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TRANSFORMATIONS BETWEEN THE HEIGHT REFERENCE FRAMES: Kronsztadt'60, PL-KRON86-NH, PL-EVRF2007-NH

The State Spatial Reference System in Poland currently includes two height reference frames: the first, PL-KRON86-NH, with the old name Kronsztadt'86, and the second, called PL-EVRF2007-NH, as a Polish implementation of the European Vertical Reference Frame (EVRF), named also NAP (Normal Amsterdams Peil). Kronsztadt'86 was supposed to replace the earlier reference system called Kronsztadt'60, but the intentions were not fully in line with reality. Kronsztadt'60 has been implemented in all geodetic and cartographic elaborations even before the computer era and will probably exist until the use of analogue maps or their duplicates in the form of raster maps. For practical purposes, transformation formulas have been developed between all three reference frames mentioned in the title of work. For this purpose, about 16,000 points of the base height network in the PL-EVRF2007-NH and PL-KRON86-NH were used and more than 7,000 points in the Kronsztadt'60. Transformational formulas were developed in two variants: in the form of polynomials approximated by the least squares method and in the form of an interpolative grid. Basic empirical relationships were implemented among others in the program TRANSPOL v. 2.06 [8], elaborated according to the assumptions of the Head Office of Geodesy and Cartography.

Keywords: height transformations, reference frames, Kronsztadt'60, Kronsztadt'86, Kronsztadt'2006, PL-KRON86-NH, PL-EVRF2007-NH

1. Historical review of Polish height reference frames and general relations between them

In the time of the Second Polish Republic, a uniform height reference frame was created, using the heights data inherited from the partitioning countries. These data referred to the average levels of the three seas: the North Sea with the mareograph in Amsterdam (the Netherlands), the Baltic Sea with the mareograph in Kronstadt (Russia) (see eg. [12]), and the Adriatic Sea with the mareograph in Trieste. Finally, after the creation of a uniform leveling network in the 1930s (Fig.1), the "Amsterdam" reference system was adopted, also known as the NAP (Normal Amsterdams Peil). A fixed point in the network was a benchmark at the town hall in Toruń, which was previously associated with the NAP system.

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Fig. 1. Sketch of the basic height network during the times of the Second Polish Republic. According to the then naming, the network consisted of lines I and II order. Traverses closed numbered with Roman numerals. The age points (in the legend named as "basic") and a mareograph in Gdynia are marked by big circles. Source: Archive WIG [25]

After the World War II, as part of the post-war measurement campaign of 1946–1955, the first class height networks were established in the areas recovered. The new areas were the Western Lands and the Northern Lands, which were part of the area of former East Prussia. In 1951 these networks were calculated in reference to the existing height network in the area of the Second Polish Republic, that is in the "Amsterdam" (NAP) system. After only two years (1953–1955) was designed and a uniform height network was established, linking with the basic heights network of neighboring countries. The average level of the Baltic Sea, measured by a mareograph in the town of Kronstadt in the Gulf of Finland (now the Russian Federation) was used as the reference level. The new heights reference frame in Poland was called the "Kronsztadt" (as the Polonized name of the town). The whole measurement-computation operation in the years (1946–1955) is called the second measurement campaign of the national height network. In the new system, the height ordinate of the benchmark on the town hall in Toruń was also designated. The difference in the heights between the "Amsterdam" and "Kronsztadt" reference frames for the benchmark in Toruń was 84 mm.

The "Kronsztadt" reference frame was introduced throughout the entire territory of Poland as the obligatory height system. The heights in this reference frame appeared in all classic (paper) cartographic elaborations, including the engineering maps and in the information bases of geodetic points. In later years, to distinguish it from other definitions, this reference frame adopted the name "Kronsztadt'60" (probably marking the epoch of its implementation).

In the years (1974–1982), another third measurement campaign was designed and implemented. After about 20 years, it was found that a significant part of the benchmarks was destroyed or not found, so the necessary supplements and modernization of its structure were carried out in the network. Connections to neighboring countries' networks have been taken into account again. The adjustment and computing of the I class network was done using the NOVA 840 digital machine. As a result, the new heights of all benchmark were obtained. This was the basis for establishing a new height system in Poland, which was called "Kronsztadt'86" (see eg: [1], [16], [17], [18]). Comparing the heights of 7679 benchmarks (surviving common points in the networks of the second and third measurement campaigns), the average difference was 48 mm ($H_{\text{Kronsztadt'60}}$ - $H_{\text{Kronsztadt'86}}$) with a spread in the range from -10 mm to +139 mm. Wyrzykowski in [17] interprets this effect as a result of vertical movements of the Earth's crust at 1.8 mm / year. Such a conclusion would be justified, provided that the same reference level adopted in both campaigns was independent of the vertical movements of the earth's crust in the studied area of Poland (analogously to the local tasks of displacement analysis, where a stable reference system is required). It is difficult to say how the above condition was realized, for example by referring to the average level of the Baltic Sea indicated by mareograph, or by uniform alignment of the main polygons of the former Eastern bloc countries network, in reference to the mereograph in Kronsztadt. The effect analyzed by Wyrzykowski may also have been partly a result of erroneous measurement and calculation processes, including reduction of observations, and changes in the geometric structure of the network. It seems, however, that inference about the vertical movements of the Earth's crust on the territory of Poland, based on the results of all measurement campaigns, despite many efforts to obtain archival materials (see: [11]), will not be fully possible due to failure to comply particular condition, maintaining a common and stable frame of reference. If it were only for local displacements, for example in the area of the Upper Silesian Industrial District, a sufficiently stable frame of reference can create external points of this area, especially points with observation of mutual constancy confirmed.

The new height reference frame (Kronsztadt'86) was used only in geodetic elaborations of the post-analogue (computer) era, while height data related to classic (paper) geodetic elaborations remained unchanged in the Kronsztadt'60. Currently we have already the basic economic maps made as vector maps in all major cities and industrial areas. Such maps can be easily updated or transformed in computer technology. This applies in particular to the height reference system.

In the years 1999–2002, IV measurement campaign of the basic height network was carried out. In the calculations of this network (see [4]) it was assumed that the average height in the set of 15 age points (Fig. 2) in relation to the previous measurement campaign should not change. The author of the calculations (Gajderowicz) first made the observational adjustment of the network, based on a single point of Warsaw-Wola, and then corrected all ordinates by a fixed value so that the above condition for 15 age points was met (see Fig. 2). It can be said that the described approach does not allow to identify the vertical displacement of the Earth's crust between Campaigns II and III, because the reference level "flows" along with the network. As a result of

Fig. 2. Sketch of the I class leveling network in the IV campaign (1999–2002), with 15 age points (appointed with black quadrate) and 64 points of the Polish part of EUVN network , appointed on red (information sources: [4], [8], [10], [22])

leveling the first class network, a new reference frame was created, which called "Kronsztadt’2006”. This reference frame, although internally well proven in terms of observational reducing and close network adjustment with accuracy characteristics, was not formally adopted for practical applications. The reasons, it seems, were simple. First, geodetic practice, does not tolerate too frequent changes of spatial reference systems, because it always entails significant costs and introduces, at least in transition periods, a certain disorder. The second reason was that in the near future a return to a uniform, European, height system was planned with reference to the average level of the North Sea with a mareograph in Amsterdam.

Comparing the point heights between Campaign III and IV for 16,222 points of the basic height network, we get an average difference of 5 mm (see Fig. 3), with a distribution from -27 to + 42 mm. The reference frame Kronsztad'2006 was used only as an auxiliary (indirect) object in the conversion between various height reference frames ([8], [9], [10]). In addition, the data in Kronsztad'2006 were used to determine the heights in the new PL-EVRF2007-NH reference frame, which is the Polish implementation of the European height reference frame EVRF2007, related to the average sea level measured on the Amsterdam maretograph (NAP – Normal Amsterdam Peil). According to the information received from the Central Geodetic and Cartographic Documentation Center, the results obtained from the fourth measurement campaign were also used for local updates of heights in the reference frame Kronsztad'86, which after this modification adopted the name PL-KRON86-NH. In the last years in Poland another measurement and calculation campaign of the basic height network was carried out, but the results of studies are not ready for use yet [23].

The issues of height reference frames, as elements of the spatial reference system in Poland, are regulated by the Regulation of the Council of Ministers (see [20], [21]). In the light of this regulation, there are currently two height reference frames assigned special names (see [5], [6], [21]):

a) A height reference frame called PL-KRON86-NH, which is basically a Kronsztadt'86, but locally modified by including some observation lines from the newer (IV) measurement campaign to class 1 network.
b) The height reference frame called PL-EVRF2007-NH, which is the implementation of the European reference frame EVRF2007 in Amsterdam (NAP system - Normal Amsterdams Peil). The heights of the points of the basic height network in the European system was determined on the basis of a subset of the points of this network, which is part of the EUVN European network. It is assumed to be a system of Mołodenski's normal heights, with zero tide (see [6] and in the theoretical sense: [3], [7], [13], [14], [15], [16]). The average height difference between the PL-EVRF2007-NH and PL-KRON86-NH reference frames, determined based on over 15999 points, is 166 mm, with a dispersion from 123 to 223 mm.

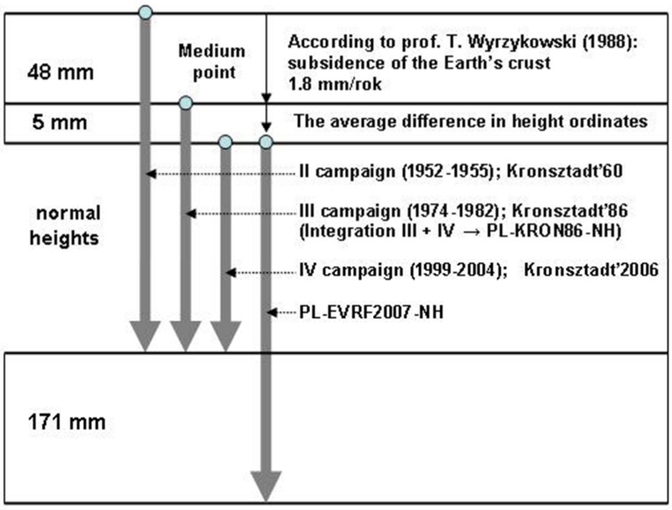


Fig. 3. Average relations between the reference frames

2. General principles of transformation between height reference frames

Fig. 4 shows symbolically the transition paths (transformations) between the previously mentioned height systems. In addition to the direct transformation between the PL-KRON86-NH and PL-EVRF2007-NH reference frames, alternative transition paths may be also by the Kronsztadt'2006. The indirect

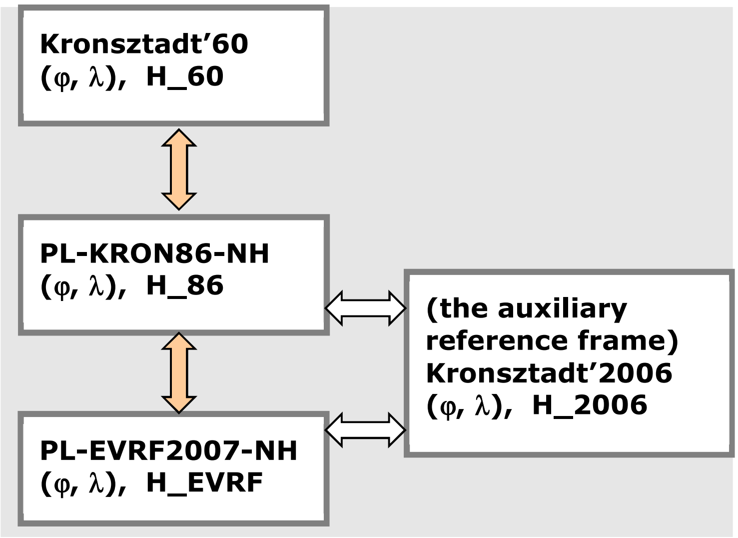


Fig. 4. Schema of transformations between the height reference frames

transition through the Kronsztadt'2006 reference frame was used in the new version of the TRANSPOL program in the height transformation module [8]. Although the "Kronsztad'2006" reference frame was not officially introduced for applications, it was the basis for determining of point heights in the PL-EVRF2007-NH reference frame for I class network.

The general formula realizing the transformation between two height reference frames can be presented in the form of:

$$H_{II} = H_I + \delta H_o + \delta H_{I\text{-}II}(u,v) \tag{1}$$

where: I, II – example markings of height reference frames (reference systems),
H_I – a height in a reference frame I,
H_{II} – a height in a reference frame II,
δH_o – average height difference between reference frames, calculated on a given set of reference points,
$\delta H(u, v)$ – local correction relative to average height increment, depended on the position of the point specified by the parameters u, v.

In particular, the parameters of position may be coordinates in any two-dimensional system, for example the Cartesian x, y or geodetic B, L coordinates. To determine the local correction $\delta H(u, v)$ we apply two alternative methods, described in detail in the following sections. One of them, called the interpolative or empirical method, based on interpolative grid. The second method, called polynomial or analytical method, expresses the local correction in the form of a polynomial of two variables, whereby polynomial coefficients are estimated based on reference points (points having heights in both reference frames). The interpolative grid, as a fixed element of the interpolation algorithm, is created on the basis of a given set of reference points. The methodological details of both methods are described in the following sections.

The general formula (1), in relation to the national height systems that we are interested in, will adopt the following detailed forms:

$$\begin{aligned} &H_{Kronsztadt'60} = H_{PL\text{-}KRON86\text{-}NH} + 0.048\text{m} + dH1(B,L), \\ &dH1(B,L) \in <-0.010,\ 0.139> \end{aligned} \tag{2}$$

(7679 common benchmarks were used in both reference frames)

$$\begin{aligned} &H_{Kronsztadt'2006} = H_{PL\text{-}KRON86\text{-}NH} - 0.005\text{m} + dH2(B, L), \\ &dH2(B,L) \in <-0.037,\ 0.062> \\ &H_{PL\text{-}EVRF2007\text{-}NH} = H_{Kronsztadt'2006} + 0.171\text{m} + dH3(B,L) \end{aligned} \tag{3}$$

$$dH3(B,L) \in <-0.017,\ 0.016> \tag{4}$$

(16222 common benchmarks were used in the following reference frames: PL-KRON86-NH, Kronsztadt'2006, PL-EVRF2007-NH) where $dH1(B,L)$, $dH2(B,L)$, $dH3(B,L)$ represent local corrections depending on the location specified by B, L coordinates. To determine these corrections both the interpolation method was used (see section 3) as well as the polynomial method (see section 4). Both methods were the subject of computer implementations ([8], [9], [10]).

3. Construction of interpolative grids for height transformations using the empirical method

The basic grid is a set of points (nodes), located regularly in a certain conventional area. In our case, we assume that the grid nodes have the geodetic coordinates B, L with the resolution of $0.01^\circ \times 0.01^\circ$. Such a base grid on the area of Poland we call PL-grid-001.

The basic grid PL-grid-001 (Fig. 5), which is the basis for creating interpolative grids (featured grids) for various transformations, will contain a total of 613,621 nodes, arranged on 601 parallel lines in the range of latitude from 49° to 55° and 1021 meridians in the geodetic length range from 14° to 24.2°

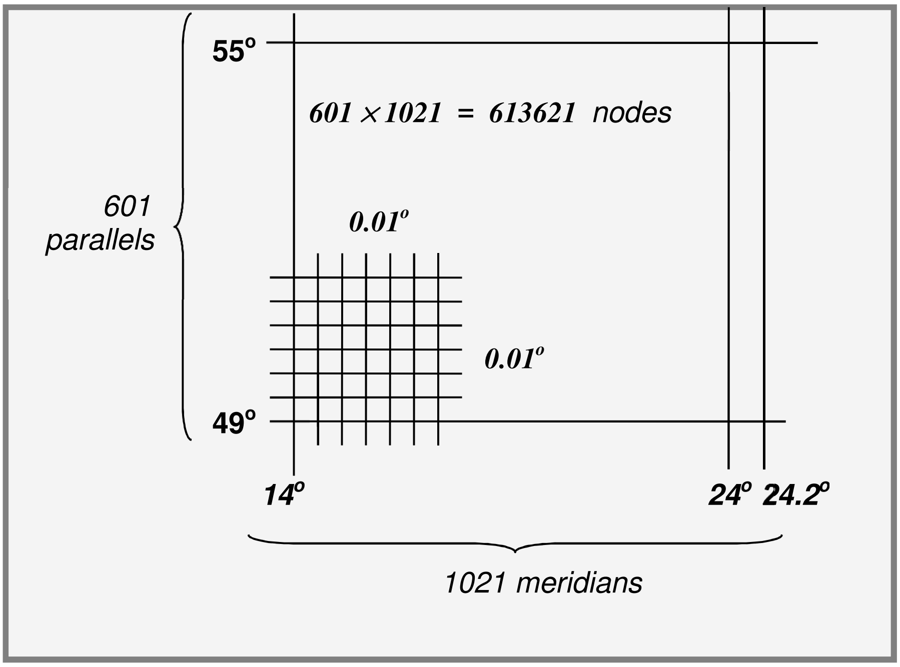


Fig. 5. Structure of the basic grid PL-grid-001

With the above definition, the PL-grid-001 basic grid covers the area of Poland with a certain margin that meets the technical requirements of the application. For each mesh node, we can specify the i and j indices of a rectangular array as follows:

$$i = (B_i - 49.00) \cdot 100; j = (L_j - 14.00) \cdot 100 \qquad (5)$$

where B_i,L_i are the geodesic coordinates of grid nodes in degrees, $i = 0,1,2, \ldots ,600$; $j = 0,1, 2, \ldots ,1020$.

Conversely, if the indicators (i, j) of a given node are known, then its coordinates are given by the formulas:

$$B_i = i \cdot 0.01 + 49.00; L_j = j \cdot 0.01 + 14.00^{\circ} \qquad (6)$$

Assigning a feature of some kind for each node of the grid, we obtain the appropriate interpolative grid (featured grid).

An interpolative grid in general is a set of certain features (for example height differences between two systems) assigned to points (nodes) of the basic grid. Of course, when transforming the height of a point that is not the node of the basic grid, we interpolate the appropriate feature (a height difference) based on the features of the four nearest nodes.

We create an interpolative grid for a given pair of height reference frames, based on a given basic grid and a given set of network points (in our case it will be the basic height network from the entire country) having heights in both systems. For each point, we create the appropriate difference (increases) in height between the reference frames. The next step is to transfer the these differences to the grid nodes. For this purpose, we have a choice of a whole range of so called geostatic methods, used to interpolate the grid nodes by a discrete set of points (see eg. [19]). Especially popular in geodesy (used, among others, in the so-called "post-transformative correction") is the method of inverse distances as weights, in short: IDW (Inverse Distance Weighting), with the exponent of the power $q \in <1, 5>$. Other methods are: triangle (triangulation) method, Thiessen's traverses, minimum curvature, polynomial interpolation, and kriging including also correlations between given points. Taking into account the geodetic applications already tested, in the creation of interpolation grids as spatial data models ([10]) the IDW method with the exponent of the power $q = 2$ was used.

Fig. 6 shows the use of the IDW method to create an interpolative grid based on distributed geodetic reference points. We assume that for interpolation of each grid node, only points located in a certain neighborhood of a given node with a radius guaranteeing correct interpolation task execution are selected. The radius length is determined numerically depending on the local density of the reference points (according to the IDW principle, points located further away from the node should not affect the value of the interpolated feature). We define the feature of the grid node as a weighted average:

$$C_{ij} = (\Sigma C_k \cdot w_k) / (\Sigma w_k),\ w_k = 1/\rho_k{}^2 = weight \tag{7}$$

(summation only for points in the interpolation area).
where: i, j – mesh node indicators,

w_k – weight for the k-th geodetic point (benchmark) in the interpolation area relative to the grid node (i, j),

ρ_k – distance of the k-th geodetic point from the interpolated node,

C_k – a feature of the k-th point (the height increase dH between two systems),

C_{ij} – the interpolated feature (the height increase) of the mesh node.

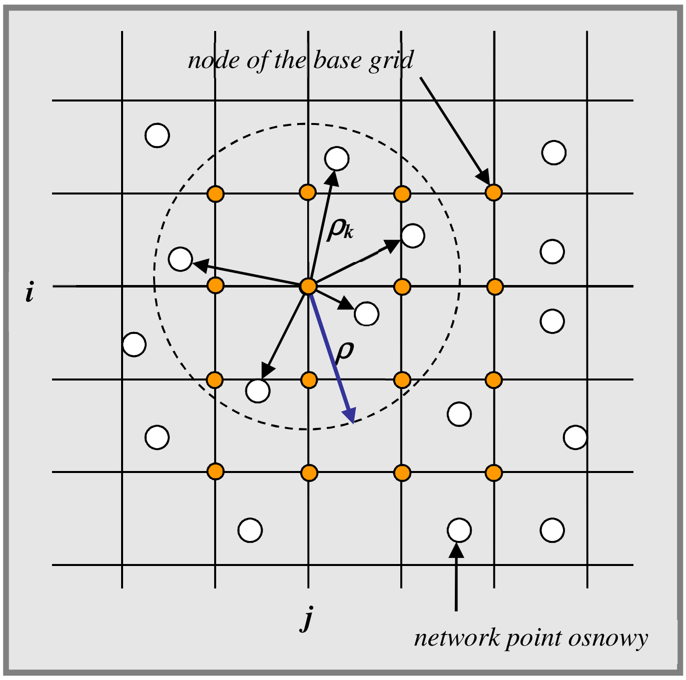


Fig. 6. Interpolation of grid nodes based on the close points of the geodetic network, by means of the weighted average method; ρ – radius of searching of points for node interpolation, ρk – distance of the network point from the interpolated node

The described empirical method, based on the interpolative grid, has been implemented in the TRANSPOL v. 2.06 program and in the national base of basic geodetic networks at GUGiK ([8], [10]). Interpolative grids saved in the form of discrete text files are published also at the address indicated. The file concerning our transformation tasks contains differences between normal heights in PL-KRON86-NH and PL-EVRF2007-NH reference frames, for all nodes of base grid with increments of 0.01° in latitude and longitude, for the whole area of Poland. In the published (in [10]) sets lack the old reference system Kronsztadt'60 but this reference frame has been included in another database software (in [9]). The polynomial transformation method presented below includes all considered reference frames in the given numerical formulas.

4. Polynomial transformations

4.1. Conversions between the reference frames: Kronsztad'60 and PL-KRON86-NH

The polynomial transformation between these reference frames have been carried out, for example, for the Cracow district [2]. The polynomial of the second degree turned out to be adequate for such an area. For larger areas, especially covering the whole of Poland, it becomes a necessity to take into account mutual inconsistencies of the reference frames represented by the corresponding heights of leveling networks. These inconsistencies may also result from vertical displacements of some benchmarks, given that the period between measurement campaigns measures about two decades. Of course, a polynomial approximation can be not define all local deviations because they are accidental. In contrast to the interpolative grid, the polynomial approximation smooths the appropriate empirical relationship. In practical applications, this may be an undesirable effect, but a smoothed transformation formula allows detection of possible gross errors (outliers phenomena) in height values of the reference points. In addition, post-transformative corrections resulting from deviations at reference points can be introduced to the result of the polynomial transformation, approaching the result of the empirical (interpolative) method.

In our case, however, there are no physical reasons to assume any particular form of the height change model. Therefore, we use the general form of the algebraic polynomial of two variables as a standard. Parameters of polynomials are estimated using the least squares method, taking into account given sets of reference points. The calculations were carried out with GEONET 2006 system programs (see: www.geonet.net.pl).

We assume that the arguments of polynomials are the flat coordinates x, y of points in the uniform PL-1992 system for whole Poland. The adoption of the PL-1992 system as a positioning platform for points is convenient, because this system is not divided into zones. Of course, it would also be possible to adopt coordinates in other systems, for example in the B, L geodetic coordinate system. The position coordinates of the point can be approximate, because the big change of a horizontal position does not cause a significant change in height. In practice, the flat coordinates of the benchmarks are usually rounded to full meters.

The following formula express the polynomial transformation of height between the reference frames PL-KRON86-NH and Kronsztad'60 estimated by 7,679 reference points:

$$H_{(Kronsztad'60)} = H_{(PL\text{-}KRON86\text{-}NH)} + \delta H_o + \delta H + \varepsilon \qquad (8)$$

$H_o = 0.0485\ m$ = average height offset,

$H = a_{00} + a_{01} \cdot v + a_{10} \cdot u + a_{02} \cdot v^2 + a_{11} \cdot u \cdot v + a_{20} \cdot u^2 + a_{03} \cdot v^3 + a_{12} \cdot u \cdot v^2 + a_{21} \cdot u^2 \cdot v + a_{30} \cdot u^3 + a_{04} \cdot v^4 + a_{13} \cdot u \cdot v^3 + a_{22} \cdot u^2 \cdot v^2 + a_{31} \cdot u^3 \cdot v + a_{40} \cdot u^4$,

$u = (x - x_o) \cdot s$, $v = (y - y_o) \cdot s$ – arguments normalized so that $|u| < 1$and $|v| < 1$,
x, y – point coordinates (in meters) in the Polish system PL-1992,

$x_o = 466{,}458\ m$, $y_o = 514{,}429\ m$ – components of the translation vector (centering coordinates),

$s = 2.51510 \cdot 10^{-06}$ = standarizing scale,

a_{ij} $(i, j = 0,1,2,4,\ \ i+j \le 4)$ – polynomial coefficients of degree n = 4 (the values in Table 1), estimated by using of Least Squares (LSQ) method,

ε – random error (the Fig. 7 shows the characteristic estimates of errors as point deviations from polynomial model).

Of course, the inverse transformation results directly from (8):

$$H_{(PL\text{-}KRON86\text{-}NH0)} = H_{(Kronsztadt'60)} - \delta H_o - \delta H - \varepsilon \qquad (9)$$

Table 1. Polynomial coefficients for transformations between the reference frames Kronsztad'60 and PL-KRON86-NH

POLYNOMIAL COEFFICIENTS AND STANDARD DEVIATIONS

a_{00}:=	1.88044E-02	3.905E-04
a_{01}:=	3.72458E-02	9.623E-04
a_{02}:=	-8.88262E-02	3.142E-03
a_{03}:=	-3.62611E-03	2.145E-03
a_{04}:=	5.11118E-02	5.261E-03
a_{10}:=	3.36639E-02	1.068E-03
a_{11}:=	9.78759E-02	3.469E-03
a_{12}:=	-1.61254E-02	3.178E-03
a_{13}:=	-1.22407E-01	7.141E-03
a_{20}:=	-6.57799E-02	3.660E-03
a_{21}:=	8.81326E-03	3.794E-03
a_{22}:=	1.42289E-01	9.288E-03
a_{30}:=	-3.09316E-02	2.893E-03
a_{31}:=	-2.52344E-02	9.762E-03
a_{40}:=	4.86396E-02	7.374E-03

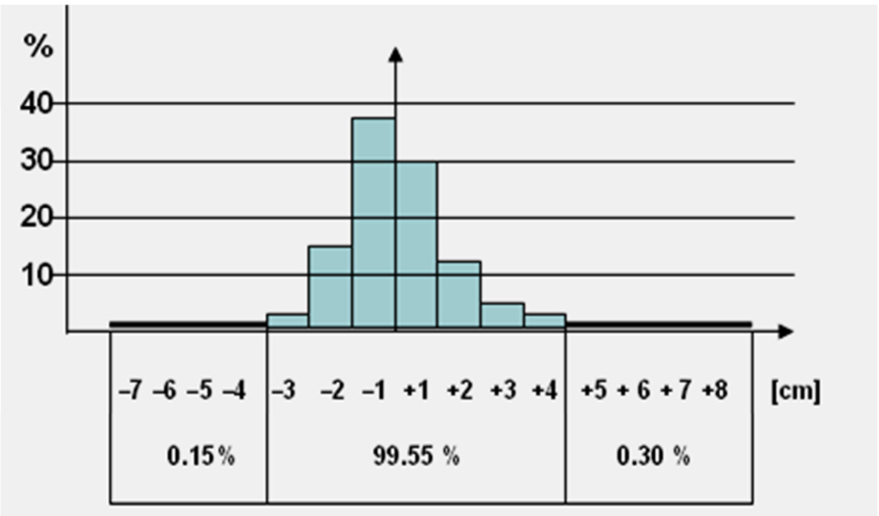


Fig. 7. Distribution of transformation deviations for 7679 reference points between the Kronsztadt-60 and PL-KRON86-NH reference frames

4.2. Conversions between PL-KRON86-NH and PL-EVRF2007-NH reference frames

The formula of the polynomial transformation between PL-KRON86-NH and PL-EVRF2007-NH reference frames, estimated by 15999 reference points, is expressed as follows:

$$H_{(PL\text{-}EVRF2007\text{-}NH)} = H_{(PL\text{-}KRON86\text{-}NH)} + \delta H_o + \delta H + \varepsilon \qquad (10)$$

$\delta H_o = 0.1659 =$ the average height offset,

$\delta H = a_{00} + a_{01} \cdot v + a_{10} \cdot u + a_{02} \cdot v^2 + a_{11} \cdot u \cdot v + a_{20} \cdot u^2 + a_{03} \cdot v^3 + a_{12} \cdot u \cdot v^2 + a_{21} \cdot u^2 \cdot v + a_{30} \cdot u^3$,

$u = (x - x_o) \cdot s$, $v = (y - y_o) \cdot s$ – arguments normalized so that $|u| < 1$ and $|v| < 1$,

x, y – point coordinates (in meters) in the Polish system PL-1992,

$x_o = 469{,}175\ m$, $y_o = 513{,}591\ m$ – components of the translation vector (centering coordinates),

$s = 2.55611 \cdot 10^{-06}$ = standardizing scale,

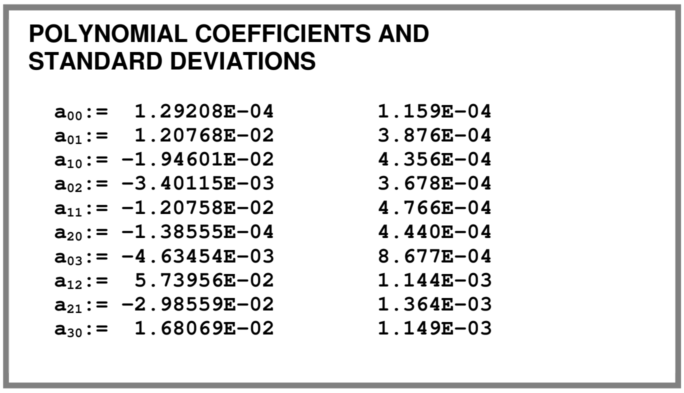
a_{ij} $(i, j = 0,1,2,3,\ i+j \leq 3)$ – polynomial coefficients of degree n =3 (the values in Table 2),estimated by using of Least Squares (LSQ) method,

ε– random error (the Fig. 8 shows the characteristic estimates of errors as point deviations from polynomial model).

Of course, the inverse transformation results directly from (10):

$$H_{(PL\text{-}KRON86\text{-}NH)} = H_{(PL\text{-}EVRF2007\text{-}NH)} - \delta H_o - \delta H - \varepsilon \qquad (11)$$

Table 2. Polynomial coefficients for transformations between the reference frames PL-KRON89-NH and PL-EVRF2007-NH



POLYNOMIAL COEFFICIENTS AND STANDARD DEVIATIONS

a_{00}:=	1.29208E-04	1.159E-04
a_{01}:=	1.20768E-02	3.876E-04
a_{10}:=	-1.94601E-02	4.356E-04
a_{02}:=	-3.40115E-03	3.678E-04
a_{11}:=	-1.20758E-02	4.766E-04
a_{20}:=	-1.38555E-04	4.440E-04
a_{03}:=	-4.63454E-03	8.677E-04
a_{12}:=	5.73956E-02	1.144E-03
a_{21}:=	-2.98559E-02	1.364E-03
a_{30}:=	1.68069E-02	1.149E-03

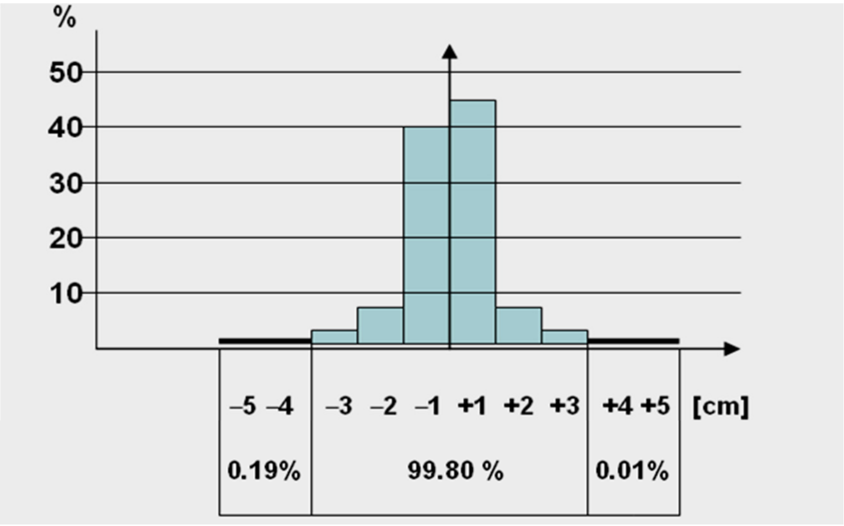


Fig. 8. Distribution of transformation deviations for 15999 reference points between the reference frames PL-KRON86-NH and PL-EVRF2007-NH

5. Illustration of differences between reference frames in the area of Poland

The Figures 9–11 show the contour line of constant differences between height reference frames in the Polish area. It seems very likely to say that in every newer measuring era there is a qualitative improvement in measurement methods and calculation techniques. In this sense, there are several conclusions (interpretations) that we note below.

In the Fig. 9, we note that deviations between the Kronsztad'60 and PL-KRON86-NH reference frames increase towards the north-east and are the smallest in the western areas. In the interpretation of these differences, it is important to note that in the measurement campaigns, which resulted in the Kronsztadt-60 system, three independent networks have been integrated: from the area of the Second Republic of Poland, newly connected western and northern territories. Of course, the cause-and-effect aspects of the differences between the reference frames may be the subject of separate studies. Meanwhile, according to the purpose of this work, the subject of our research was specific metric compounds that can be practically used to transform heights between systems (polynomials, interpolative grids).

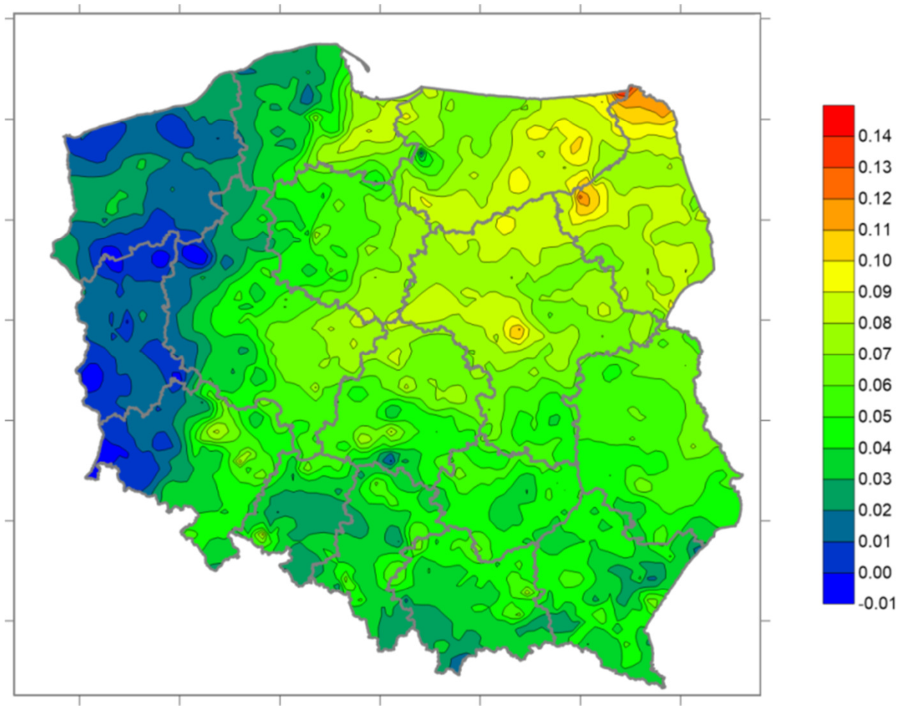


Fig. 9. Image of height differences δ = HKronsztad’60 – HPL-KRON86-NH (in meters)

In turn, the Fig. 10 shows analogous contour lines of equal spacing between the PL-KRON86-NH and Kronsztad'2006 reference frames. We see, first of all, that the differences between normal heights of both frames in the whole area of Poland are already small, however small areas appear with the absolute maximum values of these differences reaching 3.5 cm.

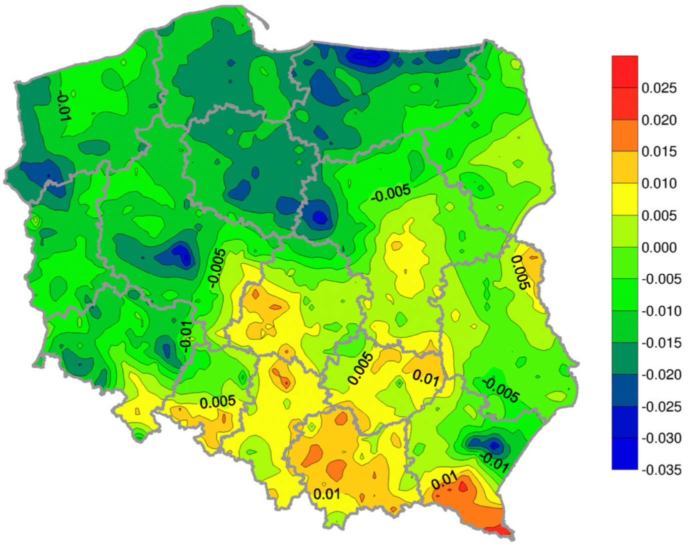


Fig. 10. Image of height differences δ = H Kronsztad’2006 – HPL-KRON86-NH in meters (source: [8])

In the Fig. 11 we have an image of contour lines of equal height differences between the Kronsztadt’2006 and PL-EVRF2007-NH reference frames. The regularity of this dependence in comparison to the previous ones, shown in Figures 9 and 10, results from the fact that the heights in the PL-EVRF2007-NH system come from the direct conversion of heights in the Kronsztadt’2006 (computed by Gajderowicz [4]), i.e. from the same leveling network.

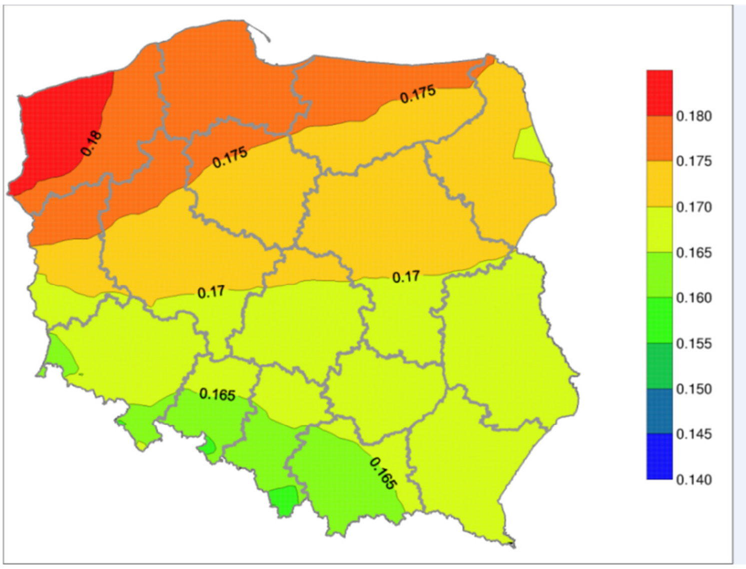


Fig. 11. Image of height differences $\delta = H_{\text{PL-EVRF2007-NH}} - H_{\text{Kronsztadt'2006}}$ in meters (source: [8])

If we add the height differences from Fig. 9 and 10, then according to their definitions given in the description of drawings, we get the deviations between two currently applicable state reference frames: PL-EVRF2007-NH (new "Amsterdam" reference frame), PL-KRON86-NH (Fig. 12). Local irregularities are the result of differences in normal heights determined independently in the framework of the 3rd and 4th measurement campaign.

It is known that the next 5th measurement campaign of the basic height network was already carried out along with the necessary epochal modernization of the network. Unfortunately, the results have not yet been practically used, as some network inconsistencies have been identified in the areas south-eastern of Poland.

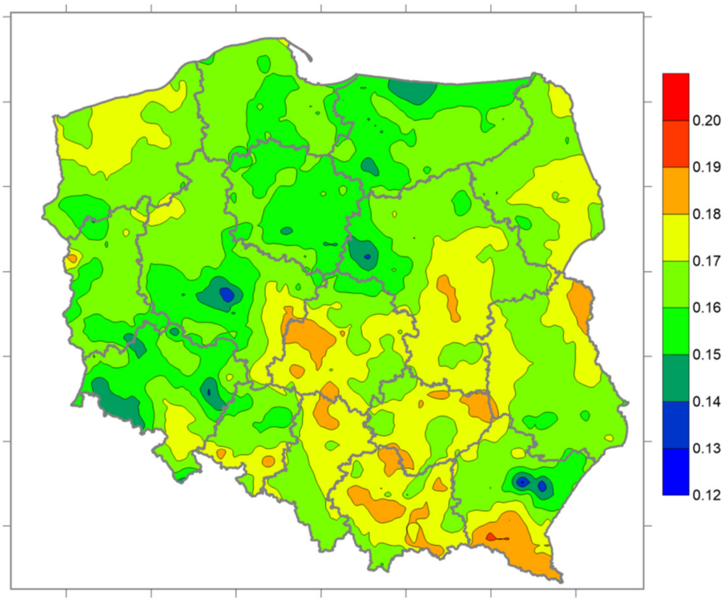


Fig. 12. Image of height differences δ = HPL-EVRF2007-NH – HPL-KRON86-NH (in meters)

6. Conclusions

Now in Poland normal heights in two reference frames: PL-KRON86-NH (modernized Kronsztadt '86) and the PL-EVRF2007-NH (the Polish implementation of the European height reference frame EVRF2007) are used. Regardless of official legal regulations (see: [20], [21]), there is also (materially, in various geodetic and cartographic documents) the former Kronsztad'60 reference frame. In addition, a certain theoretical significance has the Kronsztad'2006 reference frame, which was the original object for the calculation of the heights in PL-EVRF2007-NH reference frame. Kronsztad'2006 is also present in the height transformation module in the

TRANSPOL 2.06 program [8]. This publication considers transformations between all the mentioned height reference frames, describing methods and algorithms adopted in the special elaborations for the Central Office of Geodesy and Cartography (including: [8], [9], [10]).

The basic method of transformation between height reference frames, recommended for use in geodetic practice is the empirical (interpolative) method using interpolative grids. For a given pair of reference frames, grid nodes have features (increases in height between two reference frames) determined on the basis of known values of features in the network points, by the weighted average method. The weight is defined as a number inversely proportional to the square of the distance of the node from a given point of the network. In order to interpolate a given grid node, only network points located in a circular area around the node are selected (see Fig. 6). The radius of the area depends on the local density of reference points. For any transformed point, which is not a grid node, the determination of the feature (the corresponding increase in height) boils down to the known bi-linear interpolation in a single cell of the grid.

In addition to the empirical method, the possibility of height transformation is presented using polynomials describing the "smoothed" relationship between height reference frames used in Poland. The polynomial transformation does not accurately fit into the local leveling network, but may have special applications. For example, it can be used to detect local warp errors in the original or current system. The signal of the large local error will be the deviation of the polynomial transformation from the known from the measurement of the height of a given control point. Of course, between empirical and polynomial transformation there will be differences, which a distribution in area of Poland can be identified in Fig. 9–12.

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