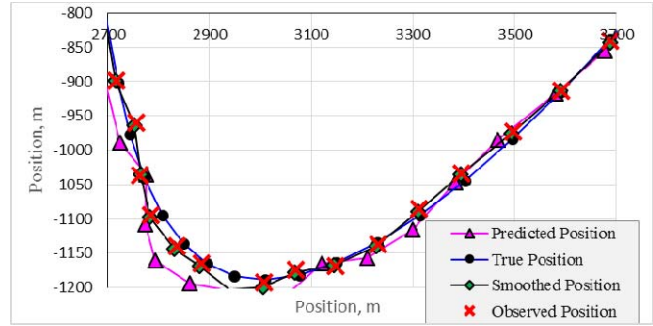

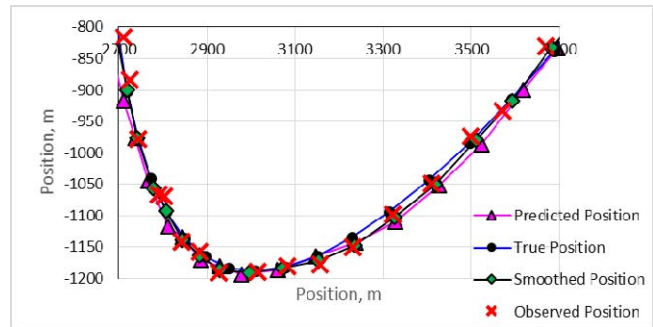Figure 8. Target's True, Observed, Predicted and Smoothed Position, Benedict- Bordner model.

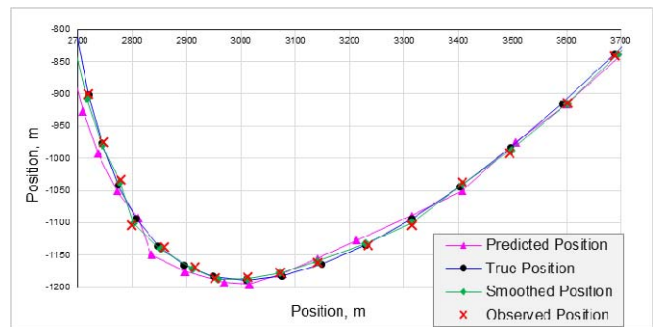Figure 9. Target's True, Observed, Predicted and Smoothed Position, Gray-Murray model.

Figure 10. Target's True, Observed, Predicted and Smoothed Position, fading memory model.

Table 5 shows the total prediction and estimation errors obtained using the different *α-β-γ* filters. Estimation error is obtained by computing the deviation of the estimated data from the true position for each sample. Similarly, prediction error indicates how far the predicted position deviates from the true position hence it is the tracking error. These results show that the fading memory model has the highest accuracy in both tracking and estimation of the position of the high dynamic target among the *α-β-γ* filters as can be seen from the small error values obtained, followed by the Gray-Murray model. The Benedict-Bordner model performs the worst in terms of tracking and estimation noise reduction for both prediction and estimation as indicated by the resulting large errors values. This can be explained by the fact that the design of this filter is based primarily on the requirement for satisfying a good transient response. And since performance of a filter is a tradeoff between a good transient response and noise reduction, the filter then performs poorly when applied to meet the requirement for tracking error reduction.

Table 5. Summary of the total tracking and estimation accuracy obtained from the α-β-γ filters.

Filter type	Tracking error	Estimation error
	m	m
Benedict-Bordner	26,326	11,677
Gray-Murray	21071	11,693
Fading memory	19,622	10,653

5.2 *Comparison of α-β-γ filter results with the jerky model*

Figure 11 shows the true, observed, predicted and smoothed trajectories obtained using the jerk model. The curves can be observed to easily follow the highly maneuvering target with greater sensitivity as indicated by the steadiness in the predicted and smoothed trajectories and a reduction of fluctuations that were observed in the trajectories obtained using the fading memory α-β-γ filter. However, the Gray-Murray model still maintains a greater sensitivity to target maneuvers and has a higher stability in its output data leading to steadier trajectories.
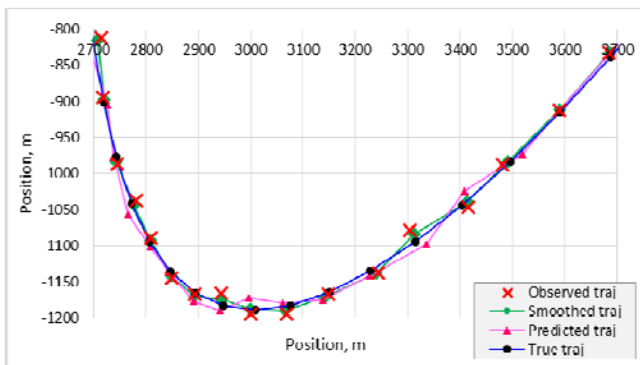

Figure 11. Target's True, Observed, Predicted and Smoothed Position, jerky model.

The total tracking error and total estimation error are obtained as shown in Table 6. The results indicate an improvement in both tracking and estimation accuracies on applying the jerk model in comparison with the fading memory α-β-γ filter model. The accuracy in tracking is therefore improved by 1,733.27 *m* equivalent to approximately 9%. Similarly, the estimation accuracy is increased by 419.49 *m* on employing the jerk model filter.

Table 6. Summary of the total tracking and estimation accuracy obtained from the α-β-γ filters.

Filter type	ξ	Tracking error	Estimation error
		m	m
α-β-γ filter	0.62	19622	10,653
α-β-γ-η filter	0.74	17,859	10227

6 CONCLUSION

This study investigated the performance of three conventional α-β-γ filter models under the same initial conditions to track a high dynamic target undergoing random velocity changes. The performance of the filters was evaluated based on

ability to follow the maneuvering target steadily and closely with minimum jerky motions and without loss of target. It was also a function of noise reduction in the estimation and prediction results.

Of the three filters, the Benedict-Bordner filter performed the worst as the resulting trajectories were characterized by overshooting at various points of the target's curves.

The critically damped filter, on the other hand, performed efficiently in terms of noise reduction in both prediction and estimation which is visibly clear from the high accuracy obtained compared to the Gray-Murray filter. In addition to demonstrating a good capability of following the maneuvering target with ease and steadiness, the critically damped filter was also easy to implement due to its simplicity and low computational load. However, the Gray-Murray filter depicted a better sensitivity to target maneuvers which was visible from the obtained smooth curves of the position trajectories indicating a higher efficiency in following the highly maneuvering target.

On applying the jerk model, an improvement was realised in both noise reduction and ability to follow the maneuvering target with less fluctuations on the trajectories. Tracking accuracy was improved by approximately 9% compared to the constant acceleration filter. The jerky model was therefore a further enhancement of the constant acceleration filter in terms of increasing data stability through a reduction of fluctuations especially at points of sudden speed and course changes.

Future studies will investigate the tracking performance of the filter while both the observing ship and the high dynamic target are on motion.

REFERENCES

Benedict, T. R. & Bordner, G. W. 1962. Synthesis of an Optimal Set of Radar Track-While-Scan Smoothing Equations. *IRE Trans. on Automatic Control*. AC-7: pp.27-32.

Brookner, E. 1998. *Tracking and Kalman filtering made easy*. New York: Wiley.

Gray, J. E. & Murray, W. J. 1993. A Derivation of an Analytic Expression for the Tracking Index for the α-β-γ Filter. *IEEE Trans. On Aerospace and Electronic Systems* Vol. 29 (3).

Jury, E. I. 1964. *Theory and application of the z-transform method*. New York: Wiley.

Kalata, P. R. 1984. The tracking index: A generalized parameter for α - β and α - β - γ target tracker. *IEEE Trans. on Aerospace and Electronic Systems* AES-20 (2): pp. 174-181.

Pan B.F., Njonjo A.W. & Jeong T.G. 2016. Basic Study of the Optimization of the Gain Parameters α, β and γ of a Tracking Module for ARPA system on Board High Dynamic Warships Proceedings, *Journal of Navigation and Port Research* Vol.40 (5): pp.241-247.

Pan B.F., Njonjo, A.W. & Jeong, T.G. 2016. A Study of Optimization of α-β-γ-η filter for tracking a high Dynamic target. *Proceedings, 2016 Asia Navigation conference*.

Simpson, H. R. April 1963. Performance Measures and Optimization Condition for a Third Order Sampled-Data Tracker. *IEEE Transactions on Automatic Control* Vol. AC8: pp. 182–183.