Control and Cybernetics

vol. **44** (2015) No. 3

A more direct demonstration on the EOQ model under retailer partial trade credit policy in supply chain*

by

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Abstract: Huang and Hsu (2008) investigated the inventory system as a cost minimization problem, with the objective to determine the retailer's optimal inventory policy under the supply chain conditions. Then, Chung (2008) presented the comments to this problem, with the aim to overcome the shortcomings and present complete proofs for the results, relative to Huang and Hsu (2008). However, the proof, proposed by Chung is cumbersome, and can be followed with difficulty. The main purpose of this paper is to develop another proof, much more easy to comprehend.

Keywords: EOQ, inventory, trade credit, permissible delay in payments, supply chain

1. Introduction

Huang and Hsu (2008) investigated the retailer's inventory policy under two levels of trade credit, meant to reflect the supply chain management situation. They assume that the retailer has a powerful decision-making right. So, they extend the assumption that the retailer can obtain the full trade credit offered by the supplier and the retailer just offers the partial trade credit to his/her customer. Then, they investigate the retailer's inventory system as a cost minimization problem to determine the retailer's optimal inventory policy and two easy-to-use theorems are developed to efficiently determine the optimal inventory policy for the retailer. Subsequently, Chung (2008) presented the comments, with the aim to overcome the shortcomings and present the complete

*Submitted: April 2010; Accepted: January 2016.

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proofs for the proposals of Huang and Hsu (2008). However, the proof, developed by Chung is excessively cumbersome and cannot be easily understood. The main purpose of this paper is to develop another easy-reading proof for the readers.

2. Analysis and explanation

We adopt the same notation and assumptions as Huang and Hsu (2008) in this paper, namely:

D = demand rate per year,
A = ordering cost per order,
c = unit purchasing price,
s = unit selling price, $s \geqslant c$,
h = unit stock holding cost per year excluding interest charges,
α = the customer's fraction of the total amount owed payable at the time of placing an order offered by retailer, $0 \leqslant \alpha \leqslant 1$,
I_e = interest earned per \$ per year,
I_k = interest charged per \$ in stocks per year by the supplier,
M = the retailer's trade credit period offered by supplier in years,
N = the customer's trade credit period offered by retailer in years,
T = the cycle time in years,
TRC(T)= the annual total relevant cost, which is a function of T,
$T*$ = the optimal cycle time of *TRC(T)*,
$Q*$ =the optimal order quantity = DT*.

In both Huang and Hsu (2008) and Chung (2008), the retailer's inventory annual total relevant cost consisted of the elements as outlined in what follows. Two situations may arise, i.e. (I) $M \geqslant N$ and (II) $M < N$. And the annual total relevant cost for the retailer can be expressed as

TRC(T) = ordering cost + stock-holding cost + interest payable − interest earned.

Case I: Suppose that M $\geqslant$ N.

$$TRC(T) = \begin{cases} TRC_1(T) & \text{if} \quad T \geqslant M \\ TRC_2(T) & \text{if} \quad N \leqslant T \leqslant M \\ TRC_3(T) & \text{if} \quad 0 < T \leqslant N \end{cases} \tag{1}$$

where

$$TRC_1(T) = \frac{A}{T} + \frac{DTh}{2} + cI_k D(T-M)^2/2T - sI_e D[M^2 - (1-\alpha)N^2]/2T, \tag{2}$$

$$TRC_2(T) = \frac{A}{T} + \frac{DTh}{2} - sI_e D[2MT - (1-\alpha)N^2 - T^2]/2T \tag{3}$$

and

$$TRC_3(T) = \frac{A}{T} + \frac{DTh}{2} - sI_eD[M - (1-\alpha)N - \frac{\alpha T}{2}]. \quad (4)$$

From (2) – (4), we obtain

$$TRC_1(M) = TRC_2(M) \text{ and } TRC_2(N) = TRC_3(N). \quad (5)$$

Notice that $TRC_1(T)$ is not equal to the right-hand-side terms of (2) if $T < M$. Hence, it does not make sense to redefine $TRC_1(T)$ on $T > 0$ as shown in Chung (2008). Likewise, $TRC_2(T)$, $TRC_3(T)$, and $TRC(T)$ cannot be redefined on $T > 0$. In fact, only one of the following three mutually exclusive events can occur: (1) $T > M$, (2) $N < T < M$, and (3) $T < M$. In Chung (2008), all of the magnitudes: $TRC_1(T)$, $TRC_2(T)$, $TRC_3(T)$, and $TRC(T)$ are redefined on $T > 0$, which, in turn, implies that all the three mutually exclusive events can occur at the same time.

Case II : Suppose that M<N

$$TRC(T) = \begin{cases} TRC_4(T) & \text{if} \quad T \geqslant M \\ TRC_5(T) & \text{if} \quad 0 < T \leqslant M \end{cases} \quad (6)$$

where

$$TRC_4(T) = \frac{A}{T} + \frac{DTh}{2} + cI_kD(T-M)^2/2T - sI_eD\alpha M^2/2T, \quad (7)$$

and

$$TRC_5(T) = \frac{A}{T} + \frac{DTh}{2} - sI_eD[\alpha M - \frac{\alpha T}{2}]. \quad (8)$$

By using (7) and (8), we derive

$$TRC_4(M) = TRC_5(M). \quad (9)$$

Again, it is obvious that $TRC_4(T)$ is not equal to the right-hand-side terms of (7) if $T < M$. Hence, it does not make sense to redefine $TRC_4(T)$ on $T > 0$, as shown in Chung (2008). Similarly, $TRC_5(T)$ and $TRC(T)$ cannot be redefined on $T > 0$.

Now, let us discuss the first case, i.e. the one of $M \geqslant N$, and then the case of $M < N$. To minimize the annual total relevant cost, taking the first-order and the second-order derivatives of $TRC_1(T)$, $TRC_2(T)$, and $TRC_3(T)$ with respect to T, we obtain

$$TRC_1'(T) = -\left[\frac{2A + cDM^2I_k - sDI_e(M^2 - (1-\alpha)N^2)}{2T^2}\right] + D(\frac{h + cI_k}{2}), \quad (10)$$

$$TRC_1''(T) = \frac{2A + cDM^2 I_k - sDI_e(M^2 - (1-\alpha)N^2)}{T^3}, \tag{11}$$

$$TRC_2'(T) = -\left[\frac{2A + sD(1-\alpha)N^2 I_e}{2T^2}\right] + D(\frac{h + sI_e}{2}), \tag{12}$$

$$TRC_2''(T) = \frac{2A + sD(1-\alpha)N^2 I_e}{T^3} > \quad 0, \tag{13}$$

$$TRC_3'(T) = \frac{-A}{T^2} + D(\frac{h + s\alpha I_e}{2}) \tag{14}$$

and

$$TRC_3''(T) = \frac{2A}{T^3} > 0. \tag{15}$$

Equations (13) and (15) imply that $TRC_2(T)$ and $TRC_3(T)$ are strictly convex on $T > 0$. Consequently, we obtain the corresponding unique optimal cycle times T_2^* and T_3^* as given by

$$T_2^* = \sqrt{\frac{2A + sD(1-\alpha)N^2 I_e}{D(h + sI_e)}} \tag{16}$$

and

$$T_3^* = \sqrt{\frac{2A}{D(h + s\alpha I_e)}}. \tag{17}$$

To ensure that $N \leqslant T_2{}^* \leqslant M$, we substitute equation (16) into $N \leqslant T \leqslant M$, and then we can obtain that

$$\begin{gathered}\text{if and only if } \Delta_1 = -2A + DM^2 h + sDI_e[M^2 - (1-\alpha)N^2] \geqslant 0 \\ \text{and } \Delta_2 = -2A + DN^2(h + s\alpha I_e) \leqslant 0, \text{then } T_2^* \text{is as shown in (16).}\end{gathered} \tag{18}$$

Notice that we can easily prove that $\Delta_1 \geqslant \Delta_2$. Similarly, in order to ensure that $T_3{}^* \leqslant N$, we substitute equation (17) into $T \leqslant N$, and then we can obtain that

$$\text{if and only if } \Delta_2 \geqslant 0, \text{ then } T_3^* \text{ is as shown in (17).} \tag{19}$$

If $2A + cDM^2 I_k - sDI_e(M^2 - (1-\alpha)N^2) > 0$, then we know from (9) that $TRC_1(T)$ is strictly convex on $T > 0$. Therefore, we can easily obtain the unique optimal cycle time $T_1{}^*$ as expressed through

$$T_1^* = \sqrt{\frac{2A + cDM^2 I_k - sDI_e[M^2 - (1-\alpha)N^2)]}{D(h + cI_k)}}. \tag{20}$$

It is obvious from (20) that if $2A + cDM^2 I_k - sDI_e(M^2 - (1-\alpha)N^2) < 0$, then $T_1{}^*$ does not exist. To ensure $T_1{}^* \geqslant M$, we substitute equation (20) into $T \geqslant M$, and then we can obtain that

$$\text{if and only if } \Delta_1 \leqslant 0, \text{ then } T_1 * \text{ is as shown in (20).} \tag{21}$$

From the above arguments and the fact of $\Delta_1 \geqslant \Delta_2$, we obtain the following results.

Theorem 1 *For $M \geqslant N$,*
A. If $\Delta_2 \geqslant 0$, then $T^* = T_3^*$.
B. If $\Delta_1 > 0$ and $\Delta_2 < 0$, then $T^* = T_2^*$.
C. If $\Delta_1 \leqslant 0$, then $T^* = T_1^*$.

Proof:
(A) If $\Delta_2 \geqslant 0$, then $\Delta_1 \geqslant 0$,

$$TRC_1'(T) = -\frac{1}{T^2}\left[\frac{2A + cDM^2 I_k - sDI_e(M^2 - (1-\alpha)N^2)}{2}\right] + D(\frac{h + cI_k}{2})$$

$$\geqslant (1 - \frac{M^2}{T^2})[\frac{D(h + cI_k)}{2}] > 0, \quad \text{if } T > M; \tag{22}$$

and

$$TRC_2'(T) = -\frac{1}{T^2}\left[\frac{2A + sD(1-\alpha)N^2 I_e}{2}\right] + D(\frac{h + sI_e}{2})$$

$$\geqslant (1 - \frac{N^2}{T^2})[\frac{D(h + sI_e)}{2}] > 0, \quad \text{if } T > N. \tag{23}$$

Consequently, if $\Delta_2 \geqslant 0$, then both $TRC_1(T)$ and $TRC_2(T)$ are strictly increasing functions for all $T > M$ or N, respectively. From the arguments forwarded in this section, we know that if $\Delta_2 > 0$, then T_3^* is the optimal solution of $TRC_3(T)$. Therefore, we have

$$TRC_3(T_3*) < TRC_3(N) = TRC_2(N) < TRC_2(T) < TRC_2(M),$$
$$\text{for all } N < T < M.$$

Likewise, we obtain

$$TRC_3(T_3^*) < TRC_2(M) = TRC_1(M) < TRC_1(T), \text{ for all } T > M.$$

This completes the proof of the proposition that if $\Delta_2 \geqslant 0$, then $T^* = T_3^*$.

(B) If $\Delta_1 > 0$, we know that $TRC_1(T)$ is a strictly increasing function for all $T > M$ from the discussion presented in point (A). And now, for $\Delta_2 < 0$,

$$TRC_3'(T) = \frac{-A}{T^2} + D(\frac{h + s\alpha I_e}{2})$$

$$< (1 - \frac{N^2}{T^2})[\frac{D(h + s\alpha I_e)}{2}] < 0, \quad \text{if } T < N. \tag{24}$$

Consequently, if $\Delta_2 < 0$, then $TRC_3(T)$ is a strictly decreasing function for all $T < N$. From the arguments provided before in this section, we know that if $\Delta_1 > 0$ and $\Delta_2 < 0$, then T_2^* is the optimal solution of $TRC_2(T)$. Therefore, we have

$TRC_2(T_2{}^*) < TRC_2(N) = TRC_3(N) < TRC_3(T)$, for all $T < T_2{}^*$;

Likewise, we obtain

$TRC_2(T_2^*) < TRC_2(M) = TRC_1(M) < TRC_1(T)$, for all $T > T_2^*$.

This completes the proof of the proposition that if $\Delta_1 > 0$ and $\Delta_2 < 0$, then $T^* = T_2^*$.

(C) If $\Delta_1 \leqslant 0$, then $\Delta_2 < 0$, and we know that $TRC_3(T)$ is a strictly decreasing function for all $T < N$ from the discussion in the preceding point, (B). And, with $\Delta_1 \leqslant 0$,

$$TRC_2'(T) = -\frac{1}{T^2}\left[\frac{2A + sD(1-\alpha)N^2 I_e}{2}\right] + D(\frac{h + sI_e}{2})$$

$$\leqslant (1 - \frac{M^2}{T^2})[\frac{D(h + sI_e)}{2}] < 0, \quad \text{if } T < M. \tag{25}$$

Consequently, if $\Delta_1 \leqslant 0$, then $TRC_2(T)$ is a strictly decreasing function for all $T < M$. From the arguments provided in this section, we know that if $\Delta_1 \leqslant 0$, then T_1^* is the optimal solution of $TRC_1(T)$. Therefore, we have

$TRC_1(T_1{}^*) < TRC_1(T)$, for all $T > T_1{}^*$;

Likewise, we obtain

$TRC_1(T_1^*) < TRC_1(M) = TRC_2(M) < TRC_2(N) = TRC_3(N) < TRC_3(T)$, for all $T < T_1^*$.

This completes the proof of the statement that if $\Delta_1 \leqslant 0$, then $T^* = T_1^*$.

Next, let us discuss the last case, in which $M < N$. To minimize the annual total relevant cost, by taking the first-order and the second-order derivatives of $TRC_4(T)$ and $TRC_5(T)$ with respect to T, we obtain

$$TRC_4'(T) = -\left[\frac{2A + DM^2(cI_k - s\alpha I_e)}{2T^2}\right] + D(\frac{h + cI_k}{2}), \tag{26}$$

$$TRC_4''(T) = \frac{2A + DM^2(cI_k - s\alpha I_e)}{T^3}, \tag{27}$$

$$TRC_5'(T) = \frac{-A}{T^2} + D(\frac{h + s\alpha I_e}{2}) \tag{28}$$

and

$$TRC_5''(T) = \frac{2A}{T^3} > 0. \tag{29}$$

Equation (29) implies that $TRC_5(T)$ is a strictly convex function on T >0. Consequently, we obtain the corresponding unique optimal cycle time T_5^* as

$$T_5^* = \sqrt{\frac{2A}{D(h + s\alpha I_e)}}. \tag{30}$$

To ensure that $T_5^* \leqslant M$, we substitute equation (30) into $T \leqslant M$, then we can obtain that

if and only if $\Delta_3 = -2A + DM^2(h + s\alpha I_e) \geqslant 0$, then T_5^* is as shown in formula (30).

Equation (27) implies that $TRC_4(T)$ is strictly convex on T >0 when $2A + DM^2(cI_k - s\alpha I_e)$>0. Likewise, we can easily obtain the unique optimal cycle time T_4* as

$$T_4* = \sqrt{\frac{2A + DM^2(cI_k - s\alpha I_e)}{D(h + cI_k)}} \quad . \tag{31}$$

It is obvious from (31) that if $2A + DM^2(cI_k - s\alpha I_e)$>0, then T_4* exists. Otherwise, T_4* does not exist. To ensure $T_4* \geqslant M$, we substitute equation (31) into $T \geqslant M$, then we can obtain that

$$\text{if and only if } \Delta_3 \leqslant 0, \text{ then } T_4^* \text{ is as shown in (31).} \tag{32}$$

From the above arguments, we can obtain the following results.

Theorem 2: *For $M < N$,*
A. *If $\Delta_3 \geqslant 0$, then $T^* = T_5^*$.*
B. *If $\Delta_3 < 0$, then $T^* = T_4^*$.*

Proof:
(A) If $\Delta_3 \geqslant 0$,

$$TRC_4'(T) = -\frac{1}{T^2}\left[\frac{2A + DM^2(cI_k - s\alpha I_e)}{2}\right] + D(\frac{h + cI_k}{2})$$

$$\geqslant (1 - \frac{M^2}{T^2})[\frac{D(h + cI_k)}{2}] > 0, \qquad \text{if } T > M. \tag{33}$$

Consequently, if $\Delta_3 \geqslant 0$, then $TRC_4(T)$ is a strictly increasing function for all $T > M$. We know that if $\Delta_3 \geqslant 0$, then T_5^* is the optimal solution of $TRC_5(T)$. Therefore, we have

$TRC_5(T_5{}^*) < TRC_5(T)$, for all $T < T_5{}^*$.

Likewise, we obtain

$TRC_5(T_5{}^*) < TRC_5(M) = TRC_4(M) < TRC_4(T)$, for all $T > T_5^*$.

This completes the proof of the proposition that if $\Delta_3 \geqslant 0$, then $T^* = T_5^*$.

(B) If $\Delta_3 < 0$,

$$TRC_5'(T) = \frac{-A}{T^2} + D(\frac{h + s\alpha I_e}{2})$$

$$< (1 - \frac{M^2}{T^2})[\frac{D(h + s\alpha I_e)}{2}] < 0, \qquad \text{if } T < M. \tag{34}$$

Consequently, if $\Delta_3 < 0$, then $TRC_5(T)$ is a strictly decreasing function for all $T < M$. We know that if $\Delta_3 < 0$, then T_4^* is the optimal solution of $TRC_4(T)$. Therefore, we have

$TRC_4(T_4{}^*) < TRC_4(T)$, for all $T > T_4{}^*$.

Likewise, we obtain

$TRC_4(T_4{}^*) < TRC_4(M) = TRC_5(M) < TRC_5(T)$, for all $T < T_4{}^*$.

This completes the proof of the proposition that if $\Delta_3 < 0$, then $T^* = T_4^*$.

3. Conclusions

Huang and Hsu (2008) investigated the inventory system in terms of a cost minimization problem, with the aim to determine the retailer's optimal inventory policy, under the conditions of the supply chain. They developed two easy-to-use theorems, meant to locate the optimal inventory policy for the retailer. However, Chung (2008) pointed out that their way of proceeding with the arguments to derive those theorems, serving to find the optimal inventory policy is not complete. So, Chung (2008) proposed to overcome these shortcomings and presented the complete proofs for the proposals of Huang and Hsu (2008). However, Chung's proof appears to be too cumbersome to be easily understood. In this note, we present another easy-reading proof to the readers.

Acknowledgement

The authors would like to thank to anonymous referees for their valuable and constructive comments that have led to a significant improvement in the original paper.

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