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MULTI-OBJECTIVE OPTIMIZATION MODEL FOR A MULTI-DEPOT MIXED FLEET ELECTRIC VEHICLE SCHEDULING PROBLEM WITH REAL-WORLD CONSTRAINTS

Summary. This paper presents the problem of public transport planning in terms of the optimal use of the available fleet of vehicles and reductions in operational costs and environmental impact. The research takes into account the large fleet of vehicles of various types that are typically found in large cities, including the increasingly widely used electric buses, many depots, and numerous limitations of urban public transport. The mathematical multi-criteria mathematical model formulated in this work considers many important criteria, including technical, economic, and environmental criteria. The preliminary results of the Mixed Integer Linear Programming solver for the proposed model on both theoretical data and real data from urban public transport show the possibility of the practical application of this solver to the transport problems of medium-sized cities with up to two depots, a heterogeneous fleet of vehicles, and up to about 1500 daily timetable trips. Further research directions have been formulated with regard to larger transport systems and new dedicated heuristic algorithms.

1. INTRODUCTION

Urban sprawl and numerous limitations of transport systems make the development of public transport increasingly important. This entails considerable resources and requires careful planning. In view of the limited, often tight, budgets of many public transport organizers, optimal public transport planning presents a serious challenge. Public transport organizers often face the need to reduce the frequency of trips due to the extension of lines and the insufficient number of vehicles in the fleet. At the same time, global requirements to reduce exhaust emissions, including for mass transit vehicles, significantly complicate the problem of designing vehicle schedules. The often-practiced approach of the manual allocation of vehicles to routes and trips is ineffective and even impossible for larger fleets.

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Therefore, the need to automatically search for optimal solutions, or at least close to optimal ones, ensuring cost reduction and better use of the available fleet of vehicles, is becoming increasingly apparent.

The above-mentioned problem is referred to in the literature as “vehicle scheduling” and, along with route planning with a timetable design, is a typical problem in public transport system design. As confirmed by many authors, both these problems can be considered from various points of view (operational and strategic) depending on the managing entity. For example, Ceder [3], Liebchen [13], and Schobel [18], followed by Kisielewski [10] (among others, argued that the most common approach is to break down the planning process in public transportation into several phases such as line route planning, timetabling, and vehicle and crew scheduling.

In this paper, we consider the vehicle scheduling problem with multiple depots and a heterogeneous fleet of vehicles with particular emphasis on electric buses. Moreover, we take into account multiple criteria, including economic, technical, and environmental criteria. To the best of our knowledge, the proposed approach has not yet been discussed in the literature in such a comprehensive manner [1–2, 8–9, 11–12, 14–18, 20–23], covering so many aspects of the problem (e.g., a number of different criteria in the objective or various scheduling preferences presented in our problem definition).

2. PROBLEM STATEMENT AND RELATED WORKS

2.1. Problem Statement

Planning in public transportation systems typically involves the following processes: 1) network design, 2) line planning and frequency setting, 3) timetable development, 4) vehicle scheduling, 5) crew scheduling, and 6) crew rostering [10, 13]. These processes are usually performed sequentially. The first three belong to strategic planning, whereas the next three are operational planning problems (see Fig. 1). In this paper, we consider the vehicle scheduling process with complex real-world assumptions and constraints.

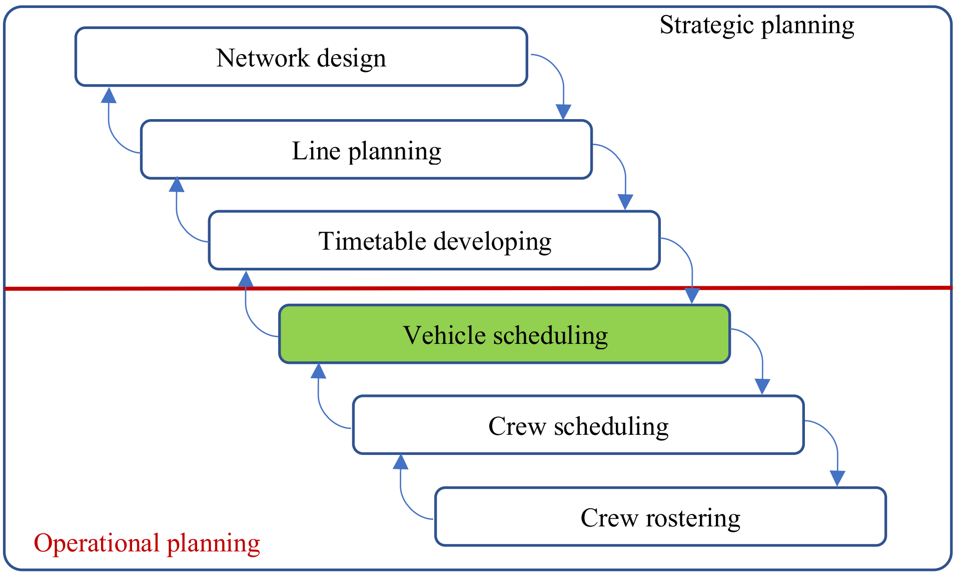

Fig. 1. Planning in public transportation systems

In the literature, the above-described problem is known as the vehicle scheduling problem (VSP) and has many variants. Currently, we can see an increased interest in the electrification of bus fleets because of climate damage from CO_2 emissions. The electrification of bus fleets, however, makes it necessary to change the transport infrastructure and correct the approaches to planning transport systems. New decision problems that arise with the use of electric fleets include [17] 1) the issues of investment in the electric bus fleet and charging infrastructure, 2) the number and location of charging infrastructure, 3) the problem of scheduling electric vehicles, and 4) the problem of scheduling charging operations (e.g., use of different energy tariffs during charging).

The basic VSP is concerned with combining a number of trips, which are defined by timetables, into blocks. These blocks are sequences of trips made by one vehicle from an initial or home depot to the same depot. The objective is to minimize the operational costs of vehicle usage. Solving this problem is of great importance for public transport operators. The most important (often depending on the company) features that define the structure of the VSP include the following:

- Number of depots – two basic cases are distinguished: single depot (SD VSP) or multiple depots (MD VSP). The MD VSP has been proven to be NP-hard [2, 15].
- Number of line trips – determined by a given timetable and is usually constant. In large public transport systems, the number of trips is over 5,000.
- Multiple vehicle types and their assignment to individual depots – there may be restrictions on the allocation of specific vehicle types to lines/trips. In practice, some additional restrictions may arise.
- Limitations on depots or charging points capacity – number of possible vehicles stationed.
- Parameters and constraints related to different durations of layovers between two line trips.
- Minimum and maximum block length.
- Preference for changing lines within a block.
- Other parameters due to specific in-company constraints.

In this paper, we consider the VSP in a public transport network with multiple depots and different types of vehicles, including electric vehicles (i.e., the multi-depot multi-vehicle type electric vehicle scheduling problem (MD MVT E-VSP) [18]). We take into account the following assumptions:

- The transport network is composed of a set of depots (bus depots), a set of starting and ending points for line trips, and a set of charging points for electric vehicles of different types (plug-in chargers, pantograph chargers, etc.).
- The heterogeneous vehicle fleet is composed of different types of vehicles.
- The fleet of electric vehicles requires additional maintenance (recharging the batteries between trips).

A simplified diagram of the transportation network under consideration is shown in Figure 2.

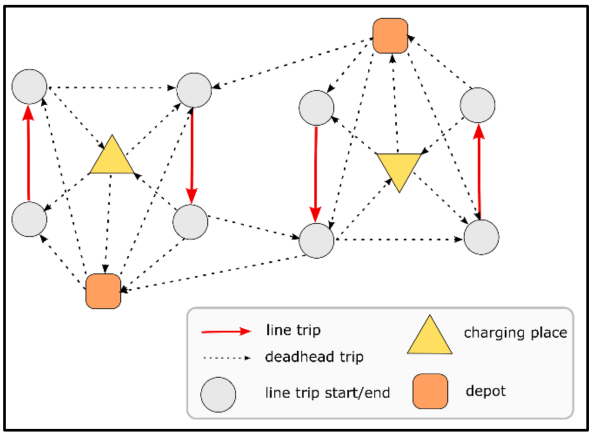

Fig. 2. Simplified transportation network diagram – nodes and links (line and deadhead trips)

2.2. Related works

In recent years, the number of research papers related to scheduling in transportation systems has significantly increased. In particular, electric vehicle scheduling is a rapidly growing area of research. Most of the existing works are concerned with vehicle scheduling problems [1–3, 7–11, 14, 18, 21, 22] and emissions and energy consumption issues [1, 4]. Some papers address the complex (especially for real-world problems) issue of integrated vehicle and crew scheduling [6, 16, 21].

Perumal et al. [17] reviewed the literature on constraints in the scheduling of electric vehicles and proposed approaches to handle them. Aspects of rescheduling or considering robust scheduling for electric vehicles were identified as a future research area. The shift in research towards integrated vehicle and crew scheduling has been highlighted. This topic has also been addressed in [6, 16, 21].

Ge et al. [6] provided a model and formulations containing days-off patterns. They also investigated whether solvability with standard solvers is currently possible and to what extent incorporating other aspects could make the problem richer but still solvable. In contrast, Perumal et al. [16] compared different approaches to solving the integrated electric vehicle and crew scheduling problem (E-VCSP). They developed the adaptive large neighborhood search (ALNS) algorithm, which uses branch-and-price (B&P) heuristics, and identified the potential benefits of integrating these two scheduling problems compared to the traditional sequential approach. The algorithm was verified for different instances of the real problem (the maximum problem size was 1109 line trips). Wang et al. [21] presented a bi-level multi-criteria programming model for the problem of collaborative vehicle and crew scheduling on a bus route served by a mixed bus fleet.

Though a lot of work has been devoted to the VSP with electric vehicles, a continuing challenge is to develop efficient methods for solving the VSP (including electric vehicles) for large real-world problems. This topic has been addressed, among others, in [9, 21,22]. Xu et al. [22] proposed an improved model and algorithm for a multi-depot vehicle scheduling problem (MD VSP) with constraints on departure time. The developed large neighborhood search (LNS) algorithm achieved results with a low quality gap for test instances with different numbers of trips (500, 1000, 1500, and 2000).

Issues related to environmental concerns were presented by Corazza et al. [4]. The authors proposed an integrated, user-friendly iGREEN methodology to determine emission characteristics for different vehicle fleet parameters. Bie et al. [1], on the other hand, proposed an optimization model that aimed to minimize delays in running times, energy consumption, and bus purchase costs. In [18], Reuer et al. applied a time-space network-based approach using six different strategies for flow decomposition and developed an algorithm that inserted necessary charging into a given vehicle schedule.

The literature review that has been carried out concerns only specifically selected recent works in the field of public transport planning involving electric vehicle fleets (several hundred works on this subject have appeared between 2010 and 2022). Based on this review, we can state the following:

- Many works are dominated by one or two optimization criteria (usually cost and fleet size minimization).
- In most studies, the research is conducted for relatively small fleets.

Moreover, based on one of the newest and most comprehensive surveys of electric bus planning and scheduling problems and methodologies (see [17]), the following conclusions can be drawn:

- MD VSP and E-VSP – the mixed integer programming (MIP) methods, leading to the strict optimal solutions, along with the large-scale real-world instances of the problem, are rather rare, if they occur at all.
- E-VSP – the combination of such aspects of the problem as multiple depots, multiple vehicle types, recharging duration, driving range of buses and partial charging is not tackled in any of the 23 works published within the last two decades.
- E-VSP – Of the 25 datasets used, one-third concern European countries, whereas almost half concern other continents (the rest are random or not declared).

This study responds to the identified research gap. In Section 3, we formulate a mathematical model that takes into account several optimization criteria (economic, technical, and environmental). This model was solved using the MILP solver for a real public transport system problem of a medium-sized city. In Section 4, preliminary results of computational experiments with about 1200 line trips are presented.

3. MODEL

In this work, we focus on the multiple depot VSP with a heterogeneous fleet of vehicles, emphasizing electric vehicles (MD MVT E-VSP). The objective function presented in Section 3.1 takes into account various aspects of vehicle block scheduling, which are very important from a practical point of view. These aspects include the impact of municipal transport on environmental pollution, with particular emphasis on CO_2 emissions. This is of great importance due to the very high level of air pollution in the largest Polish cities. In addition, the objective function takes into account preferences regarding the possibility of changing the serviced lines by a given vehicle block. This aspect is of particular importance to drivers working for public transport companies. To the best of our knowledge, the proposed model of solving the MD MVT E-VSP is much broader than existing models. We would also like to emphasize that by taking into account all the above-mentioned aspects, the model meets the challenges of the public transport industry to a greater extent than other models found in the literature. The details of the proposed model are presented in the next section.

3.1. Nomenclature

Table 1

The basic notation used throughout this paper

Indices	
i	node representing the service trip or the starting point of a deadhead trip
(i,j)	trip connection (link) connecting two consecutive service trips or service trip and a deadhead trip
k	vehicle block
δ	depot
π	vehicle type
Sets	
V	set of all nodes in the trip graph (e.g., depots, trip endpoints, charging places)
T	set of trip nodes
A	set of all edges of the transport graph; there are four types of edges: (i) A_1 – connecting depots with service trips (pull-out trips), (ii) A_2 – connecting two service trips, (iii) A_3 – connecting service trips with endpoint depots (pull-in trips), (iv) A_4 – connecting service trips with charging points
D	set of depots
P	set of vehicles types
L	set of lines
K	set of vehicles blocks
Parameters	
e_i^{π}	energy used by vehicle of type π during trip i [%]
e_{ij}^{π}	energy used by vehicle of type π during the deadhead trip on link (i,j) [%]
r^{π}	increase in the battery state of charge (SoC) for a vehicle of type π [%/min.]
s_i	time of trip i [min.]
d_{ij}	length of deadhead trip on link (i,j) [km]
t_{ij}	time of deadhead trip on link (i,j) [min.]
z_i	scheduled time of departure for trip i
u^{min}	minimal battery SoC [%]
u^{max}	maximal battery SoC [%]
u^{charge}	maximal battery SoC at which the vehicle is recharged [%]
u_{π}	cost of usage of the vehicle of type π [PLN]
c_{π}	cost of the trip with the vehicle of type π per 1 kilometer [monetary unit/km]
f_{π}	emission (carbon footprint) for the vehicle of type π per 1 kilometer [kg CO_2/km]
c_{π}^{δ}	number of available vehicles of type π in depot δ [-]
$o_{l_1 l_2}$	preference for changing line l_1 to l_2 [-]
h	minimal working time of vehicle block [min.]

M	constant, very large positive number [-]
$g(t)$	average unit charging cost [A] according to the time-of-use tariff t [PLN/kWh]
$l(i)$	line for service trip i [-]
$\Pi(k)$	vehicle type π for vehicle block k [-]
$\Delta(k)$	home depot for vehicle block k [-]
a^{δ}	start node (depot) [-]
b^{δ}	end node (depot) [-]
Variables	
X_{ijk}	binary decision variable; X_{ijk} = 1 if vehicle block k carries out the deadhead trip on link $(i,j) \in A$ or carries out service trip j after trip i [–]
E_{ij}	binary decision variable; E_{ij}= 1 if the vehicle can be charged on link (i,j) [–]
Y_i	auxiliary variable, SoC of the battery before starting trip i [%]
W_{ij}	continuous auxiliary variable, amount of energy to be charged after completing trip i before starting trip j [kWh]
Z_k	auxiliary binary variable; Z_k = 1 if vehicle block k is used in the schedule [–]

Below, we present the mathematical formulation of the mixed integer linear programming model for the MD MVT E-VSP P. The components of the objective function are divided into four groups:

I. Cost components (α)
   1. Costs of trips – the costs of all (service, deadhead, to chargers) trips, assuming that the cost of each trip depends on the type of vehicle used.
   2. Costs of charging electric vehicle batteries.
   3. Costs of using vehicles/vehicle blocks – this covers costs related to the use (departure from the depot on a given day) of a single vehicle/vehicle block. It can be a very important tuning parameter due to the adopted criteria of bus fleet management (i.e., by setting the cost of use to a high value, a clear barrier is set, which will minimize the number of vehicles used). At the same time, by setting the cost of use to a very low (or even negative) value, solutions can be promoted that use the largest possible fleet of vehicles, which may be preferable from a business point of view, according to which it is better to use as many vehicles as possible. The discussed cost of use may also depend on the vehicle type, which allows, for example, the promotion of the use of electric vehicles.
   4. Costs related to the number of vehicles/vehicle blocks used – this limits the number of vehicle blocks and can be used for planning/limiting the fleet of vehicles.

II. Environmental component (β) taking into account CO_2 emissions – this is used to promote vehicles with lower CO_2 emission and can be of particular importance in cities affected by smog, where it is highly desirable to use vehicles with low CO_2 emissions.

III. Technical components (γ)
   1. Total length of deadhead trips – this parameter is used to minimize the length of deadhead trips (including trips to chargers), which prevents long deadhead trips to a greater extent than would result from the transport cost criterion.
   2. Total time of standstills between trips – this is used to minimize the time of standstills; this is equivalent, among other things, to the minimization of the number of vehicle blocks being used both before and after rush hours.

IV. Additional preference component (θ) taking into account preferences regarding the change of the line on the vehicle block.

A mixed integer linear programming model for the proposed MD MVT E-VSP can be formulated as follows:

minimize

$$\alpha_1 \sum_{k\in K}\sum_{(i,j)\in A} d_{ij}\, c_{\Pi(k)} X_{ijk} + \alpha_2 \sum_{(i,j)\in A} g(z_i) W_{ij} + \alpha_3 \sum_{k\in K} u_{\Pi(k)} Z_k + \alpha_4 \sum_{k\in K} Z_k + \beta_1 \sum_{k\in K}\sum_{(i,j)\in A} d_{ij}\, f_{\Pi(k)} X_{ijk} + \gamma_1 \sum_{k\in K}\sum_{(i,j)\in A} d_{ij}\, X_{ijk} + \gamma_2 \sum_{k\in K}\sum_{(i,j)\in A} (z_j - z_i - s_i) X_{ijk} + \theta_1 \sum_{k\in K}\sum_{(i,j)\in A} o_{l(i)l(j)}\, X_{ijk} \quad (1)$$

subject to:

$$\textstyle\sum_{k\in K}\sum_{j:(i,j)\in A} X_{ijk} = 1,\ \forall i \in T \quad (2)$$

$$\textstyle\sum_{j:(i,j)\in A} X_{ijk} - \sum_{j:(j,i)\in A} X_{jik} = 0,\ \forall i \in T, \forall\, k \in K \quad (3)$$

$$\textstyle\sum_{\delta\in D\setminus\{\Delta(k)\}}\sum_{i\in V} X_{a^{\delta}ik} = 0,\ \forall\, k \in K \quad (4)$$

$$\textstyle\sum_{\delta\in D\setminus\{\Delta(k)\}}\sum_{i\in V} X_{ib^{\delta}k} = 0,\ \forall k \in K \quad (5)$$

$$\textstyle MZ_k - \sum_{(i,j)\in A} X_{ijk} \ \geq 0, \forall k \in K \quad (6)$$

$$\textstyle W_{ij} - (z_j - z_i - s_i - t_{ij}) r^{\Pi(k)} \sum_{k\in K} X_{ijk} \leq 0,\ \forall (i,j) \in A \quad (7)$$

$$\textstyle Y_i - \sum_{k\in K} X_{ijk}\left(e_i^{\Pi(k)} + e_{ij}^{\Pi(k)}\right) + W_{ij} + M\left(1 - \sum_{k\in K} X_{ijk}\right) \geq Y_j, \forall (i,j) \in A \quad (8)$$

$$\textstyle E_{ij} - \sum_{k\in K} X_{ijk} \leq 0, \forall (i,j) \in A \quad (9)$$

$$\textstyle Y_i - \sum_{k\in K} X_{ijk} e_i^{\Pi(k)} + W_{ij} - u^{max} \leq 0, \forall (i,j) \in A \quad (10)$$

$$Y_i \geq u_i^{min},\ \forall i \in V \quad (11)$$

$$\textstyle Y_i - \sum_{k\in K} X_{ijk} e_i^{\Pi(k)} - u^{\mathrm{charge}} - M(1 - E_{ij}) \leq 0, \forall (i,j) \in A \quad (12)$$

$$\textstyle\sum_{j:(i,j)\in A, i\in T} s_i X_{ijk} + \sum_{(i,j)\in A} X_{ijk}\, t_{ij} \geq h, \quad \forall k \in K \quad (13)$$

$$\textstyle\sum_{k\in K, \Delta(k)=\delta, \Pi(k)=\pi} \quad Z_k \leq c_\pi^\delta, \quad \forall \delta \in D, \forall \pi \in P \quad (14)$$

$$W_{ij} \geq 0, \quad \forall (i,j) \in A \quad (15)$$

$$X_{ijk} \in \{0,1\}, \quad \forall (i,j) \in A, k \in K \quad (16)$$

$$E_{ij} \in \{0,1\}, \quad \forall (i,j) \in A \quad (17)$$

$$Z_k \in \{0,1\}, \quad \forall k \in K \quad (18)$$

The objective function (1) consists of eight weighted components: $\alpha_1, \ldots, \alpha_4$ (economic criteria), β_1 (environmental criterion), γ_1, γ_2 (technical criteria), and θ_1 (criterion incorporating preferences), which were described above in detail.

As for the constraints, (2) ensures that each trip is operated by only one vehicle. Formula (3) indicates the flow conservation constraint. Constraint (4) states that each vehicle starts at a designated depot, while (5) means that each vehicle ends at a designated depot. Constraint (6) is a condition that specifies the use of the block/vehicle. Next, constraint (7) states that charging on link (i,j) is performed only if it is selected and time is available. Constraint (8) ensures the consistency of vehicle battery energy for two consecutive trips. Constraint (9) checks whether a battery is to be charged on link (i, j). Constraint (10) ensures that the upper limit u^{max} of battery SoC will not be exceeded after recharging. Constraint (11) keeps the vehicle battery SoC above the u^{min} parameter value. Constraint (12) ensures that the vehicle does not start charging if, at the end of the trip i, the battery charge SoC is above u^{charge}. Constraint (13) defines the minimum block duration, whereas constraint (14) is the limit on the number of vehicles type π available at the depot δ. Constraints (15)–(18) define the domains of the variables.

4. COMPUTATIONAL EXPERIMENTS

4.1. Data preparation – standard benchmark set

Initial experiments were performed on one of the most popular sets of benchmark data for the MD-VSP. Their main goal was to determine the tradeoff between the cost of deadheads, the number of blocks used, and the bus fleet types (traditional and electric). The standard harvest has been slightly modified to simulate the actual cost of deadhead trips. The costs of using vehicles (pull-in and pull-out) have been reduced; in the original data sets, the benefits are, on average, 5,000 higher than other deadhead trips.

Additionally, the energy consumption of the vehicles was taken into account, assuming that electric buses account for 50% of the bus fleet. Three standard set sizes were considered as reflecting the typical number of daily trips ranging from 500 (small city or weekend schedules) to 1000 (medium city) and 1500 (big city). In this phase of the experiments, four and eight depots were taken into account.

However, to take into account the electric fleet, it was assumed that half of the depots have electric buses, while the other half have traditional vehicles with diesel engines. The purpose of the experiments in this phase was primarily to test how the MILP solver deals with different sizes of problems and where the limit is when its use can be problematic in practice.

Five objective functions were considered:

- The objective function F1 minimizes only the costs.
- The function F2 minimizes only the number of blocks.
- The function F3 minimizes only the energy consumption (which can be converted into CO_2 emissions into the environment; the exact emissions from charging an electrical vehicle depend on the electricity sources used to charge it – MPG-CO_2e is a standard way to understand and compare emissions from electric vehicles (a higher MPG-CO_2e indicates less CO_2 emission).
- The functions F4 and F5 are computed as combined-weighted objective functions. For function F4, weights of 1 were used; for function F5, the weights were 1, 100, and 10 for deadhead cost, blocks, and energy consumption, respectively.

4.2. Experimental results for the standard benchmark set

The results for the smallest problem instance are presented in Table 2.

The solver returns the results very quickly – most often in 0.5 minutes or less, except for the second objective function (number of blocks), for which it needed slightly longer than a minute.

Depending on the objective function used, the number of blocks ranged from 121 to 147, costs from 53 to 149 thousand EUR, and energy consumption from 1 to 11 thousand kWh. The most compressive feature of the F5 keeps the number of blocks at a minimum, increasing deadhead costs by 30% and energy consumption by 10%.

For the problem with 1000 daily trips (Table 3), the computation time increased significantly, ranging from less than seven minutes to 30 minutes (the upper time limit set for the computation). In addition, it is also shown how long it took the solver to achieve a result that reflected less than a 1% gap from the optimal solution (see the last row in Table 3). As can be seen for the combined objective functions (F4 and F5), this result was achieved after about 40 seconds, which should be considered very satisfactory.

There are slightly smaller fluctuations in the number of blocks (10% above the minimum) and energy consumption (412%, compared to over 1000% for the 500 trips problem) and slightly larger fluctuations in deadhead costs (204%).

Table 2

Results for four depots and 500 service trips

	F1		F2		F3		F4		F5	
Deadhead costs [EUR]	53,746	0%	149,175	178%	140,331	161%	56,083	4%	69,522	29%
No. of blocks	147	21%	121	0%	121	0%	143	18%	121	0%
Energy usage [kWh]	11,436	889%	13,168	1039%	1156	0%	6117	429%	1272	10%
Time [s]	33		71		27		10		19	

Table 3

Results for four depots and 1000 service trips

	F1		F2		F3		F4		F5	
Deadhead costs [EUR]	10,1903	0%	309,964	204%	286,985	182%	105,036	3%	148,834	46%
No. of blocks	264	10%	240	0%	240	0%	265	10%	240	0%
Energy usage [kWh]	27,415	439%	26,070	412%	5088	0%	20,833	309%	6050	19%
Time [s]	1800		897		202		532		210	
Time to 1% [s]	1325		897		202		41		39	

For the problem with 1500 service trips (Table 4), the computation execution time limit was set to 30 minutes. In such a given time, the solver was able to find solutions to all objective functions except for F2. Furthermore, it was not always able to descend to the optimal solution during this time, but the maximum difference from the optimal one was only 1.39% (for F1) – the distances from the lower bound (LB) are shown in the last row of Table 4.

Interestingly, in this case, we observed even smaller luctuations in the values of individual components of the objective function, but this may be due to the fact that it was not possible to determine the minimum value of the number of blocks (F2 did not return the result). The lowest value obtained by other functions (F3 and F5) was adopted as the minimum number of blocks of vehicles required.

In subsequent experiments, the number of depots was increased to eight. This means that we had four depots with electric buses and four with traditional buses (which can also be interpreted as four depots and two types of buses).

For the problem with 500 trips (Table 5), the solver obtained satisfactory solutions, returning the result in a time that is admittedly much longer than the probation with 500 trips and four depots but shorter than for 1000 trips. Additionally, for two objective functions (F1 and F5), the solver was able to obtain a solution reflecting less than a 1% gap from the LB in approximately 40 seconds.

The fluctuations in the value of deadhead costs, block counts, and energy consumption are similar to those observed for the problem with four depots. The compromise function of the target (F5) resulted in 35% higher costs and 7% higher energy consumption but maintained the minimum number of blocks, which can be considered acceptable.

The last experiment in this phase was run for eight depots and 1000 service trips. The results of it are presented in Table 6.

In this case, the designated 30-minute execution time limit was too demanding, and the solver either did not return the result (F2) or returned a result very distant from the LB (73–91%), meaning the solutions can be treated as unreliable. Only for the last objective function (F5) did the solver return a very good solution, decreasing with the value of the objective function to below 1% of the LB in just 20 seconds.

Table 4

Results for four depots and 1500 service trips

	F1		F2		F3		F4		F5	
Deadhead costs [EUR]	148,455	0%	no sol.	–	424,073	186%	153,849	4%	196,272	32%
No. of blocks	398	9%	no sol.	–	365	0%	384	5%	365	0%
Energy usage [kWh]	40,147	442%	no sol.	–	7411	0%	27,768	275%	7873.5	6%
Time [s]	1800	–	1800	–	1062	–	1800	–	905	–
Time to 1% [s]	–	–	–	–	1062	–	168	–	20	–
Gap to LB	1.39%	–	–	–	0	–	0.72%	–	0.01%	–

Table 5

Results for 8 depots and 500 service trips

	F1		F2		F3		F4		F5	
Deadhead costs [EUR]	46,182	0%	158,677	244%	154,194	234%	49,295	7%	62,530	35%
No. of blocks	152	23%	124	0%	124	0%	142	15%	124	0%
Energy usage [kWh]	13,286	1050%	14,822	1183%	1155	0%	6803	489%	1232	7%
Time [s]	400		370		108		681		124	
Time to 1% [s]	41		370		108		418		39	
Gap to LB	0.07%		0.00%		0.00%		0.09%		0.02%	

This is because the CPLEX solver, apart from the branch and cut technique, uses various techniques to speed up the calculations, including heuristics. In this case, the heuristics at the beginning of the computation time dropped from 73% to just 0.36%. This does not mean that it will always be the case, but one can look for different weighting factors and recalculate or extend the computation time.

Table 6

Results for eight depots and 1000 service trips

	F1		F2		F3		F4		F5	
Deadhead costs [EUR]	37,3694	201%	no sol.	–	445,489	259%	382,634	208%	124,044	0%
No. of blocks	370	62%	no sol.	–	421	85%	379	66%	228	0%
Energy usage [kWh]	24,873	767%	no sol.	–	29,762	937%	23,732	727%	2869	0%
Time [s]	1800		1800		1800		1800		1800	
Time to 1% [s]	–		–		–		–		22	
Gap to LB	75.30%				91.26%		73.07%		0.59%	

Nevertheless, it seems that without additional problem relaxation, the instance of the problem with eight depots and 1000 trips represents the limit for the use of the solver as a ready-made tool.

4.3. Data preparation – real-world cases

For the next phase of computational experiments, data were obtained from a real public transport company operating in a medium-sized city located in the western part of Poland. It has two depots, each with a mixed fleet of diesel and electric buses. Generally, buses are divided into two types: articulated (18 m) and standard (9–12 m), and each kind can be diesel or electric. This equates to eight virtual depots (2 bus types x 2 power types x 2 depots) and, therefore, corresponds to the largest size of the problems optimized in phase one.

The computational experiments included the rates from the sample week: 1257 line (service) trips for the working day, 803 for the Saturday timetable, and 635 for the Sunday timetable.

Table 7 shows the results for the smallest problem with 635 line trips. The algorithm returned a solution in a minute (for F2) or about 20 seconds (for other objective functions). The situation is analogous to the problem with 500 trips and four depots analyzed earlier, and it confirms that the model formulated in this way copes for the longest duration by minimizing the number of blocks.

The number of blocks ranges from 30–39, and the greatest deviations are related to energy consumption (the highest consumption for minimizing costs increases consumption by almost 780%). This is because there is a large redundant electric fleet that can be additionally used if the need arises to optimize energy consumption. In fact, there are additional constraints limiting the consumption of this fleet, which will be the subject of further development of the model once the relevant data are available.

Compared to the previously analyzed artificial problems, finding a compromise solution is not easy. For F5, the number of blocks increases by 31% and deadhead costs increase by as much as 227%, which may not always be acceptable to planners.

Table 7

Results for two depots and 635 real service trips (Sunday timetable)

	F1		**F2**		**F3**		**F4**		**F5**	
Deadhead costs [PLN]	1464	0%	11,047	655%	14,398	883%	4698	221%	4790	227%
No. of blocks	30	3%	29	0%	38	31%	39	34%	38	31%
Energy usage [kWh]	25,354	779%	22,873	693%	2886	0%	3288	14%	3206	11%
Time [s]	20		63		21		18		22	

Similar results were obtained for the problem with 803 trips (Table 8). The waiting time for the solution almost doubled, with the exception of the solution for the F3 function.

The obtained solution for F5 is also difficult to consider as a compromise since the number of blocks increased by as much as 46% compared to the minimum possible value.

Table 8

Results for two depots and 803 real service trips (Saturday timetable)

	F1		**F2**		**F3**		**F4**		**F5**	
Deadhead costs [PLN]	2092	0%	15,195	626%	16,796	703%	6399	206%	7335	251%
No. of blocks	42	8%	39	0%	57	46%	56	44%	57	46%
Energy usage [kWh]	31,984	740%	30,540	702%	3807	0%	4285	13%	3807	0%
Time [s]	45		105		23		32		37	

For a working day with 1257 trips (Table 9), the calculation time increased to a few minutes (to a maximum of almost eight), which can be considered an acceptable result in practice.

As there are many more service trips with the same fleet, there are no longer such large fluctuations in energy consumption (most buses, both electric and diesel, have to be used). However, the complete minimization of energy consumption requires the involvement of as many as 19 additional blocks.

Owing to the greater utilization of the available bus fleet, it is also slightly easier to obtain a compromise solution (deadhead costs no longer exceed 200% of the minimum costs).

Table 9

Results for two depots and 1257 real service trips (working day timetable)

	F1		**F2**		**F3**		**F4**		**F5**	
Deadhead costs [PLN]	4241	0%	24,293	473%	27,403	546%	9115	115%	11,156	163%
No. of blocks	68	3%	66	0%	87	32%	87	32%	87	32%
Energy usage [kWh]	40,504	266%	48,261	337%	11,053	0%	12,253	11%	11,053	0%
Time [s]	214		463		324		231		201	

5. CONCLUSIONS

This article presents a practical model of the multi-criteria optimization of work planning for a heterogeneous fleet of buses. The objective functions cover economic, technical, and environmental criteria. The model was based on the classic MD-VSP model but was extended to include different vehicle types and additional restrictions resulting from the use of a hybrid fleet comprising both electric and diesel vehicles. The results obtained from the MILP solver show that the proposed model enables control over the tradeoff between various criteria, depending on business goals or immediate needs. They also show that the solver can handle problems containing up to 1500 trips and four depots. In practice, this may not be enough because, when taking into account the types of vehicles, this means a limit of two depots and two types of buses.

Therefore, further possibilities for reducing the complexity of the model should be sought (e.g., by applying relaxation, introducing additional cuts, or simplifying the model. This will be the subject of the authors' upcoming research in parallel with the search for alternatives in the form of heuristics applied.

A separate goal is to obtain more real-world data from various public transport companies, which will allow for better model validation and a more precise assessment of the possibility of using MILP solvers in public transport planning.

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