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Analysis of selected dynamic properties of quasi-fractional-order measuring transducer used in transportation facilities

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ABSTRACT
The paper presents possibilities of using fractional calculus in dynamic measurements used in telematic equipment in cars and railway vehicles diagnostic systems. It describes a laboratory measurement system for investigating dynamic properties of accelerometers. Tests are executed in the MATLAB&Simulink programme. Properties of the examined transducers of integral and quasi-fractional-orders are compared. The authors indicate the fractional calculus advantages from the point of view of their dynamics description.

KEYWORDS: measuring transducer, accelerometer, fractional calculus, MATLAB&Simulink

1. Introduction

Dynamic development of recent research into the use of fractional calculus for the dynamic system analysis encouraged authors of this paper to attempt the use of it for the analysis and modelling of measuring transducers and measurement systems used in telematic equipment [2], [3], [9] and [12].

The work's main objective is an implementation of a fractional calculus-based method allows for a description of dynamic properties of signal processing of measuring transducers with integer-order and quasi-fractional-order [1], [4], [5], [6], [7], [8], [10] and [11].

Measuring transducers used in transportation facilities, especially telematic equipment in cars and railway vehicles diagnostic systems – accelerometers [10], [11] are tested, treated as a representative group of measuring transducers. In the classic notation, accelerometers are described with second-order differential equations, like many other groups of measuring transducers, such as: RLC circuits, mechanical vibrating systems, displacement measurement sensors, systems including tensometric and piezoelectric transducers. In addition, linear transducers of higher than second orders, when in transitional states, behave in ways similar to second-order linear transducers.

The tested 2nd-order accelerometers were a point of reference for the modelled transducers of quasi-fractional-orders.

2. Model of the 2nd order measuring transducer

A measuring transducer comprising three types of elements characteristic for linear systems, i.e.: elements storing kinetic energy, elements storing potential energy and elements causing energy losses, are referred to as second-order measuring transducers [4], [7], [8]. Simulation and laboratory testing of a second-order measuring transducer - accelerometer has been tested in this paper, treated as a representative group of measuring transducers (Fig. 1.).

A differential equation describing the absolute motion of a second-order measuring transducer's seismic mass can be expressed as:

$$\frac{d^2}{dt^2}y(t)+2\zeta\omega_0\frac{d}{dt}y(t)+\omega_0^2 y(t)= \omega_0^2 x(t)+2\zeta\omega_0\frac{d}{dt}x(t) \quad \textbf{(1)}$$

where: $\omega_0=\sqrt{\frac{k_s}{m}}$, $\zeta=\frac{B_t}{2\sqrt{k_s m}}$ and $k=\frac{1}{k_s}$.

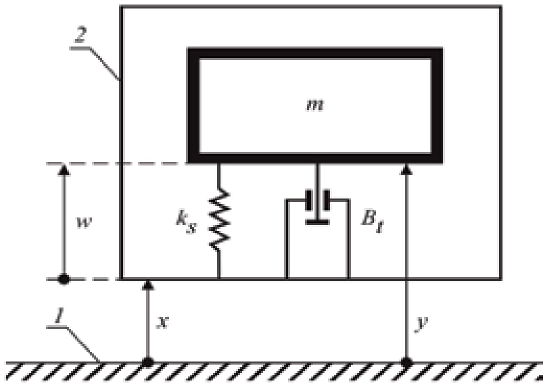

Fig. 1. Kinetic diagram of an accelerometer [4], [6], [11]

Where: m – seismic mass, k_s – spring constant, B_t – damping coefficient, x – object motion relative to a fixed system of coordinates, y – motion of a vibrating mass relative to a fixed system of coordinates, w – motion of a vibrating mass relative to a vibrating object.

Considering the motion of the vibrating mass relative to the vibrating object (Fig. 1):

$$\frac{d^2}{dt^2}w(t)+2\zeta\omega_0\frac{d}{dt}w(t)+\omega_0^2 w(t)= -\frac{d^2}{dt^2}x(t) \quad \textbf{(2)}$$

Depending on selection of k_s, m and B_t, a transducer can serve to measure displacements as a vibrometer assuming low k_s and a B_t, and high m (Fig. 2b), or acceleration as an accelerometer assuming a high k_s, low m and B_t (Fig. 2a).

In practical vibration measurements used in cars and railway vehicles diagnostic systems, acceleration-measuring transducers, the so-called accelerometers, are employed. For purposes of simulation testing, a measuring transducer was assumed of a frequency f=350Hz, that is, circular frequency of free vibrations $\omega_0=2200\frac{rad}{s}$ and degree of damping ζ=0.2. Dynamics of such a transducer, characterised by means of a 2nd-order differential equation (2), are described by operator transmittance:

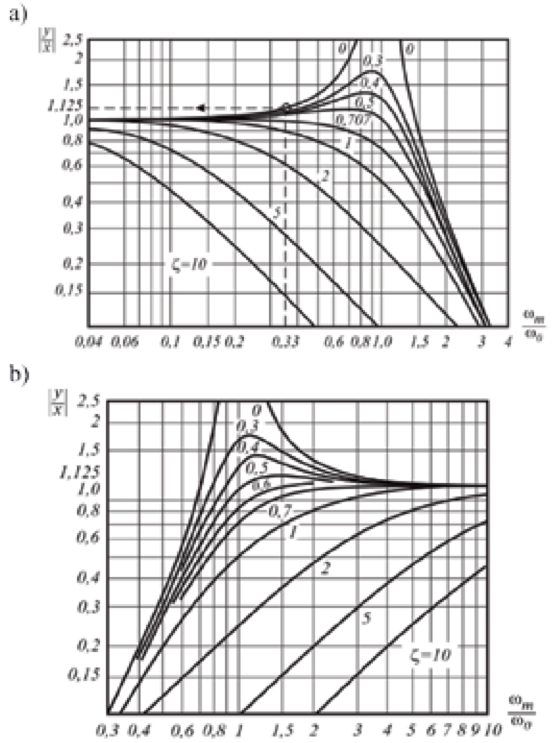

Fig. 2. Graphic solution to the equation of measuring transducer's seismic mass motion [10]: a) for $\omega_m < \omega_0$: ω_m – measured frequencies, ω_0 – circular frequency of free vibrations, y – real mass displacements, $\ddot{x}$ – displacement of the housing; b) for: $\omega_m > \omega_0$: ω_m – measured frequencies, ω_0 – circular frequency of free vibrations, y – real mass displacements, x – displacement of the housing

$$G(s)=\frac{W(s)}{\ddot{X}(s)}=\frac{1}{s^2+880s+4.84\cdot10^6} \quad \textbf{(3)}$$

Fig. 5 illustrates amplitude and phase frequency characteristics of a measuring transducer with operator transmittance (3).

3. Quasi-fractional model of the measuring transducer

Mathematical models describing dynamic performance of telematic devices (measuring devices, automation objects, sensors, etc.) are widely used like in different disciplines of science. Their task is to reproduce the real behaviour of the examined device in the simulation environment. They are most frequently used at early stages of research, prior to the real examination of the problem, or construction of a device as a quick tool of fast prototyping. They allow for simulation testing of an object's behaviour and testing it under normal and extreme working conditions. In this way onerous and costly preliminary investigations of real objects that aim at early assessment of their usefulness (the method investigated or the object) for concrete applications are ignored. What is more models are also the basic tool allowing us to get acquainted with mathematical or physical foundations of a given object or phenomenon's performance.

Dynamic behaviour of accelerometers used in telematic equipment (in general – objects, sensors and measuring devices for different applications) is written down in a form of differential equations or operator transmittances. In the process of determining a model of an accelerometer's (object's) dynamic behaviour, physical phenomena are taken into account which result from external influences and specific properties of an accelerometer being an effect of their design. Thus, accuracy of reproducing its real dynamic behaviour is first of all connected with this phenomenon.

Equation (1), describing the measuring transducer, can be expressed as a difference equation [11]:

$$a_2 w_k + a_1 w_{k-1} + a_0 w_{k-2} = b_2 x_k + b_1 x_{k-1} + a_0 x_{k-2} \tag{4}$$

or a matrix equation:

$$\begin{bmatrix} a_2 & a_1 & a_0 \end{bmatrix} \begin{bmatrix} w_k \\ w_{k-1} \\ w_{k-2} \end{bmatrix} = \begin{bmatrix} b_2 & b_1 & b_0 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-1} \\ x_{k-2} \end{bmatrix} \tag{5}$$

Equation (5) can have the following derivative-integral expression:

$$\begin{aligned} & A_2 \Delta_k^{(2)} w_k + A_1 \Delta_{k-1}^{(1)} + A_0 w_{k-2} = \\ & = B_2 \Delta_k^{(2)} w_k + B_1 \Delta_k^{(1)} x_{k-1} + B_0 w_{k-2} \end{aligned} \tag{6}$$

where $\Delta_k^{(n)}$ is the discrete function's reverse difference [9], defined as:

$$\Delta_k^{(n)} f(k) = \sum_{j=0}^{k} a_j^{(n)} f(k-j) \tag{7}$$

When (7) is taken into account, (6) has the matrix expression below:

$$\begin{aligned} & \begin{bmatrix} a_2 & -a_1 - 2a_0 & a_2 + a_1 + a_0 \end{bmatrix} \begin{bmatrix} \Delta_k^{(2)} w_k \\ \Delta_k^{(1)} w_k \\ \Delta_k^{(0)} w_k \end{bmatrix} = \\ & = \begin{bmatrix} b_0 & -b_1 - 2b_0 & b_2 + b_1 + b_0 \end{bmatrix} \begin{bmatrix} \Delta_k^{(2)} x_k \\ \Delta_k^{(1)} x_k \\ \Delta_k^{(0)} x_k \end{bmatrix} \end{aligned} \tag{8}$$

On comparing responses of the measuring transducer to the input sinusoid signal, it was described by means of three models using the above method:

1. Classic model (Fig. 3) (transfer function of measuring transducer model) described with operator transmittance (9):

$$G(s) = \frac{1}{s^2 + 880s + 4.85 \cdot 10^6} \tag{9}$$

2. Classic discrete model (discrete transfer function of measuring transducer model), derived from the operator transmittance model (9), described by means of discrete transmittance (10) (Fig. 3):

$$G(z) = \frac{1.67 \cdot 10^{-15} z^2 + 6.67 \cdot 10^{-15} z + 1.67 \cdot 10^{-15}}{z^2 - 2z + 0.99} \tag{10}$$

Response of a continuous object to a discrete input depends not only on values of this signal at a discrete moments of time but also on sampling time and the extrapolator used.

3. The quasi-fractional discrete model (discrete transfer function of fractional transducer model) is expressed with derivative integrals and described by discrete transmittance (11) (Fig. 3):

$$G(z) = \frac{z^2}{1 \cdot 10^{14} z^2 - 2 \cdot 10^{14} z + 1 \cdot 10^{14}} \tag{11}$$

Models' responses were tested in the programming environment MATLAB&Simulink. It can be noted that the model described by means of the discrete transmittance (11) correctly reproduces values of the input signal amplitude, like the model of transmittance (10). It can be noted in Bode frequency diagrams (Fig. 5.) that the measuring transducer model determined by the derivative-integral method presents the dynamics of the classically determined model (the diagrams of the models coincide). This confirms that integral-order differential-integral calculus is a special case of differential-integral calculus of non-integral orders.

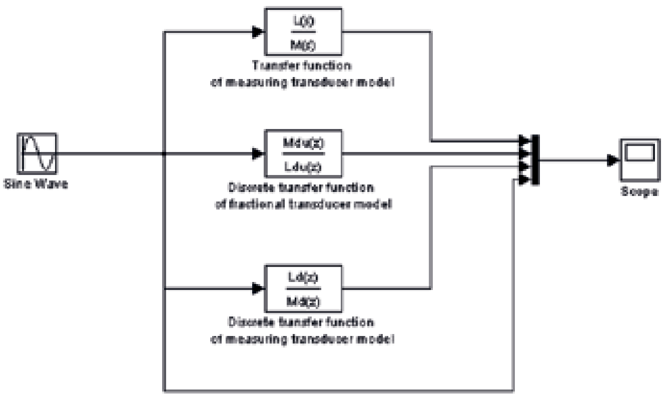

Fig. 3. Simulation diagram of the system comparing measuring transducer models [10]: Transfer function of measuring transducer model – transducer model of operator transmittance (9), Discrete transfer function of measuring transducer model – discrete transducer model of transmittance (10), Discrete transfer function of fractional transducer model – quasi-fractional discrete model (11)

Measuring transducer models (10) and (11) have only been subject to simulation testing and do not fully represent real models but the simulations indicate that the quasi-fractional model (12) exhibits the same dynamics as the classic model (Fig. 4).

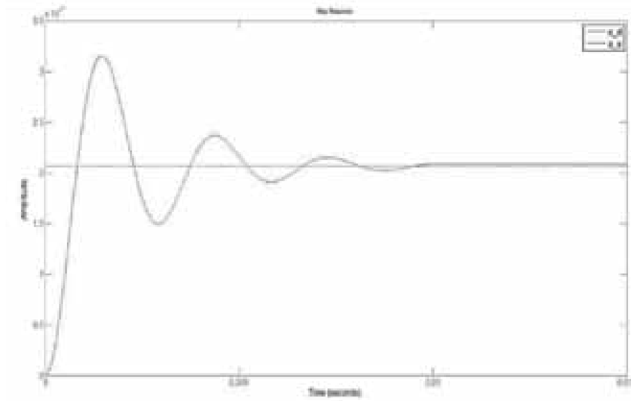

Fig. 4. Comparison of responses by measuring transducer models (10) and (11) to step functions (diagrams of the models overlap) [5]

The 'apparent' time of stabilization of time diagrams – the time after which a model's description is independent from time – for the quasi-fractional model is the same as for the classic model.

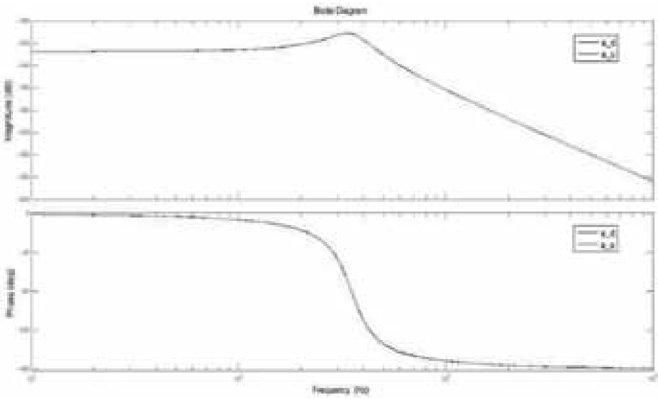

Fig. 5. Comparison of Bode diagrams of measuring transducer models (10) and (11) (diagrams of the models overlap) [5]

4. Model of a laboratory system of acceleration measuring transducer

an overview of the measurement system is shown in Fig.6.:

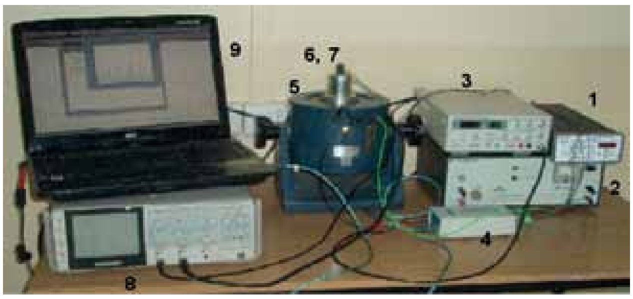

Fig. 6. Laboratory measurement system for testing of mechanical vibration transducers [10]: 1 – conditioner, 2 – generator, 3 – amplifier, 4 – measurement card µDAQ USB-26A16, 5 – inductor, 6, 7 – model and tested measuring transducers, 8 – oscilloscope, 9 computer/MATLAB&Simulink

In order to determine measuring transducer's operator transmittance, a system comprising two accelerometers (6), (7), conditioner (1) and µDAQ USB-26A16 measurement card (3) was modelled. Accelerometer (7) DeltaTron by Bruel&Kjaer type 4507, sensitivity 10.18mV/ms^{-2} and the range of frequency measurements from 0.4Hz to 6kHz was tested. The conditioner's operating range was between 1Hz and 20kHz. The transducer was mounted on an electrodynamic inductor (5). A model accelerometer (6) by VEB Metra, type KB12, sensitivity 317mV/ms^{-2} was aligned with the tested transducer.

The operator transmittance (12) describing dynamics of the measurement system was determined by identification with an external ARX (AutoRegressive with EXternal input) [4], [7] and [10]. The voltage signal from the end of the tested measurement track is the identified signal, signal from the model accelerometer in response to the generator's sinusoid function (2) of 100Hz is the comparative signal.

The ARX identification method [6], [7] produced the operator transmittance ***G(s)*** describing the system's dynamics:

1. Classic model:

$$G(s) = \frac{0.03215s^2 + 1319.6s + 1.338 \cdot 10^6}{s^2 + 4.678 \cdot 10^4 s + 2.309 \cdot 10^7} \tag{12}$$

2. Discrete transfer function of the model was determined on the basis of the operator transmittance (12):

$$G(z) = \frac{0.03215z^2 - 0.05368z + 0,02163}{z^2 + 1.625z + 0.6264} \tag{13}$$

The classic discrete model (13) was produced by discretising the classic model (12) by means of the 'Zero-Order-Hold' method with the sampling time T_p=10^{-4}s, for which Nyquist theorem of sampling frequency selection obtains.

3. Discrete transfer function of fractional models was determined with a method implemented in MATLAB&Simulink. For varying increment of *h*, quasi-fractional transducer models become discrete transmittances which is presenting in Table 1.:

Table 1. Discrete transmittances of measuring transducer models for varying increment *h* [10]

Increment variation h	Discrete transmittance $G(z)$
10^{-7}	$G_{f_1}(z) = \frac{3.228z^2 - 6.443z + 3.215}{100.5z^2 - 200.5z + 100}$
10^{-6}	$G_{f_2}(z) = \frac{3.347z^2 - 6.562z + 3.215}{104.7z^2 - 204.7z + 100}$
10^{-5}	$G_{f_3}(z) = \frac{4.548z^2 - 7.75z + 3.215}{104.7z^2 - 246.8z + 100}$
10^{-4}	$G_{f_4}(z) = \frac{1.775z^2 - 1.963z + 0.322}{59.09z^2 - 66.78z + 10}$
10^{-3}	$G_{f_5}(z) = \frac{2.69z^2 - 1.384z + 0.032}{70.87z^2 - 48.78z + 1}$

The model of the real measurement system in the form of discrete transmittance and models expressed by means of a differential-integral equation were then compared. Both types of the models were based on the classic model derived by ARX identification method (Fig. 7). The simulations were carried out by ode3 integration method for a 100Hz sinusoid input signal.

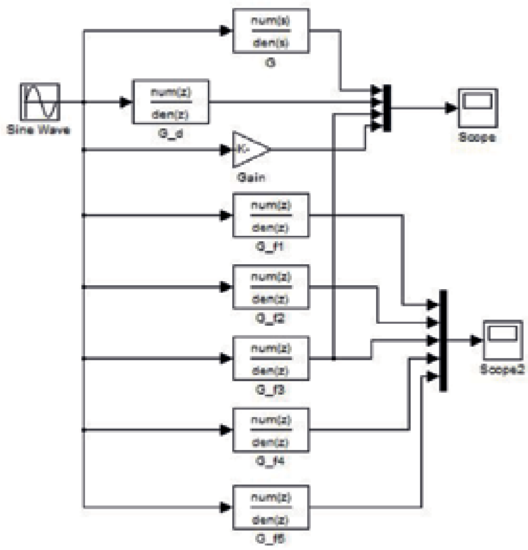

Fig. 7. Simulation diagram of the system comparing measuring transducer models (models from Table 1) [10]

Fig. 8 shows logarithm frequency amplitude and phase diagrams of the measurement system models. It can be observed that, for the adopted increment of *h*, measure of differentiation accuracy, the diagrams clearly diverge. This means that other *h* increments, far lower than the sampling frequency, must be adopted.

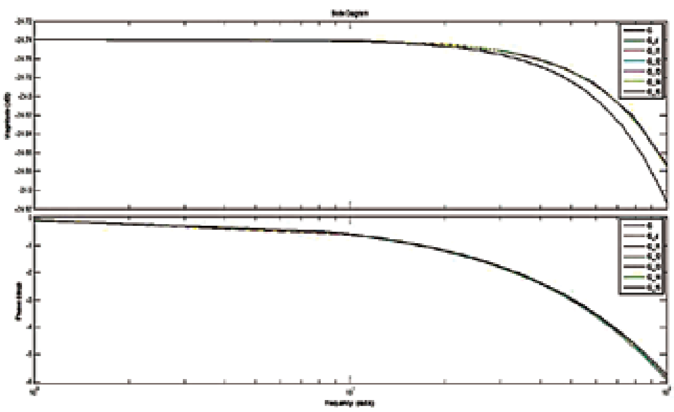

Fig. 8. Bode frequency diagrams of measuring transducer models of transmittance (models from Table 1) [10], [11]

5. Conclusion

The comparison of classic and quasi-fractional models' responses to the step function implies the time after which diagrams stabilise for quasi-fractional models is the same as for the classic model. The same applies to frequency diagrams for quasi-fractional discrete models, which have the same course in the tested frequency ranges as the classic models (Fig. 4 and Fig. 5). This means that non-integral order differential-integral calculus is a generalisation of integral-order differential calculus – this is confirmed by laboratory testing of dynamic systems.

Application of the quasi-fractional method of describing dynamic properties of measuring transducers discussed in this paper, based on non-integral-order differential calculus, will help to undertake analyses of simulated dynamics of various objects and processes in telematic problems which, due to their complexity, must be described by means of differential equations of any orders.

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